

Design of a Nature-like Fractal Celebrating Warp-knitting

Loe M.G. Feijs¹, Marina Toeters², Jun Hu¹, and Jihong Liu³

¹Department of Industrial Design
Eindhoven University of Technology
l.m.g.feijs@tue.nl

²by-wire.net, Utrecht, ³Jiangnan University,

Abstract

In earlier work we created a new textile pattern which was derived from the well-known houndstooth pattern which originates from weaving with twill binding. The new pattern became interesting, both mathematically and aesthetically because it was a fractal. Now we are turning our attention to another basic fabric construction method: warp-knitting. We develop a recursive algorithm and explore the properties of the result. We also develop an attractive fashion item based on the new pattern, to be presented at Bridges.

Introduction

First we explain what warp-knitting is and in which sense we take inspiration from it. The typical characteristic of knitting is that the threads form loops, each loop being pulled through an existing loop. Roughly speaking, there are two main approaches to knitting, called weft-knitting and warp-knitting. The well-known hand-knitting is a special case of weft-knitting, for example a single yarn being knitted from left to right and then from right to left. In warp-knitting however, the yarn moves in the length-direction of the fabric in a zigzag manner [6]. So, unlike weft-knitting, a warp knitted fabric is composed of many yarns, not just one.

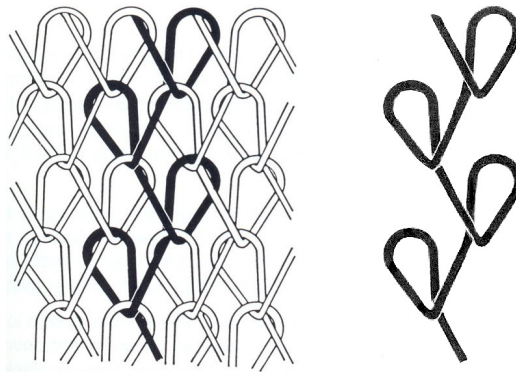


Figure 1: Warp knitted fabric (left) and one thread thereof (right).

Earlier work by Bernasconi, Bodie and Pagli on algorithmic knitting [1] demonstrates the power of recursion as a programming technique for knitted fractals (we use recursion as an essential tool too). The work of the present paper is the result of a new cooperation between TU/e, Jiangnan University and by-wire.net which was initiated during the DeSForM2013 conference in Wuxi. Whereas the Industrial Design Department in Eindhoven has strength in wearable senses and in generative design, the Engineering Research Center of Knitting Technology at Jiangnan University, Wuxi is specialised in warp-knitting. We share an interest in textile design and algorithmic pattern design, witnessed by results such as [4, 3, 2].

In Figure 1 (source of left figure: Wikimedia Commons) the basic principle of warp-knitting is given. One yarn is singled out and this one yarn with its loops is taken as the inspirational source for the fractal to be designed. But first we explain a bit of fractal theory.

How to make a fractal

We take inspiration from line fractals such as the Koch fractal and the dragon curve. Lindenmayer systems [5] are often used to describe the growth of fractal plants. This works with substitution, e.g. a forward move F can be replaced by F-F++F-F. As a formal rule: $F \rightarrow F-F++F-F$. The idea is to apply the rule repeatedly (to all F simultaneously). Starting from F we get F-F++F-F, then F-F++F-F-F-F++F-F-F-F++F-F-F-F++F-F-F-F++F-F-F-F, and so on. Interpreting the symbols as turtle graphics commands, one gives F the meaning of drawing forward, + to turn right 60° , and - to turn left 60° and then this Lindenmayer system describes the Koch fractal.

The warp-knitting fractal

We show the approximations of our new fractal for nesting levels $N = 0, 1, 2, 3$ and 4 in Figure 2. These have been created using a recursive algorithm and a turtle-graphic system, in a similar way in which one makes the Koch fractal. The lines in Figure 2 are drawn starting at the bottom of the figure with the turtle pointing upward. For the second line of Figure 2, the turtle made two loop pairs. In this way we get loops similar to the single yarn of Figure 1 (the loops are not nicely rounded yet, but we will repair that later).

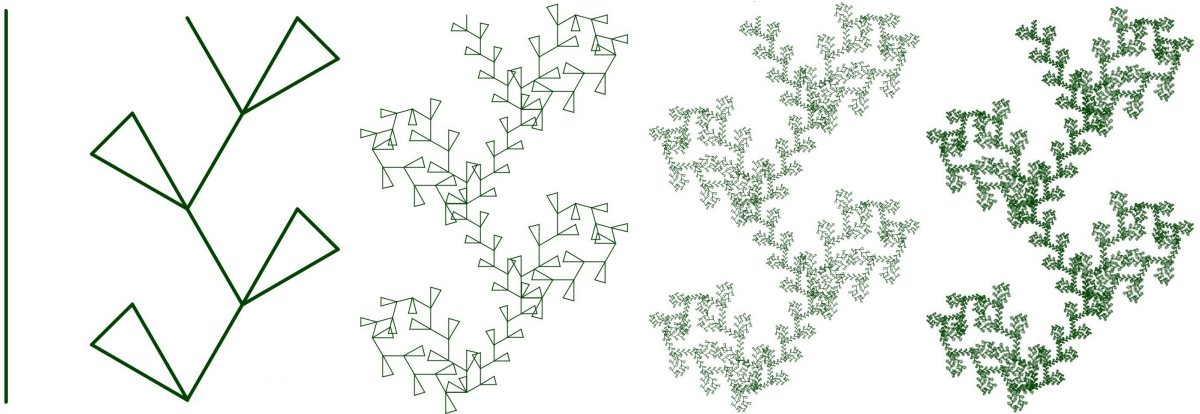


Figure 2: Approximations of the warp-knitting fractal for $N = 0, 1, 2, 3$ and 4.

