Loopy Dances

Karl Schaffer
De Anza College
Erik Stern
Weber State University
Dr. Schaffer and Mr. Stern Dance Ensemble
E-mail: karl_schaffer@yahoo.com
estern@weber.edu

Abstract

This workshop for Bridges participants will explore integrating mathematics and dance in the classroom as well as on stage. This workshop involves mathematical topics related to the use of oversize loops of rope. These include play with dance sequences involving the loops, and construction of polygons and polyhedra, including several Platonic solids. Participants will create, practice, and perform short dance phrases, and simultaneously explore mathematical principles and critique the work from the point of view of both the mathematics and the artistry involved.

"Math Dance"

In [3] we pointed out the following: "Dance and mathematics share many essential concerns: (1) Both deal with the manipulation and exploration of patterns. (2) Both involve defining a problem and seeking a solution. (3) Both begin with concrete problems and progress to abstract ideas – or vice versa. (4) Both involve aesthetics and are integrally connected to cultural values and biases. (5) Both can make you sweat!"
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Purpose. This workshop will give participants a concrete experience of the connections between mathematics and choreography. It is in a certain sense a follow-up to a workshop given by Karl Schaffer and Erik Stern at the 2010 Bridges conference, and one given by Karl Schaffer at the 2012 Bridges conference, in that it involves further explorations with a simple prop, dance, and polyhedra; that workshop was itself based on many years of such workshops, as outlined in [3]. The introduction and overall philosophy of this workshop are similar to those workshops. Prior experience in either mathematics or dance is not necessary. The workshop alternates between creative problem solving and reflection/discussion. As in [3], the workshop participants will (1) solve problems physically in small groups, (2) discuss problems in smaller groups and as a class, (3) will both explore creatively the subject matter and examine the formalization of the concepts, (4) Discuss the use of these activities in K-12 and college classes.

Background. Mathematicians often view their work as involving creativity and imagination. Dancers/choreographers utilize analytical thinking and attend to the forms and stylistic rules inherent in their art. The two disciplines work well together, and we will see that dance compositions may be developed as "embodied mathematics," or mathematical principles may be found embedded in dance and rhythm. Several of the activities in this workshop are related to workshops created by the authors as part of their work with the Kennedy Center Partners in Education program.

The activities do not require special training in dance or mathematics (although training or interest in these subjects can be helpful). They begin by addressing the most basic universal elements of dance and mathematics: recognizing and playing with patterns, moving or locomoting simply, counting, making shapes. The activities are designed to be flexible and can be extended to suit the level of the participants.

Loopy Dance. Participants make shapes with loops of rope and with their bodies in space, and perform for each other. They start with simple movement games and explorations involving two dancers and one loop of rope. Then the loops are used to build and analyze 2-dimensional figures, and participants explore transformations between the figures, as well as provocative questions for educational applications of the material. This material is incorporated into the earlier movement sequences with the loops. Next, several polyhedral loop figures will be practiced and explored, and finally a sequence will be learned involving all five Platonic solids [1],[4].

Description of Activities. In the initial activity, participants become familiar with manipulating large loops of rope with their bodies. Using one loop, they make shapes with a partner, explore transitions, and perform the shapes. One partner creates a large open shape with the loop, while the other moves through it slowly and without touching the rope or the person holding it; the "holder" changes the shape and the partner again finds a movement through it, then gently takes the rope and the partners repeat with roles switched. These introductory activities are accessible, give participants confidence, and engender discussion about how we instinctively judge size and shape constantly and the role that instinct can play in mathematical understanding. Working in groups of four, participants next find ways to smoothly manipulate one loop into the form of various polygons, including triangles, quadrilaterals, and pentagons. Finally, they find ways to construct simple polyhedral. Interesting classroom questions are explored, for example:
How might the group form an equilateral triangle, and prove it is equilateral?
How might the group form a square and prove it is a square?
Can the group form a convex pentagon and smoothly flex it into a five-pointed star?
Can the group form a convex octagon and flex it into the skeleton of the tetrahedron?
What are some other ways to construct a tetrahedron?
Why can a tetrahedron not be formed without doubled edges, and how is this related to the concept of Euler circuits?
Can the group find ways to smoothly construct an octahedron or a cube?
Simple methods for these constructions will be demonstrated, as shown in [2] and [4]. The top row in figure 1 shows several simple polygons and one polyhedron that participants will explore. The second row shows several polyhedral they will make.

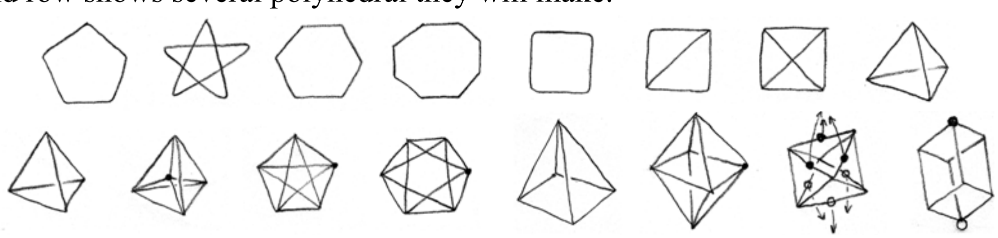
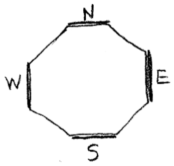
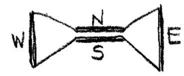


Figure 1.

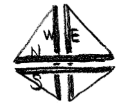
The participants will also be asked to create an unusual shape, and then be challenged to create a sequence of shapes with the loop involving their group members, rehearse the sequence so that it can be performed smoothly, and then perform the sequence for other participants. The diagram in figure 2, taken from the authors' book *Math Dance* [2], shows one way to make a sequence of basic polyhedra. The sequence of Platonic solids in [1] will also be practiced.



