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Right-Angle Preference in Impossible Objects and Impossible Motion

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Abstract

There is a class of pictures called pictures of impossible objects. Although they are named "impossible," they are not necessarily impossible. Some of them can be realized as three-dimensional solid objects. In this study, we show that the impression of impossibility comes from the human preference for right angles. This preference can also explain another visual illusion called impossible motions.
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Introduction

There is a class of pictures called pictures of impossible objects. Typical examples are Penroses' triangle and an endless loop of steps [2]. The latter became famous in that it was used in Escher's "Ascending and Descending" (1960) [3]. Pictures of impossible objects are usually considered as just images that generate an impression of three-dimensional structures and cannot be realized in three-dimensional space.

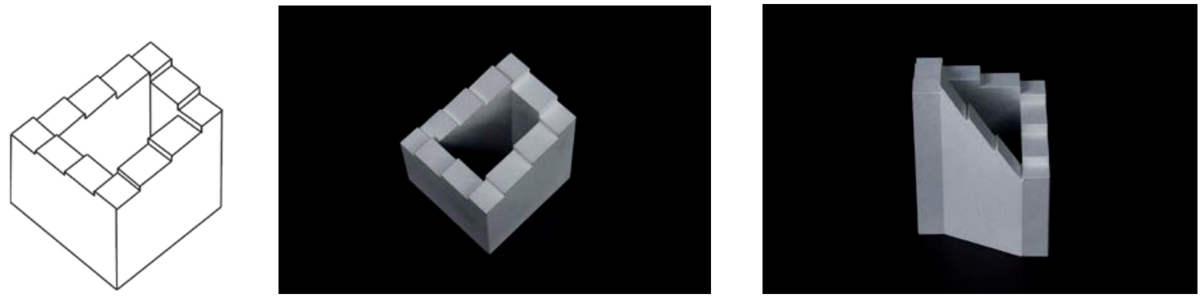
However, some of these so-called impossible objects can be actually constructed as three-dimensional objects. There are two well-known tricks by which this can be contrived [1]. One is to make gaps in depth in such a way that faces look connected when seen from a particular viewpoint. The second is to use curved surfaces that look planar from a particular viewpoint.

There is a third trick, in which neither gaps nor curved surfaces are used, but by which three-dimensional impossible objects can still be constructed [4]. This trick is based on an inverse problem of projection, in which we are shown a picture and search for a three-dimensional object whose projection generates the given picture. A similar trick can also be used to generate an impression that we perceive physically impossible motions [5].

When we consider what kind of three-dimensional objects can generate this illusion effectively, our answer is that we should use non-right angles in such a way that they look like right angles when seen from a specific viewpoint. This is because the human visual system prefers right angles much more than other angles. In this paper, we discuss this observation in detail.

Impossible Objects and Impossible Motions

Let us start our discussion with an example. Fig. 1 shows an example of an impossible object. Fig. 1(a) is a drawing of an impossible object called the endless loop of steps. What is represented by this drawing seems impossible, but it can be constructed as shown in Fig. 1(b). The actual shape of this object is such that, as shown in Fig. 1(c), three of the four walls have normal steps, but the fourth wall has distorted steps that absorb the difference in the heights of the start step and the end step of the normal steps.



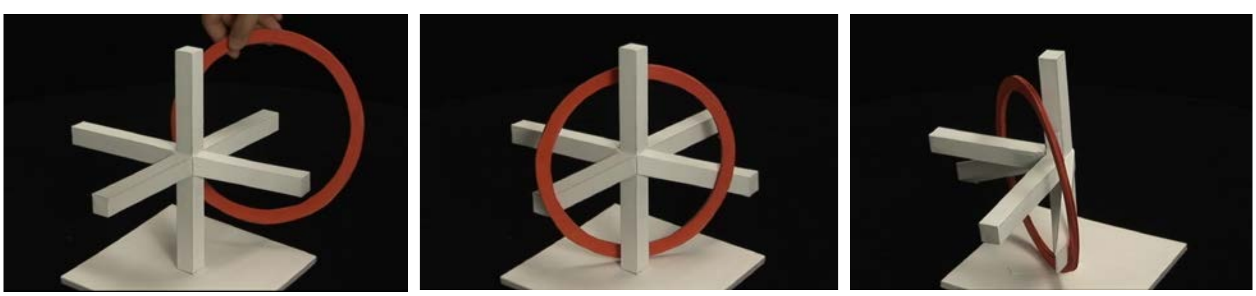
(a) (b) (c)

Figure 1: *Impossible object "Endless Loop of Steps": (a) sketch of the impossible object; (b) solid object realizing the sketch; (c) another view of the object.*

Note that in this structure we do not use the depth-gap trick or the curved-surface trick, but we still can construct the object. So our question is as follows: although we can actually realize the object as shown in Figs. 1(b) and (c), why do we feel the sketch in Fig. 1(a) to be impossible? One possible answer is that this is because our visual perception system thinks that the structure is composed of horizontal and vertical faces, in other words, we think that the object planes meet at right angles.

Fig. 2 shows an example of an impossible motion titled "Eccentric Ring Toss". When we see the image shown in Fig. 2(a), we usually perceive an object consisting of a vertical pole and four horizontal perches connected at the middle of the pole at right angles. However, a flat ring can be inserted into the structure as shown in Fig. 2(b). This motion of the ring seems impossible because it is behind the vertical pole while it is in front of all four perches. In fact, as shown in Fig. 2(c), the four perches all extend toward the rear. An interesting observation is that, even though we know the true shape of an object, we perceive right-angle horizontal perches again when we go back to Fig. 2(a) or (b).

