Tria-Tubes

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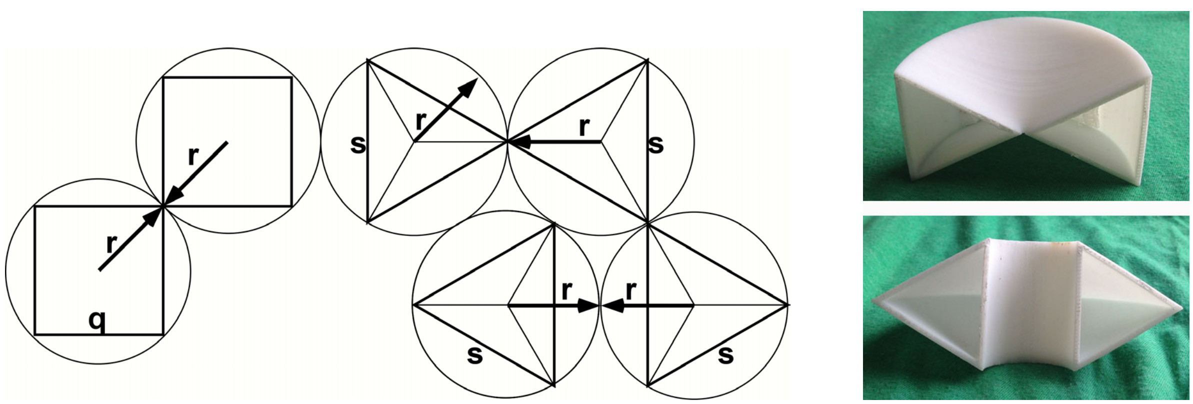
Abstract

This is an extension of the modular building-block system LEGO®-Knots, described earlier in these proceedings, to geometrical sculptures based on sweep geometries with a constant, triangular cross-section. Many of the extensions and further developments of the original "*Borsalino*" shape by Henk van Putten, which inspired this work, can also be realized with this new cross section and also result in a variety of attractive sculptures. The prototype parts were designed to be easily manufacturable on low-end, fused-deposition-modeling (FDM) machines.

1. "*Tria-Borsalinos*"

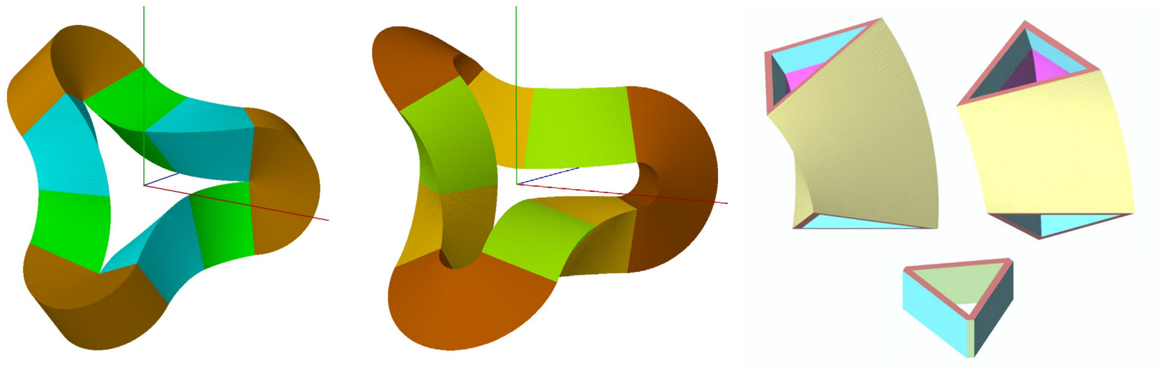
The LEGO®-Knot system [3] is based on a square cross-section swept along composite space curves composed of segments of circular arcs. In this paper we investigate what happens when this square cross-section is replaced with an equilateral triangle. For the prototype parts used in this study, we chose a size for this cross-section that has the same circumcircle as a square cross-section with side length q based on the LEGO®DUPLO [2] geometry (Fig.1a). The tightest bending radius r for an end-cap turning through 180º becomes: $r = q/\sqrt{2}$; and the side length s for the triangular cross section becomes: $s = r*\sqrt{3}$.



