Drawing with Elliptical Arcs

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Abstract

Inspired by elliptical shapes that occur in nature, I create planar images that use entire ellipses or proper arcs of ellipses as their single graphic element. I construct collections of ellipses or of elliptical arcs of varying absolute sizes, eccentricities, positions, rotational orientations, and angular extents that are abstractions of natural objects. Specifically, I use abstractions of a single ginkgo leaf as basic graphical element along planar curves in my images.

Introduction

This paper has two parts; in the first part I will describe images of collections of entire ellipses that are laid out on a modified rectangular grid; in the second part I will start with a triangularly shaped area formed by overlapping ellipses, create an abstraction of a ginkgo leaf (ginkgo biloba, see [3]), based on these triangular shapes, and then place the abstractions of ginkgo leaves of varying geometric and graphical properties on planar grids or along planar curves.

I create all my images with Mathematica and use its graphics primitive `Circle[{x,y},{a,b},{s,t}]` to define elliptical arcs; the meanings of its parameter pairs are: coordinates of the center (x, y), semi-axes (a, b), initial and final angles (s, t); or parametrically $f(\varphi) = (a \cos(\varphi) + x, b \sin(\varphi) + y)$ for $s \leq \varphi \leq t$. I refer to the closed elliptical arc as an a×b-ellipse. It is essential to me to be able to specify in my drawings graphics properties such as color and thickness as well as geometric operations such as translations and rotations for each individual elliptical arc. Treating an arc as a 2D-graphics object rather than as a parametric function in the plane also helps me conceptualize geometric operations and programming code in creating images composed of many different collections of elliptical arcs.

Collections of Ellipses on Modified Rectangular Grids

This project started with experiments of placing an axes-parallel ellipse on the points of a rectangular grid. Changing the spacing of the rectangular grid relative to the semi-axes of the ellipse just produces different rectangular tiling patterns. However, when I replace the single ellipse with a collection of ellipses of varying eccentricities and sizes, and also change to a non-equidistant rectangular grid the resulting images are very different from tilings. Figure 1a shows a collection of ellipses based on a 5×2-ellipse. The range of the horizontal (vertical) semi-axes is between about 1/4 (2/3) and 1/10 (1/80) of the horizontal (vertical) extent of the grid so that the underlying grid is still visible in the image. In Figure 1b I modify the grid with an exponential function such as f(t) = e^(1/t^2) and overlay three collections of ellipses with different ranges of their semi-axes; the collections use a 5×1-, a 3×3-, and a 1×5-ellipse as their respective base ellipses. Since the exponential function changes rapidly in size each of the collections presents the illusion of being tightly clustered about a point rather than over an extended rectangular grid. In both images the ellipses are colored according to their position in the grid, and linear functions determine the geometric and graphics properties of the ellipses.

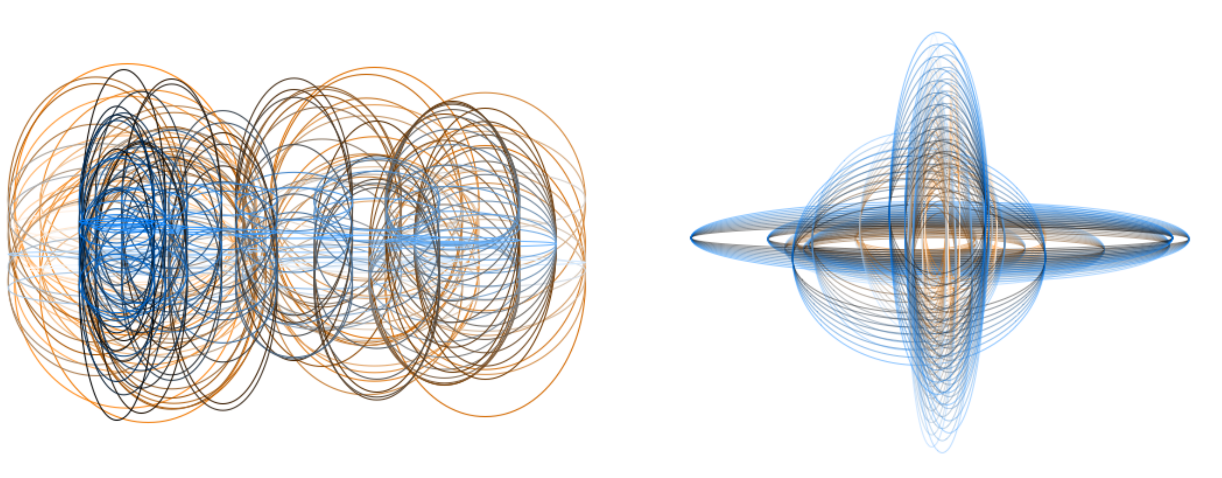

Figure 1: *Collections of Ellipses on Modified Rectangular Grids.*
(a) Trigonometric & (b) Exponentially Modified.