This gives us a recipe for a fractal: draw a looped line, but whenever the basic recipe tells us to move forward, we move forward while doing a few loop pairs. More precisely: we do 3 loop pairs for the first “forward”, 2 for the next (it is shorter by a factor of $2 \sin 15^\circ$), then 3 again, and 4 for the last “forward”. And then we repeat in a glide-mirrored fashion. The numbers are chosen after experimentation: 2 for the shortest line, 3 inside the loops (where the corners would become messy otherwise) and 4 for the last move. The recipe is related to the Lindenmayer rule $F \rightarrow -F^3-F^2-F^3-F^4F^3+F^2+F^3+F^4+$ where F^2 abbreviates FF, F^3 abbreviates FFF and so on and where the four minus signs represent left turns of $30^\circ, 105^\circ, 105^\circ$ and 90° respectively; the plus signs represent right turns of $105^\circ, 105^\circ, 90^\circ$ and 30° (to specify the exact lengths we would need the more powerful formalism of parametric L-systems). In practice we use the Oogway library in Processing [2]. This also allows us to fine-tune the scaling factors of subfigures and explore aesthetic effects.

Fractal dimension

Replacing a line of length 1 by a loop pair, it turns into six segments of length $\frac{1}{3}\sqrt{3} \approx 0.577$ and two of length $\frac{2}{3}\sqrt{3} \sin 15^\circ \approx 0.299$. If we replace it by three loop pairs, it turns into 18 segments of length $\approx 0.577/3 \approx 0.19$ and 6 of length $\approx 0.299/3 \approx 0.10$. So one line is replaced by 24 segments of a (weighted) average of length 0.17. In the fractal, most lines are replaced by three loop pairs, but there are

also those which are replaced by two loop pairs or by four. To estimate the dimension we pretend each line is replaced by three double loops, so it is broken up in 24 segments of length ≈ 0.17 . Writing n for the number of line segments, s for the scaling factor, $n = 24$ and $\frac{1}{s} = 1/0.17 = 5.9$ so $D \approx (\log 24)/(\log 5.9) = 1.8$. The fractal is almost two-dimensional, which is what we see in the rightmost line of Figure 2: the line almost appears to fill certain areas. This gives the line its natural appearance, like a plant. If we insist on avoiding approximations, we solve $n_1 \times s_1^D + n_2 \times s_2^D + n_3 \times s_3^D = 1$ where $n_1 = 12$, $s_1 = (\frac{1}{3}\sqrt{3})/3$, $n_2 = 8$, $s_2 = (\frac{1}{3}\sqrt{3})/4$, $n_3 = 4$, and $s_3 = (\frac{2}{3}\sqrt{3} \sin 15^\circ)/2$. Using Mathematica's FindRoot we get $D = 1.79659$.

Back to fashion

We promised to make rounded loops, which we achieve using `beginSpline` and `endSpline` in Oogway [2]. This strengthens the nature-like appearance and even for low N it resembles a vine plant now (Figure 3, left). The next step is designing a real fashion item: an elegant lady's dress. We used a combination of knitting (the jersey substrate) and textile printing (the fractal line); special thanks go to Pauline Klein Paste of HKU (Utrecht School of Arts). The pattern can be seen in Figure 3 (center) and the dress in Figure 3 (right) and Figure 4. An interesting question is whether the new pattern can be really machine-knitted. It will also be interesting to see what happens if we involve multiple threads (we leave these questions as options for future research). We shall bring the dress to Bridges Seoul.

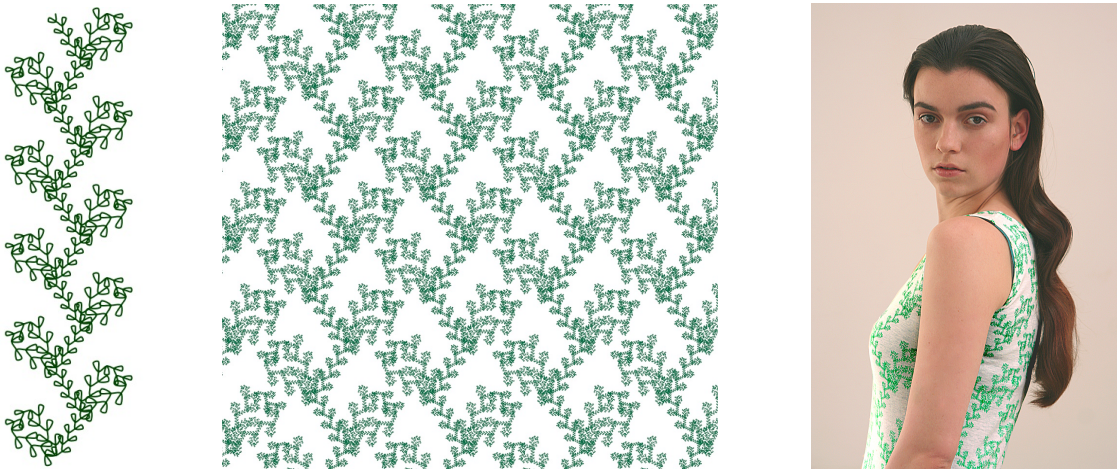


Figure 3: *Spline-based line (left), pattern (center) and lady's dress with pattern of line fractal(right), (Model Charlotte Geeraerts, Make-up artist Lana Houthuijzen, Photographer Katinka Feijs).*

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Figure 4: *Lady's dress with green fractal (Model Charlotte Geeraerts, Make-up artist Lana Houthuijzen, Photographer Katinka Feijs).*