The Mathematics behind the Art of the Death Spiral

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Abstract

In light of the recent 2014 Winter Olympic Games, mathematical modeling problems involving algebra, geometry, trigonometry and calculus are presented via dynamic geometry software in the context of pairs figure skating. An aesthetically pleasing and athletically demanding pairs figure skating element, the Death Spiral, is discussed. Activities related to the pairs Death Spiral which are suitable for middle and high school students are provided in this workshop. Workshop participants use Geometer's Sketchpad on their laptops as an educational platform to conduct interactive investigations related to these modeling problems.

Introduction

Professional development can expose teachers to models, strengthen teachers' subject matter knowledge, and enhance teachers' specialized content knowledge. Research in situated cognition has shown that middle school students learn better when they interact with contexts which they find motivating [1]. The overarching goal of this workshop is to have participants understand underlying mathematics related to the Death Spiral context. In this workshop, we provide a modeling situation involving pairs figure skating, which can serve as a motivating context since many students are interested in sports contested in the Winter Olympic Games. Figure skating is one of the most attractive Winter Olympic sports to watch because it combines artistry and athleticism; in fact, figure skating is the most popular sport to watch among females older than 12 years in the United States [3].

In-service elementary, middle and high school teachers took a graduate-level problem solving class during the summer 2013 or fall 2013 semester at a public university in the United States and completed the activities described in the workshop. We discuss teachers' explanations to problems that involve connecting geometry, algebra, trigonometry, and calculus within the context of the Death Spiral, a required element in the pairs figure skating program. While completing this activity, teachers had access to computers and were familiar with Geometer's Sketchpad software, a software which provides a dynamic geometry environment.

The Death Spiral

The Death Spiral is a required element of a pair team's routine. In order to perform the Death Spiral, the man holds one of the lady's hands (see Figure 1).

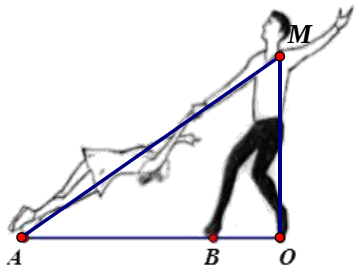

Figure 1: *Illustration of the Death Spiral, side view (yz projection)*

The man pivots in a circle with one of his skates at a fixed point, denoted by point O, on the ice while the other skate, denoted by point B, is traveling around in a circle. Simultaneously, the lady moves around him in a circular path with only one of her blades, denoted by point A, touching the ice [2]. The man's height MO decreases as the lady's torso slides horizontally such that OA increases. The lady's torso becomes low and almost parallel to the ice. When the man raises out of his pivot, the lady returns to an upright position. The success of this spiral depends on the cooperative work of both partners, since any minor error can make the Death Spiral collapse.

Our model of the Death Spiral will focus on two two-dimensional projections of the three-dimensional move (see Figure 2). The first projection, the yz projection, was explained above.