Images Based on an Elliptical Ginkgo Leaf Design

Overlapping ellipses on a rectangular grid contain pointed ovoid and crescent moon shapes bounded by two arcs as well as various triangular shapes bounded by three arcs. One of these triangular areas reminds me of the outline of a ginkgo leaf. This austere form lacks however important aspects of a ginkgo leaf: (1) the V-shaped cut that occurs frequently at the top of a leaf, (2) the veins of a leaf, and (3) its stem. The ginkgo tree has leaves with one uncommon feature, possibly because it is a living fossil documented from the Permian about 250 million years ago: the veins of the leaf originate at the stem and when they branch, all branches extend directly towards the top of the leaf. This is distinctly different from the branching of veins in leaves of contemporary deciduous trees such as oak and maple and reinforces those elliptical elements of the leaf that I use as basis for my abstract designs. The two semi-axes and the varying curvature of a single elliptical arc provide two degrees of freedom and thereby may convey "naturalness" of the abstract representation to a viewer. One could use circular arcs instead and achieve a beautifully simple design such as in the signature symbol of the capital city of Tokyo (see [4] and Figure 2a below for one version of the symbol); however, in collages of many leaves the mathematical complexity of the ellipse is essential for my designs.

Figures 2b shows the outline of a leaf with a straight (i.e. a degenerate ellipse) stem and a V-shape cut formed by two elliptical arcs; Figure 2c shows the veins realized as eight elliptical arcs on either side of the leaf, and Figure 2d shows all elements of my abstract design. The arcs representing the veins are defined with two linear sequences of parameter values for their eccentricities and for their angular extent from stem to endpoint, respectively. I deliberately let arcs defining the shape of a leaf as well as the ends of the veins go outside the leaf boundary at the tips of the leaves while the others stay in the interior (see Figure 2d) in order to visually enhance the elliptical boundary of the leaf as abstract raggedness.

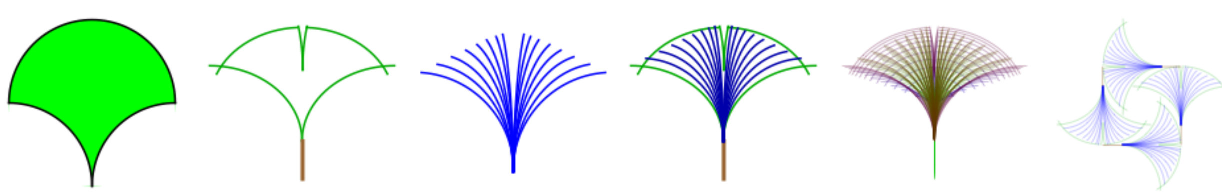

Figure 2: *Graphic Elements of an Elliptic Ginkgo Leaf and Two Compound Designs:*
(a) Circular Ginkgo, (b) Outline, (c) Veins, (d) Complete Leaf, (e) Fan of Leaves, (f) Square of Leaves

My goal in this project is to manipulate each elliptical ginkgo leaf as a whole and treat it as a basic graphic element in more complex designs. I will use affine transformations to translate, rotate and scale a base elliptical ginkgo leaf. For example, in Figure 2e I overlay a set of 16 ginkgo leaves that are sized exponentially using the Gaussian function f(t) = e^(-16/t^2), centered at their common base. Figure 2f on the right takes four leaves in a square arrangement that I use as a logo on my business card.

Next, I want to discuss two images that combine the design on an elliptical ginkgo leaf with ideas embedded in images I exhibited and discussed in my paper at Bridges Towson 2012 (see [1]). In Figure 2f I take an elliptical ginkgo leaf and draw one leaf at each of the four corners of a square oriented along the relevant side of the square. I generalize this design to arbitrary smooth planar curves where I want individual leaves to be oriented in the direction of the tangent or the normal at discrete intervals along the curves. Computations of the tangent and normal (see [2]) can best be accomplished by representing the curves as parametric equations. In addition, I use another differentiable function of the parameter in the parametric equations, usually the distance from the origin, to determine the size of the leaves. Finally, I use random colors for the three elements of a leaf - outline, veins and stem.