(a) (b)

Figure 1: *(a) The size of the new cross-section is derived from the LEGO®DUPLO geometry. (b) Two symmetrical end-caps with triangular cross section: (top) Type I, (bottom) Type II.*

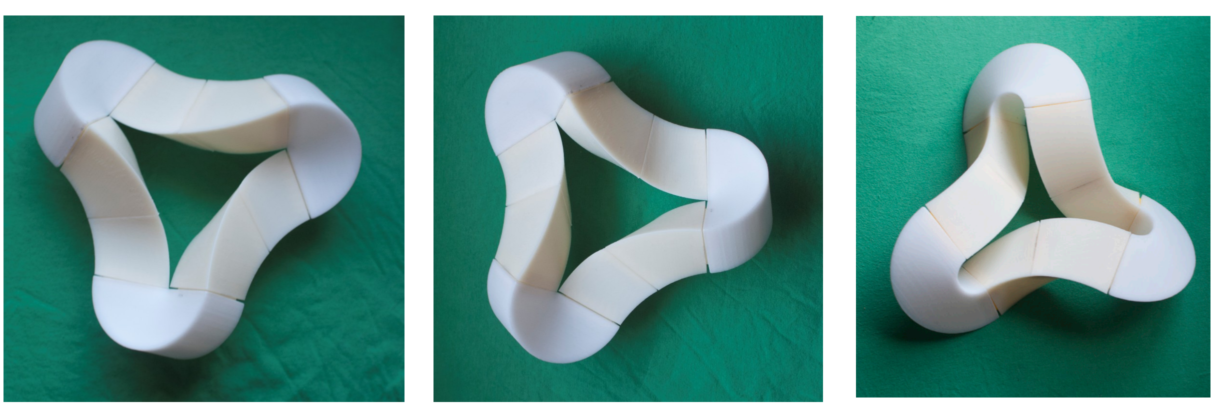
By adjusting the azimuth angle of the cross section with respect to the osculating plane of the sweep curve, we can readily generate two highly symmetrical end-cap geometries: *Type I,* with two triangle vertices pointing inward to the rotation center (Fig.1b, top), and *Type II,* with those vertices pointing outwards (Fig.1b, bottom). They will lead to two different *Borsalino* geometries as shown in Figure 2. In the basic *Borsalino* geometry, as introduced by Henk van Putten [4], the prism sides experience a cyclic permutation through 90º as they sweep from one end-cap through two curved connector pieces to the next end-cap. A square cross-section can accommodate this role-change without any geometrical twisting of the connector pieces; they are just joined together with their azimuth orientation incremented by 90º. The new triangular connector pieces still bend through 45º, and their bending radius is still ($1+\sqrt{2}$) times larger than the bending radius of the end-cap. But now these connectors also need to twist through 15º each, in order to yield an overall smoothly connected surface ribbon. Ignoring for the moment the means of coupling together adjoining building modules, the green and cyan parts in Figure 2a have the same geometry but are used in an end-to-end reversed manner. However, the curved connector geometry needed in Figure 2b is different; its azimuth angle with respect to its sweep curve differs by 180º from the *Type I* connector part. Figures 2a and 2b both have full D_3 symmetry. Both of these shapes can also be realized in mirrored versions.



(a) (b) (c)

Figure 2: *Basic Tria-Borsalino geometries: (a) Type I; (b) Type II; (c) the two corresponding types of curved connector pieces and a triangular coupling sleeve.*

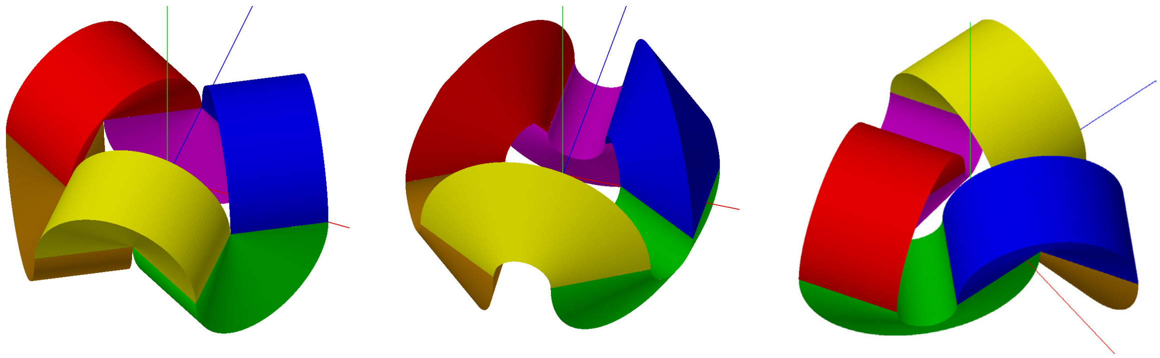
If we were to use a polarized system for coupling adjoining parts together, where each building block has a "female" sleeve at one end and a "male" extrusion at the other, then the green and the cyan parts would be different, and we would have to fabricate twice as many different curved connector parts. To avoid this undesirable diversity, and also to make the parts easier to build on low-end rapid-prototyping machines, we provide all parts with female connections at both ends, and separately fabricate a large number of triangular coupling sleeves (Figure 2c, bottom) that can be inserted as needed. Just like the LEGO®-Knot parts, these Tria-Tube pieces are not just simple sweeps with constant cross sections. They all have 0.2″ long prismatic internal sleeves at both ends to assure that the triangular coupling modules hold together adjacent tube segments with proper tangential alignment. With this approach we can build these parts on inexpensive rapid prototyping machines, such as the Afinia_H479 3D Printers [1], using only minimal support scaffolding, which needs to be pried away manually after the parts have been built in a vertical direction, as indicated in Figure 2c. Figure 3 shows some realized Tria-Borsalino models. They are all composed of three end-caps, six curved connector pieces, and nine coupling sleeves.