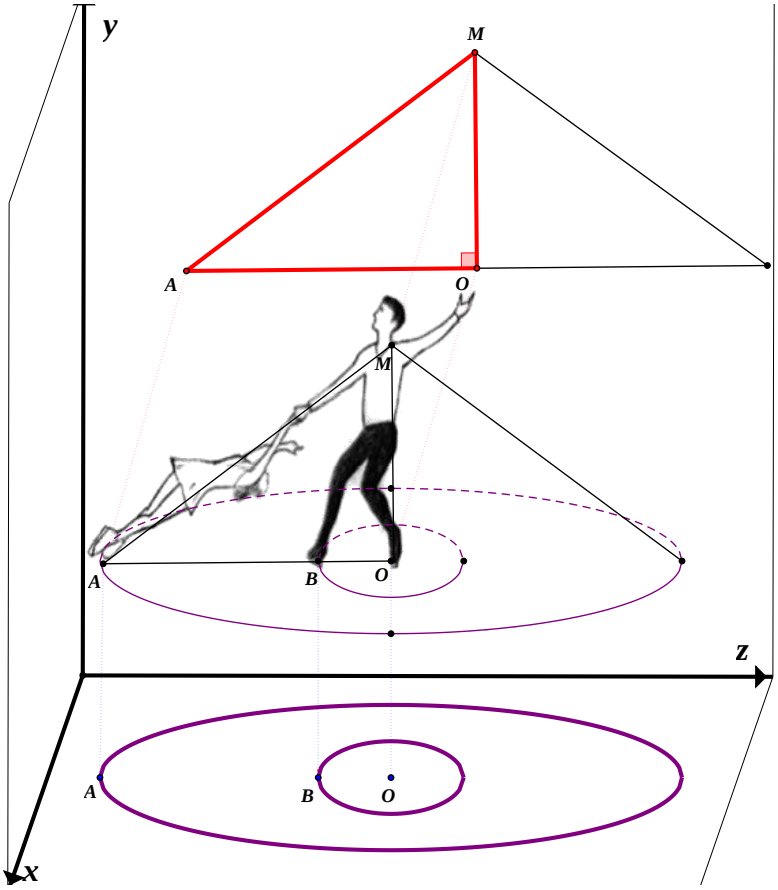

Figure 2: *Two two-dimensional projections of three-dimensional Death Spiral*

The second two-dimensional projections is the bird's eye view, or xz projection, of the tracings which the man's and lady's blades form on the ice when they rotate a full revolution. The man, whose center of mass is above point O on the ice, pivots in a small circle on the ice with radius OB while the lady glides in a circle with an increasing radius around the man until she reaches her fully extended position with radius OA. While the tracings of the man's and lady's blades are in fact spirals, for the purposes of this activity, we treat the tracings of the blades as two concentric circles centered around point O, as shown in Figure 3.

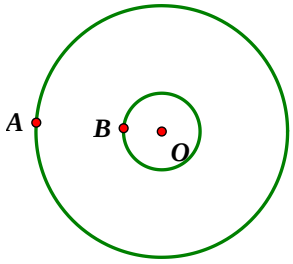

Figure 3: *Concentric circles: Bird's eye view (*xz *projection) of Death Spiral blade tracings*

Description of Activities: yz Projection

In the Death Spiral, the straight-arms distance (MA) remains constant over time, whereas the vertical (MO) and horizontal (OA) distances change simultaneously over time. Triangle MOA is a right triangle. We will ask participants to describe this phenomenon using coordinate geometry and the Pythagorean theorem. Given that MA = 9 feet and MO = 6 feet, and given that point O is at the origin (0,0) on the coordinate plane, we will discuss solutions to the following questions:

a) Find coordinates of points M and A.
b) Where possible, calculate the slopes of MA, MO and OA.
c) Find the equations of MA, MO and OA.

After some time, the man lowers his body by 2 feet, affecting the positions of points M and A accordingly. We will discuss solutions to the following questions:

d) Find the new coordinates of points M and A.
e) Calculate the new slope of MA.
f) Find the new equation of MA.

We will also discuss the related rates that are involved as the man's height lowers and the lady slides outwards horizontally. Participants will need to draw upon their knowledge of implicit differentiation of the Pythagorean theorem and of trigonometric relationships. Given that by the time the man's height MO is 6 feet, the lady is sliding at a rate of 1 foot per second horizontally, questions we will discuss are as follows:

g) At what rate does the man lower his body vertically? Will the partners' position change at the same or different rate? Explain.
h) Determine how fast angle MAO changes.
i) If the distance MO decreases as the man lowers his body, does angle MAO increase or decrease? How do you know? Draw a graph illustrating the relationship between the man's height and angle MAO.

Description of Activities: xz Projection

We will pose qualitative and quantitative questions regarding the concentric circles that are formed by the pairs skaters' blades during the Death Spiral. One conclusion that participants need to reach in order to successfully complete the activity is that the man and lady (named Rockne and Keauna below) take the same amount of time in the Death Spiral, since they are interdependent. Additionally, in order to make conclusions about the distance, rate and time, participants need to use the equation, $Distance = Rate \times Time$ and find the circumference of circles to determine the length of the distances traveled. Questions whose solutions we will discuss are as follows:

a) Does Keauna or Rockne travel faster during the Death Spiral? Explain.
b) If OA = 10 feet, and OB = 2 feet, answer the following questions:
   - How much further does Keauna travel than Rockne? Explain.
   - If the Death Spiral took 8 seconds to complete, what is the difference in speeds of Keauna and Rockne? Explain.
   - Keauna and Rockne's friend, Lisa, is trying to answer previous two questions and she first calculates the difference between the lengths of OA and OB. Is this a valid first step to solve the above two questions? If so, how could she use it, and if not, explain why not.
   - If Rockne's rate is 1.05 feet per second, then how much time did the Death Spiral take? What is Keauna's speed during the Death Spiral? Explain.
c) Keauna and Rockne completed a Death Spiral that lasted 4 seconds. Keauna's rate was 12.56 feet per second. What is the length of OA? Explain.

Additionally, given a table indicating a proportional relationship between angle rotated (in degrees) vs time (in seconds) during the Death Spiral, we will have participants discuss the following questions:

d) What variable is independent in this situation? Why is it independent?
e) How long does it take the pairs team to complete one full rotation? Explain.
f) Find the graph of the best fit for the data points. What kind of function is it?
g) What is the slope of the graph? Find two different ways to calculate the slope.
h) What does the slope represent?
i) What is the y-intercept? What does this represent?
j) Find the equation of the function.
k) What factors might restrict the domain of the function?

Participants will have the opportunity to generate additional questions related to the Death Spiral which will be investigated collectively during the workshop.

References

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