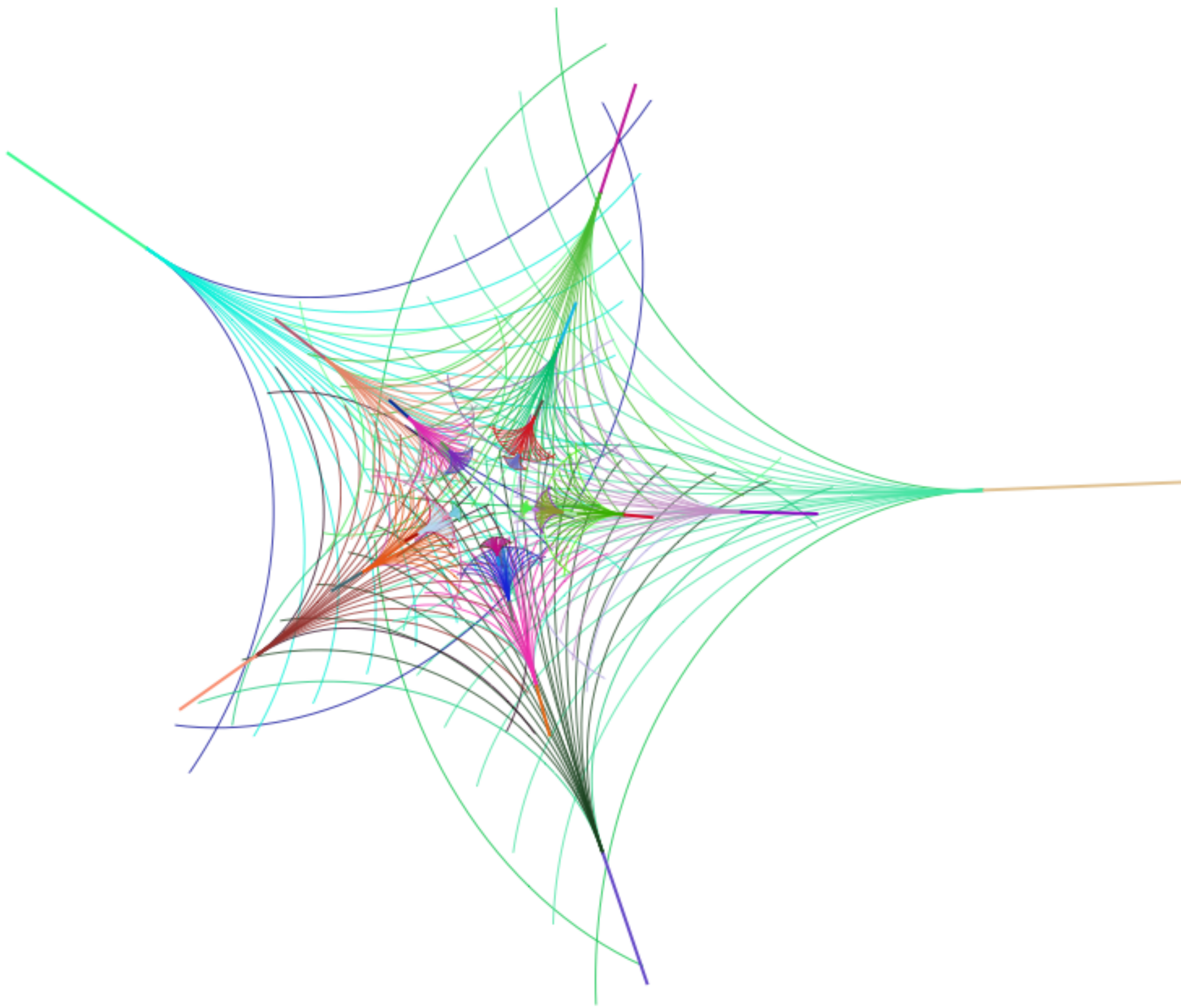

Figure 3: *A 5-Pointed Star of Ginkgo Leaves - An Illusion with a Logarithmic Spiral.*

In Figure 3 I show 21 ginkgo leaves using an equidistant subdivision of the parameter interval on the underlying logarithmic spiral r(φ) = e^(-φ/25) and shrink their sizes as they approach the center; however, the image appears to be a 5-pointed star-like design. The reasons for this illusion are twofold:

(1) I set the rotational interval between successive ginkgo leaves at 2.5 radians or about 143.24 degrees and (2) the fact that 4×pi - 5×2.5 equals about 0.06637 radians or about 3.8 degrees; in other words: going around five time placing ginkgo leaves gets you almost back to where you started (even after four complete cycles the total angular turn is only about 15 degrees).