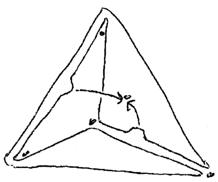
1. Each of the four people, labeled N, E, S and W in this diagram, holds two adjacent vertices.



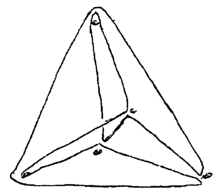
2. The north and south people bring their hands together at the top, and one person takes the strings from the other.



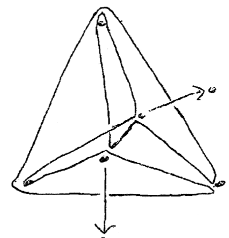
3. The east and west people do the same, bringing their hands together at the bottom, one person taking the strings from the other. Voila: **tetrahedron**!



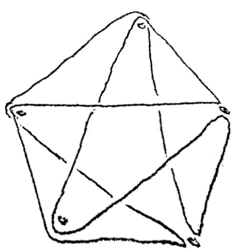
4. Notice which of the string edges of the tetrahedron are doubled. If a fifth hand pulls two of the doubled strings together into the middle of the tetrahedron (see diagram)…



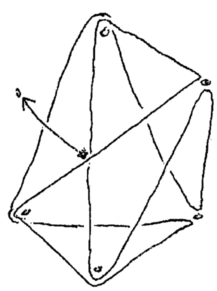
5. …a new polyhedron called the **hypertetrahedron** (or the 4-dimensional tetrahedron) results. Notice that each vertex has a string going to each of the other vertices.



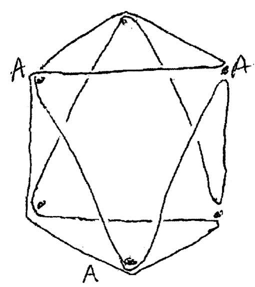
6. If all vertices are now pulled to the "outside," and the figure is flattened into the plane, the result will be a…



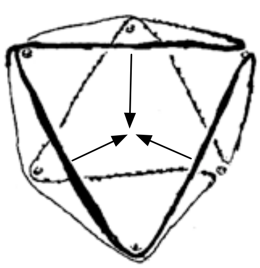
7. **…pentagon with a five-pointed star** in the middle, known as the **pentagram**.



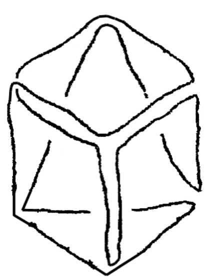
8. If someone grabs a pair of strings where they cross inside the pentagram and pulls it to the outside, the result should be a …



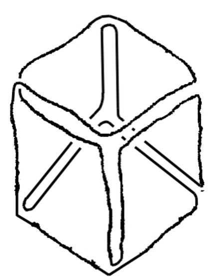
9. …**six-pointed star inside a hexagon**. In some cases three of the vertices will lie above the other three. If the three vertices labeled A in this diagram are raised upwards the result is an…



10. …**octahedron**. Notice that this figure has no doubled strings. To transform the octahedron into a cube, a fourth person gathers the midpoints of the three sides of the top triangle with one hand, as shown by the arrows…



11. …into a point and pulls the point up. In the same way, the other hand gathers the midpoints of the three sides of the bottom triangle into a point with the other hand…



12. … and pulls down. Behold: **a cube** standing on one vertex!

Figure 2.

Everyday Movement. Everyday movements are a great way to involve non-dancers in doing and creating movement sequences. In [2] and [3] we noted that, “Although many types of dance, such as classical ballet, take years of practice to master, others are built on everyday movements that anyone can do. For instance the musical performance Stomp creates dance out of everyday actions like sweeping the floor. Many hip-hop moves began as everyday gestures. All over the world folk dances are made out of the everyday movements of work and play.”

Background. The authors had been choreographing works together for three years when we began to discuss the similarities between the processes that underlie mathematics and dance. The performance which resulted, "Dr. Schaffer and Mr. Stern, Two Guys Dancing About Math," premiered in 1990, was performed over 500 times throughout North America, and led to the creation of numerous other performances exploring the connections between mathematics and dance. In 1993 Schaffer and Stern collaborated with Scott Kim, noted puzzle designer and mathematician, on the performance "The Secret Life of Squares." As a natural outgrowth of this work on stage, Schaffer, Stern and Kim created workshops which allowed students to experience in the classroom the connections between dance and mathematics.

The goal of these activities is to provide a jumping off place for teachers and artists to use to create classroom activities and/or performance works that simultaneously explore mathematics and dance. Participants or readers might consider dance forms with which they are familiar, and look for connections between mathematical concepts and the ways that dancers form shapes with their bodies, connect with other dancers, follow spatial paths, make use of rhythm, or perform sequences of movements.

References

[1] Karl Schaffer, “A Platonic Sextet for Strings,” in *Martin Gardner in the Twenty-First Century*, ed. by Michael Henley and Brian Hopkins, Math. Assoc. of America, 2012.

[2] Karl Schaffer, Erik Stern, and Scott Kim, *Math Dance with Dr. Schaffer and Mr. Stern: Whole-Body Math Movement Activities for the K-12 Classroom,* published by MoveSpeakSpin, 2001.

[3] Karl Schaffer and Erik Stern, *Workshop on Mathematics and Dance*, Bridges, 2010.

[4] Karl Schaffer and Scott Kim, films by George Csicsery, 2011:
“Polyhedra on a Shoestring,” http://youtu.be/dwyR95c_jQ0
“String quartet,” http://youtu.be/RM-SRGnJUT8
“Octaflex,” http://youtu.be/SqOHUoqse74