Engaging Groups with Large-Scale Construction Events

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Abstract

In large arenas, it's possible to get the public personally engaged with mathematics, by coordinating large scale construction events. Many people can get physically involved in a mathematical activity, leading to a real sense of accomplishment when all the pieces come together. Participants see how mathematics can create large, beautiful, complex structures out of very simple building blocks that go together in orderly ways. People leave the construction events having had a very positive experience with mathematics, and feeling empowered.

Overview

MoMath is an institution with a mission to change public perceptions of mathematics. When most people think of math, they think of columns of numbers, of rote recipes used to find an answer, or of memorizing numerical formulas and facts that carry little meaning. There are many ways MoMath tries to change such perceptions. The *Math Encounters* series brings in research mathematicians to talk about what moves them, in a way that gets the audience actively involved and engaged. *Family Fridays* at MoMath, presented by Time Warner Cable, brings children and adults together to share in fun and learning. MoMath's exhibits highlight real math: the study of pattern and structure; the power of inquisitive thinking; the wonder of "what would happen if…"; and the magical "a-ha" moment of enlightenment.

In larger arenas, there's another way to get the public personally and physically engaged with math: coordinate large scale construction events, where there is a real sense of accomplishment when all the pieces come together. This is a methodology brought to the Museum by George Hart, who continues to innovate in this arena [1]. Participants see how mathematics can create beautiful, complex structures out of very simple building blocks. People leave the construction events feeling empowered with respect to math. The goal of this paper is to inspire others to create their own construction activities and spread excitement and engagement with math. To that end, the remainder of the paper describes concisely the materials and methods for two recent large-scale constructions: TetraColor Tower, a multicolored Sierpinski tetrahedron, and Parabolic Falls, a glowing celebration of the hyperbolic paraboloid.

TetraColor Tower

The Sierpinski tetrahedron is the three-dimensional analogue of the famous Sierpinski triangle. If one considers a solid tetrahedron in which, on each face, the centers of the edges are joined with straight line segments, these segments will form the edges of an octahedron (see fig. 1).

If the interior of the octahedron is deleted, the remaining object consists of four, one-eighth size solid tetrahedra, one at each of the original vertices. Iterating this process indefinitely on these resulting smaller tetrahedra generates the Sierpinski tetrahedron. The familiar Sierpinski triangle can be seen in each 'outer face' of the tetrahedron. See Segerman [3] for a clear description of this process.

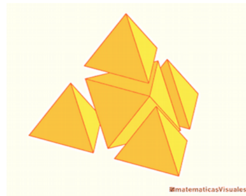

Figure 1: *Decomposing a tetrahedron into four tetrahedra and an octahedron.*

Choosing a finite number of steps (in this case, three) allows a group to construct a physical approximation of the Sierpinski tetrahedron. Working from the inside, four tetrahedra of the same size can be attached together to form a larger 'hollow' tetrahedron of twice the height at each step. The final structure contains 64 of the smallest tetrahedra, and stands eight times their height.

Materials needed:
450 1.5” x 24” Kraft mailing tubes with end caps; 10” white paper twist ties; 80 packs 8-32 light duty eye bolts; 9 rolls 18” x 24' contact paper; Loctite Blue Threadlocker; Elmer's glue; clear packing tape. (About $675 worth of supplies in all.)

Preparing the tubes:

1. Insert one eyebolt (loop out) through the center of each end cap, securing the nut with Loctite.
2. Use plain Elmer's glue to glue the end caps into both ends of all the cardboard tubes.
3. Cut the contact paper into 6” x 18” pieces. Wrap 384 tubes, each with a piece of contact paper.
4. Cut 21 tubes into three 8” sections for collars, slicing each along its entire length to make a slit.

Building the sculpture:

1. Make a level 1 tetrahedron with six tubes (fig. 2). Use the twist ties through the eye bolts to attach the tube ends to each other. Twist together tightly. A total of 64 level 1 tetrahedra are needed.

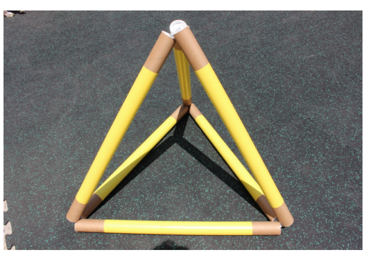

Figure 2: *A level 1 tetrahedron*

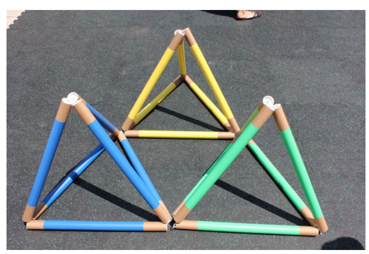

Figure 3: *Level 1 tetrahedra in place.*

Figure 4: *Collar detail.*

2. Arrange three level 1 tetrahedra in a triangle (fig. 3) and attach adjacent corners with twist ties through the eye bolts. Before attaching these bottom eye bolts, slide one of the collars onto one tube at each attachment. Twist tie the tubes together, then slide the collar over the second tube so that it straddles the two sides evenly. Secure the collar firmly with tape or twist ties (See fig. 4.)
3. Place the fourth level 1 tetrahedron on top of the other three, in the center, and attach to those three with twist ties. Make a total of 16 level 2 tetrahedra. (See fig. 5.)
4. Do steps 2 and 3 again with four level 2 tetrahedra to make a level 3 tetrahedron, to be the top.
5. The remaining level 3 tetrahedra are built beneath the corners of the tetrahedron from step 4. Put one level 2 tetrahedron near each of the three bottom corners of that top piece. Raise the top piece, slide the three level 2 tetrahedra into place, and attach their top corners very securely.
6. Place three more level 2 tetrahedra near each of those attached in step 5. Put collars on where these will be connected at the bottom, so that you can later slide them over the junctions.
7. Lift the sculpture up, slide the remaining tetrahedra into place, and attach with twist ties. Fasten the collars securely. Add collars to any horizontal attachments that do not yet have them.