(a) (b) (c)

Figure 3: *Realized Tria-Borsalino shapes: (a) Type I, (b) a mirrored version, (c) Type II.*

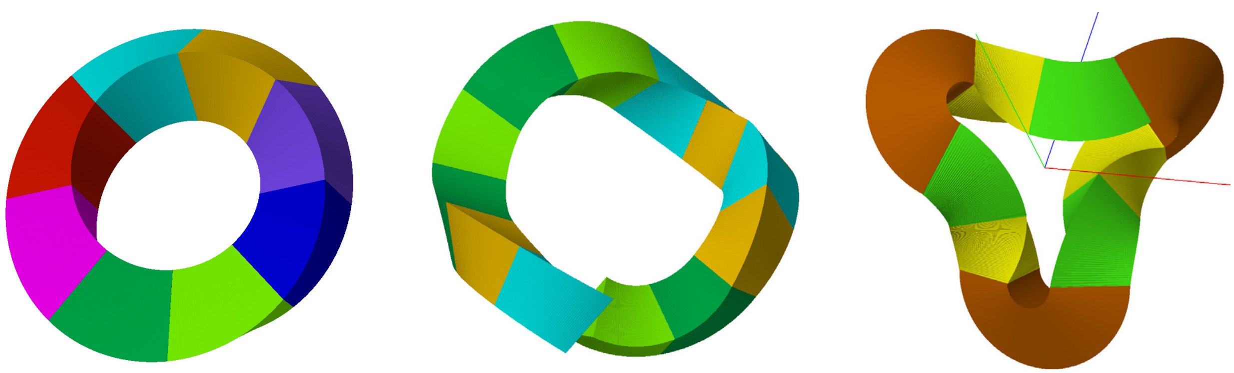
Of course, we want to fabricate more building blocks than just the minimum number needed to make the basic *Tria-Borsalinos*. If we have at least six end-caps of each type, then we can also build the two hexagonal rings shown in Figures 4a and 4b. Alternatively, using three end-caps of each type in an alternating sequence allows us to make a triangular loop as shown in Figure 4c.



(a) (b) (c)

Figure 4: *Loops composed of end-cap: (a) Hexagonal ring of Type I; (b) hex-ring of Type II; (c) triangular loop alternatingly using Type I and Type II end-caps.*

Using only the curved connector pieces, we can try to make a twisted Möbius prism ring as shown in Figure 5a. But it turns out that for this simple, smoothly twisting ring, we will have to fabricate two more curved connector types, in addition to the two types that we already have. To make this ring, we need curved connectors with starting azimuths of 0º, 90º, 180º, and 270º in both their forward and their backward orientation. Using only the two types that we already have, we can make a wiggly "chair"-type ring consisting of six 90º arcs; however, a mismatch of 180º in azimuth will prevent proper closure (Fig.5b). The coloring of the pieces corresponds to Figure 2a,b and shows which part is being used where.



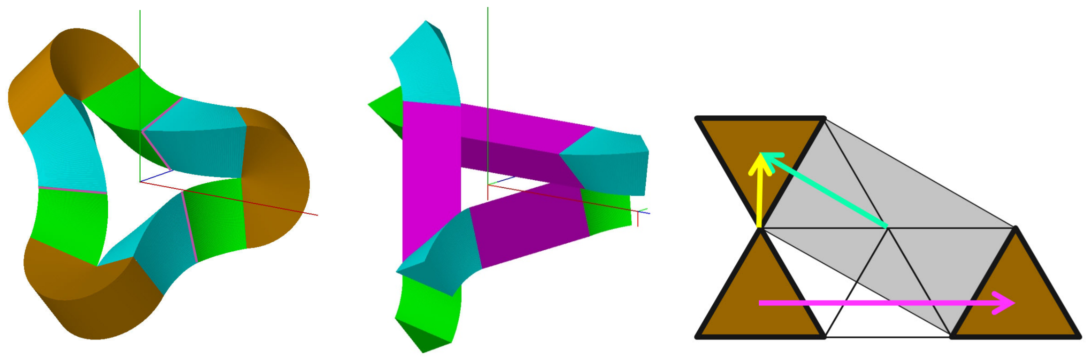
(a) (b) (c)