Typical impossible motion illusions can be created by modifying impossible object illusions. Note that if we observe Fig. 2(b) as a static scene, it is an impossible object. We first remove the ring from this object. Then the remaining part, i.e., Fig. 2(a), looks an ordinary solid. We next insert the ring again in front of the audience. Then, the audience have an impression of impossible motion.



(a) (b) (c)

Figure 2: *Impossible motion "Eccentric Ring Toss": (a) solid seen from the special viewpoint; (b) impossible motion of a ring; (c) solid seen from a general viewpoint.*

Right-Angle Preference Hypothesis

In both Fig. 1 and Fig. 2, our perception system seems to interpret the images as objects composed of planar faces meeting at right angles. Hence this observation might be explained by the next hypothesis.

Hypothesis 1 (Preference for right angles) The human vision system is more likely to choose a right-angle interpretation than other angles.

The preference for right angles is straightforward in the examples in Fig. 1 and Fig. 2. However, this preference is less direct in some cases. An example of this is “Magnet-Like Slopes” shown in Fig. 3. Fig. 3(a) is an image of an object seen from a particular viewpoint; from this viewpoint it looks like the four slopes extend from the center of the structure, the highest point, in four directions, but balls placed on the slopes look as if they roll uphill, defying gravity, as shown in Fig. 3(b). This is an illusion. In reality, as shown in Fig. 3(c), the four slopes have different lengths and the point at which they meet is, in fact, the lowest point. Hence, the balls roll downhill obeying the laws of gravity. Thus, this three-dimensional solid object generates the illusion of the impossible. This illusion won first prize in the 6th Best Illusion of the Year Contest held in Florida, USA, in 2010.

Because the slopes are slanted, they do not form right angles. However, this illusion might involve the preference for right angles in the following two aspects. First, as shown in Fig. 3(a), the five columns supporting the slopes appear to be parallel, and hence the human vision system interprets them to be vertical (in other words, they form right angles with respect to the horizontal base plate), and that the column supporting the central meeting point is the highest. Second, the human vision system seems to understand that the four roads meet at the center at right angles when seen from above. Because of these two aspects, the illusion in Fig. 3 can be explained by the right angle preference hypothesis.

Many other impossible object illusions and impossible motion illusions can be explained by this hypothesis.

It is not clear whether the preference of right angles is a general property of the vision system or something resulting from our daily experiences in the developed world with many industrial products having right angles. However, it seems to me that the illusion is stronger for adults than for children. So I feel it is acquired.

(a) (b) (c)

Figure 3: *Impossible motion “Magnet-Like Slopes”: (a) solid seen from a special viewpoint; (b) balls rolling uphill; (c) solid seen from a general viewpoint.*

Highest-Symmetry Preference Hypothesis

There is another way to explain the illusion phenomena in impossible objects and impossible motions. The three-dimensional shapes we perceive when we look at Fig. 2(a) and Fig. 3(a) are highly symmetric.

Both shapes are symmetric for rotations by 90, 180, and 270 degrees with respect to the vertical axis at the center and for mirroring with respect to four vertical planes passing through the center.

On the basis of these observations, we consider the next hypothesis.

Hypothesis 2 (Preference for highest symmetry) The human vision system is more likely to choose the highest-symmetry interpretation than other interpretations.

Hypothesis 2 is more general than Hypothesis 1, because the three-dimensional shapes we perceive in Fig. 2(a) and Fig. 3(a) can be modified into non-right angle shapes without loosing their symmetric nature with respect to rotation and mirroring.

However, our preliminary observation shows that the illusion becomes weak if the right-angle nature is lost. For example, we replaced the four slopes in Fig. 3 with the three slopes in such a way that their projection coincides with the symmetric structure with respect to rotations by 120 and 240 degrees around the vertical axis at the center. Then, we perceived that the center is the highest, but the structure is not symmetric. So we have to be careful to ascertain which hypothesis is more appropriate to explain the illusions.

Concluding Remarks

We have shown that the illusions of impossible objects and impossible motions can be explained in two ways. One is the right-angle preference hypothesis and the other is the highest-symmetry preference hypothesis. Although both of the hypotheses are helpful in the design of new three-dimensional solid objects, we have to study their relations in more detail. In future work, we will explore these two hypotheses in more detail to see which is more appropriate for explaining the presented illusions, the right-angle preference or the highest-symmetry preference.

Acknowledgments

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References

[1] B. Ernst, *The Eye Beguiled: Optical Illusions*, Benedikt Taschen Verlag GmbH, Köln, 1986.

[2] L. S. Penrose and R. Penrose, *Impossible objects: A special type of visual illusion*, British Journal of Psychology, Vol. 49, pp. 31-33, 1958.

[3] D. Schattschneider, *M.C. Escher: Vision and Symmetry*, Abrams, New York, 1990.

[4] K. Sugihara, *A characterization of a class of anomalous solids*, Interdisciplinary Information Sciences, Vol. 11, pp. 149-156, 2005.

[5] K. Sugihara, *Design of solids for antigravity motion illusion*, Computational Geometry: Theory and Applications, Vol. 47, pp. 675-682, 2014.