8. Add tape to joints as needed, and the level 4 tetrahedron is complete. (See fig. 6.)

Figure 5: *Multiple level 2 tetrahedra.*

Figure 6: *The completed level 4 tetrahedron.*

Parabolic Falls

This combination of four tetrahedral units with embedded hyperbolic paraboloids makes a dramatic display at an evening party or gathering. It is ideal for involving many people in the construction in a short time, because the "filling the structure" step can incorporate up to 400 people, and the action required by each of these people is modest, simply turning on a glow stick and inserting it in a tube.

Materials needed:

96 6-foot lengths of 1-7/8" x 2" x 1/16" Tenite Butyrate Tubing; twelve dozen Alliance Sterling Rubber Bands, number 107; 72 Uline 2" diameter clear tube caps; twine (about $1,550 in materials in all) and approximately 400 foam glow tubes (about $550).

Preparing the tubes:

1. Select twenty-four tubes to be the edges of the four tetrahedral units in this sculpture. Divide these tubes into ten equal segments with nine equally-spaced marks along their lengths.
2. Loop the twine through sets of six tubes at a time to produce four tetrahedral units, pulling the twine very hard and tying it off to make strong, rigid structures.
3. For three out of four of the units, add 18 tubes as follows: place a tube from the highest mark on one upright edge to the leftmost mark on the opposite horizontal edge, and secure it in place on both ends with a rubber band. Then place the next tube one mark down on the upright edge, and one mark right on the horizontal edge. Proceed in this way, placing nine tubes. Then begin again at the top of the next upright tube, crossing each of the first nine tubes, to run this new tube from the top of the next upright to the opposite horizontal. Add a total of nine more tubes, each crossing every one of the first nine, taking advantage of the doubly-ruled nature of a hyperbolic paraboloid; the effect is similar to Poole [2]. (See fig. 7 for a completed unit.)

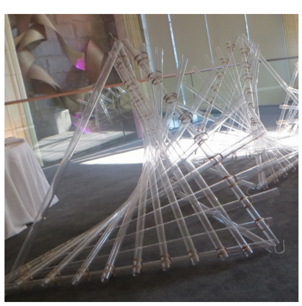

Figure 7: *One type of tetrahedral subunit (3 copies needed)*

Figure 8: *The other subunit (1 copy).*

5. For the fourth unit, place six tubes as in the first step above, to the topmost three marks and the bottommost three marks, leaving out the three in the middle. Then rotate the tetrahedron 120 degrees, and add another six in exactly the same configuration. Finally, rotate one last time and add the final six tubes. If desired, the tube ends can be moved to a more even spacing. (Fig. 8.)
6. Lash the first three tetrahedra together at their bottom vertices, creating an equilateral triangle twelve feet on a side.
7. Lift the fourth tetrahedron on top of the other three so it is touching at three vertices, and lash it into place, making certain it cannot fall. (See fig. 9.)

Figure 9: *All tubes in place.*

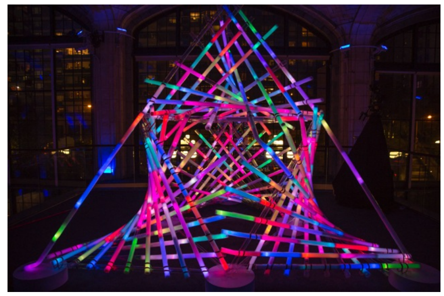

Figure 10: *The completed sculpture.*

Building the sculpture:

1. Make sure Uline tube caps are in place at the lowest point of every tube.
2. Have participants turn on the glow sticks and insert them into the clear tubes of the sculpture until all tubes are full. This will require approximately four hundred glow sticks. (See fig. 10.)

Conclusions

TetraColor Tower was built with the assistance of approximately 250 visitors to MoMath's installation at the World Science Festival in June 2013, and Parabolic Falls involved over 400 attendees of MoMath's first annual Gala in October 2013. The components of the latter were re-used at MoMath's First Birthday Party on Dec 15, engaging another 100+ participants. In both cases, participants self-reported a great deal of enjoyment and engagement. Multiple participants stayed through the entire building process, evincing great satisfaction when the sculptures reached completion. At an average investment of under $4 per participant, large group constructions appear to be an effective means of spreading interest in and engagement with math.

References

[1] George Hart, *Large Geometric Construction of Cardboard,* IMAGINARY open mathematics, http://imaginary.org/hands-on/large-geometric-construction-of-cardboard/, downloaded 2014 Apr 24.

[2] Cory Poole, *How to Make a Hyperbolic Paraboloid Using Skewers,* MathCraft Blog, http://mathcraft.wonderhowto.com/how-to/make-hyperbolic-paraboloid-using-skewers-0131751/, downloaded 2014 Mar 15.

[3] Henry Segerman, *Fractals and How to Build a Sierpinski Tetrahedron,* lecture notes available at http://www.ms.unimelb.edu.au/~segerman/talks/fractals_and_how_to_build_a_sierpinski_tetrahedron.pdf, downloaded 2014 Mar 15.