Rhythm Similarity and Symbolic Dynamics

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Abstract

Measuring the similarity between two rhythms is of crucial importance in the field of music information retrieval. There has been an examination of the mathematical closeness of rhythms by means of a number of metrics: the Hamming distance, the Euclidean interval-vector distance, the interval-difference distance measure of Coyle and Shmulevich, the swap distance, and the chronotonic distance measures of Gustafson and Hofmann-Engl. This note proposes the use of the metric, $d(s,t) = \sum_{i=0}^{+\infty} \frac{|s_i - t_i|}{2^i}$, from symbolic dynamics to explore rhythmic closeness.

Introduction

In our collaboration as mathematician and musician (drummer), we have sought to bring each other into our respective domains and to provide teaching and learning experiences that facilitate connecting intellectually and emotionally to mathematics and music. We build and traverse the bridges between these domains in the belief that by doing so we will be led to new ways of thinking about and doing mathematics and music. The seeds for this paper were sown when one of the authors demonstrated how the “same” rhythmic pattern can be heard in many different ways; he used $x's$ in a grid to represent the shifted aural perspectives behind the various perceptions. The other author was reminded of the shift map in the mathematical field of symbolic dynamics. What followed was an animated session of discussion, pattern clapping and board work. By the session’s end, we identified a shared interest in exploring the potential of symbolic dynamics for categorizing and comparing rhythms. An examination of the literature has unearthed some antecedents to analyzing music with tools from symbolic dynamics in [1] and [3]. It should be noted however that this paper appears to be the first one which specifically links rhythmic closeness and symbolic dynamics.

Preliminaries

Rhythm can be characterized by the following features: pulse, sub–pulse (meter), and cycle (measure). Pulse refers to the steady, underlying beat that organizes the sound that we hear. Sub–pulse is the manner in which a pulse is divided; it is an aspect of what gives music from different genres and traditions their distinct personalities. Cycle refers to the repetitive aspect of rhythm. Evidently, we can represent a rhythm by an infinite[1] sequence of $0's$ and $1's$ where the $0's$ denote silences and the $1's$ denote sound, i.e., onsets [4]. This space is called the sequence space on two symbols 0 and 1. In symbolic dynamics, the study of discrete dynamical systems using methods of recurrence, transitivity and graph theory [3], this space is denoted by symbol Σ_2. If we define the distance between two sequences $s = (s_0 s_1 s_2 ...)$ and $t = (t_0 t_1 t_2 ...)$ by $d(s,t) = \sum_{i=0}^{+\infty} \frac{|s_i - t_i|}{2^i}$ then since $| s_i - t_i |$ is either 0 or 1, this infinite series is dominated by the geometric series $\sum_{i=0}^{+\infty} \frac{1}{2^i} = 2$ and therefore it converges. It follows that d is a metric on Σ_2 [2]. For example, if

[1] It should be noted here that in much of the literature rhythms are represented as vectors of finite length.

$\mathbf{a} = (000\cdots)$, $\mathbf{b} = (111\cdots)$, and $\mathbf{c} = (1010\cdots)$, then $d(\mathbf{a},\mathbf{b}) = \sum_{i=0}^{+\infty} \frac{|a_i - b_i|}{2^i} = 2$ and $d(\mathbf{a},\mathbf{c}) = \sum_{i=0}^{+\infty} \frac{1}{2^{2^i}} = \frac{4}{3}$. Two sequences in Σ_2 are close [2] provided their first few entries agree. This paper proposes an investigation of the use of this metric to compare rhythms.

Playing polyrhythm mathematically

We intuitively expect that there will be some relationship between rhythmic closeness, as determined by this measure, and the rhythmic experience of a listener or performer. Our initial line of investigation will be to compute and compare the distance between different rhythms that comprise the different parts for various pieces drawn from the repertoires of African and African Diasporic traditional percussion ensembles. The question to be explored: how do the relationships among the polyrhythms, which characterize the drumming of many of these traditions, manifest themselves in relation to the distance measure? This direction is suggested by the exploration in [4] of rhythmic dissimilarity measures with respect to how much insight they provide concerning the structural interrelationships that exist within families of rhythms. For example, the music for Agbeko, one of the traditional dances of the Ewe people of Togo, Benin, Ghana, is played by an ensemble consisting of a family of drums – Atsimewu (master drum/father), Sogo (big brother), Kidi (mother), Kagan (baby) –along with Gonkoqui (double gong) and Axatse (shaker). The timeline (see [4]) is played on the gonkoqui. This part can be represented by the sequence: $(101011010101\cdots)$. One of the rhythms played by the Sogo can be represented by the sequence: $(101010100100\cdots)$. The part played by the Kagan can be represented by the sequence: $(011011011011\cdots)$. The measure of the distance between the Gonkoqui and Sogo rhythms is $\frac{226}{4095} = 0.05518925519$. The distance between the Gonkoqui and Kagan parts is $\frac{2092}{4095} = 0.5115995116$ and for the Sogo and Kagan parts the distance is $\frac{6398}{4095} = \frac{914}{585} = 1.562393162$. We see value in investigating these outcomes from the perspective of human perceptual and cognitive models of rhythmic dissimilarity and music information theory [4].

Conclusion

We end by returning to our catalyzing experience linking the shift map in symbolic dynamics with hearing the same rhythmic pattern in a number of different ways. The shift map $\sigma(s_0 s_1 s_2 \cdots) = (s_1 s_2 s_3 \cdots)$, a key ingredient in symbolic dynamics, simply "forgets" the first entry in the sequence, and shifts all other entries one place to the left. It is a two–to–one continuous map in the metric defined above [2]. The periodic points of σ correspond exactly to repeating sequences, i.e., sequences of form $(s_0 s_1 \cdots s_{n-1} s_0 s_1 \cdots s_{n-1} \cdots)$ and they form a dense subset of Σ_2. These are precisely our rhythms! In future work we will explore the connections between rhythmic similarity and the shift map on Σ_2.

References

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