Figure 5: *New configurations: (a) a smoothly twisting Möbius prism; (b) a wiggly "chair" ring; (c) Type II Tria-Borsalino loop with connectors that twist through –45º.*

2. Additional Parts → New Sculptures

We may ask what new interesting geometries become possible, if we add just one or two strategically chosen new part types, as we did for the *Borsalino* shapes with a square cross-section. Since we already have a certain amount of twist in the connector pairs between the end-caps, we may consider increasing or reversing this twist; this will then change the interconnectivity of the prism faces. Topologically, the *Tria-Borsalino* in which the connector pieces twist through –45º (Fig.5c) is no longer a *twisted* prism, since all faces connect to themselves after one lap around the loop.

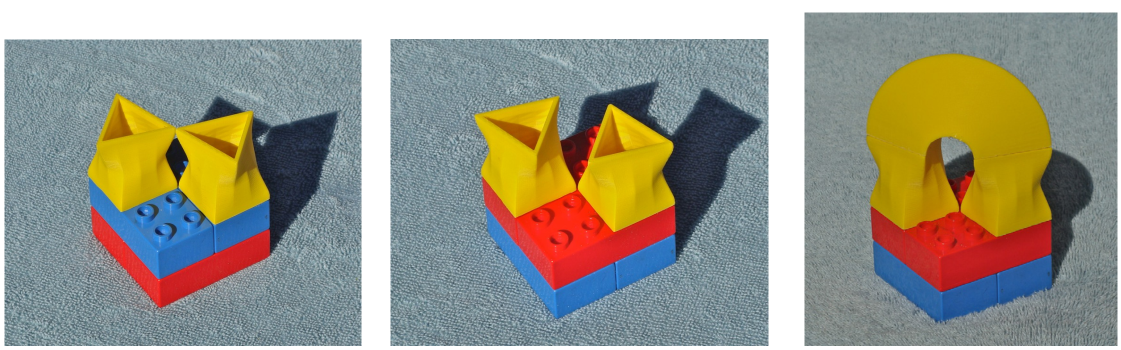
Another promising modification is to create "flipped-lobe" *Tria-Borsalinos* by inserting straight extender pieces in the middle between each pair of curved connectors (Fig.6a,b). This shifts the end-cross sections to which the end-caps are connected out of their aligned position. If the extender piece is of length $s*\sqrt{8}$, the cross-sections are shifted by $2s$ in their shared plane, and this brings them into a new aligned position (Fig.6c). The tight *Type I* end-cap (yellow radius) is transformed into a much looser *Type II* end-cap with a bending radius (shown in green) that is twice as large.



(a) (b) (c)

Figure 6: *(a) Type I Tria-Borsalino with cut-lines; (b) extender pieces inserted between connectors; (c) effect of extender pieces: shifting the positions of the end-cap cross sections.*

It appears that any trick that we applied successfully to the *Borsalinos* with square cross sections has an equivalent representative in this new triangular world! This suggests the idea that those two worlds may even be combined. This could be achieved with a suitable branching piece as suggested in [3], or it could be done with a simple converter piece, morphing from a square, via a regular 12-gon, into a triangle. We have fabricated such a piece and show it in Figure 7 in the context of some LEGO-DUPLO pieces. Now we can indeed make sculptures with parts belonging to both cross-sectional families.



(a) (b) (c)

Figure 7: *DUPLO-to-Tria converter piece: (a) in Type I configuration; (b) in Type II configuration; (c) in Type II configuration with end-cap applied.*

References

[1] Afinia, *H479 3D Printer*. -- http://www.saelig.com/product/RO023.htm?gclid=CJ75oJTQ27sCFZKGfgod50gASw

[2] LEGO® Answers, *Exact dimensions of a DUPLO brick*. -- http://bricks.stackexchange.com/questions/1588/what-are-the-exact-dimensions-of-a-duplo-brick

[3] C. H. Séquin and M. Galemmo, *LEGO-KNOTS*. Proceedings Bridges, Seoul, Aug 14-19, 2014.

[4] H. van Putten, *Borsalino*. Bridges 2013 Exhibition of Mathematical Art. -- http://gallery.bridgesmathart.org/exhibitions/2013-bridges-conference/henk-van-putten