Figure 4 shows 144 leaves along a closed curve that is an instance of the general family of parametric functions $f_{n,k}(t) = (\sin^n(n\ t) + \cos^k(k\ t)) \times (\cos t, \sin t)$ where the step element is $\pi/72$ and the size of the leaf is proportional to its distance from the center.

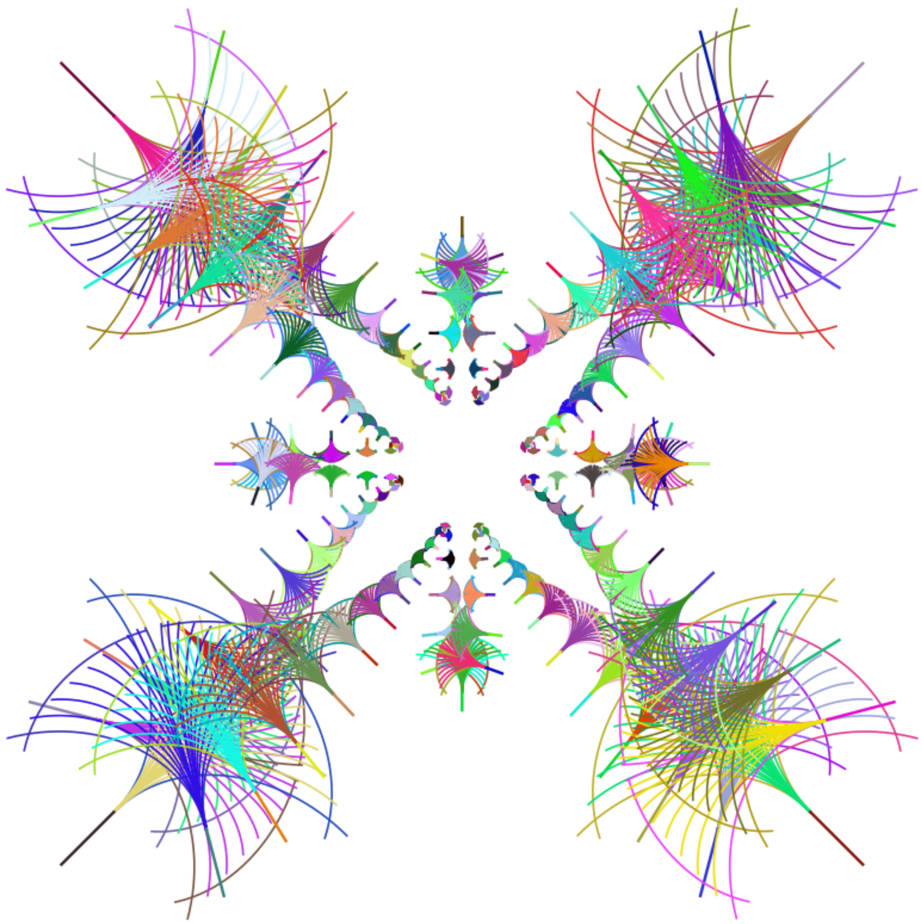

Figure 4: *A Phantom Spiky Diatom of Ginkgo Leaves.*

References

[1] Hartmut F. W. Höft, *Images and Illusions from Orthogonal Pairs of Ellipses,* Bridges Towson Proceedings 2012, pp. 447-8, Tessellations Publishing, Phoenix, Arizona, 2012.

[2] James Stewart, *Calculus, 6-th edition,* Chapter 11: Parametric Equations and Polar Coordinates, pp. 656-709, Thomson Learning Inc., 2008.

[3] Website Title: *Ginkgo biloba*, accessed January 22, 2014, http://en.wikipedia.org/wiki/Gingko_biloba.

[4] Website Title: *Tokyo City Profile and Government*, accessed January 22, 2014, http://www.metro.tokyo.jp/ENGLISH/PROFILE/index.html.