Random Processes and Visual Perception

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Abstract

The object of this presentation is to explore in visual terms a model of recursive thinking applied to a stochastic problem. Stochastic processes are associated with the concepts of uncertainty or chance. They are a major focus of studies in various scientific disciplines such as mathematics, statistics, finance, artificial intelligence/machine learning and philosophy. Visual Art too depends on elements of uncertainly and chance. To explore the commonality of concern between Science and Art and better understand stochastic processes, I used a graph theory reference model called the 'shortest route problem' and added additional elements specific to the art-making process to highlight a specific occurrence of randomness in visual perception.

Introduction

Randomness by nature is challenging to define and is often associated with unpredictability. Greek axiomatic geometry explored the logic of shape, quantity and arrangement. Mathematicians Richard Courant and Herbert Robbins stated that Mathematics offers Science both a foundation of truth and a standard of certainty based on precision and rigorous proof [1]. However, more recently, the theory of probability, to which the concept of random processes is attached, opened mathematical research to broader and more complex investigation in the area of applied mathematics, mathematical physics, mathematical biology, control theory, and engineering. In the visual arts, the perception and appreciation of an artwork depends also on random elements pertaining to light, optical alertness and various other physical and cultural parameters.

To illustrate the mathematical concept in visual terms, I selected a model used by professor Evan D. Porteus for a demonstration of stochastic random processes calculation [2]. I broke down each element of the model (or numbers) into separate objects and recombined them according to the scientific narrative while adding distinct components pertaining to visual communication methodology. Finally, to insure the validity of the process, I informally tested the results with colleagues from the scientific and artistic communities to underscore the common interest that joins scientific and artistic research in this field.

Random processes and mathematics

A random process, also called a stochastic process, is a collection of random variables defined on an underlying probability space. The study of stochastic processes was first attributed to botanist Robert Brown who described the physical trajectories of pollen grains suspended in water [3]. In 1905 Albert Einstein, using a probabilistic model, provided a satisfactory explanation of the Brownian motion. From 1930 to 1960 J. L. Doob and Kolmogorov, transformed the study of probability to a mathematical discipline and set the stage for major developments in the theory of continuous parameter stochastic processes. Probability is mathematics, Doob clearly stated in the preface of his 1953 book 'Stochastic processes' [4]. More recently, Wendelin Werner's research accomplishments in the field of probability led to his being awarded a Fields Medal for outstanding discoveries in Mathematics in 2006.

I selected a series of templates used for a demonstration of model optimization by Professor of Management Science E. Porteus to illustrate in visual terms the mathematical problem solving process. In this example, Dr Porteus approaches random processes from the recursive perspective and decomposes a complex problem into a series of smaller problems, after the Bellman principle called dynamic programming. E. Porteus' demonstration breaks the problem into two parts: first, a study of a recursive model, based on a plan of 4 horizontal and 4 vertical squares. (Fig. 1-A), and second, a similar surface incorporating elements of uncertainty, represented by circles. Square nodes are called decision nodes; circle nodes are named chance nodes. (Fig.1–B).

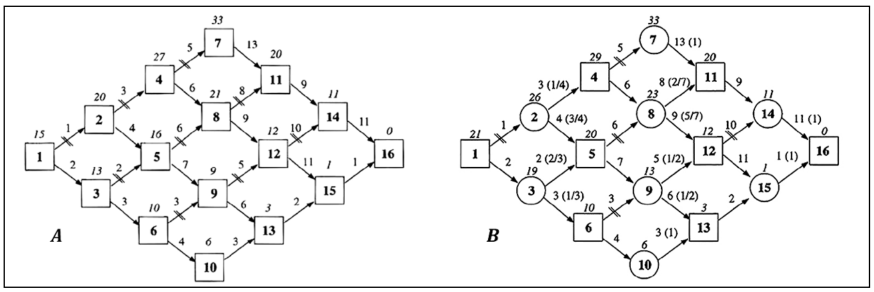

Figure 1: *The shortest route problem. A) Problem solving by simple recursive calculation. B) Introduction of element of uncertainty defined as circles on the template.*

The shortest route problem visualization

I recreated Dr Porteus' "Shortest route" model in a graphic editor software program and traced the outline of the 4 succeeding templates illustrating the mathematical reasoning process in a vector-based application to insure line sharpness and clear definition of each individual object.

The initial black and white design allowed me to focus on the object placement and shape dynamic. I created 3 different copies of the same pattern in 3 different sizes to draw attention to the recursive aspect of the demonstration (Fig.2-A). The resulting surfaces were allocated a distinct identity and set apart from each other by filling them with multiple variation of gray. Each set of square/circles objects was given specific parameters of shades and texture to differentiate one from another (Fig.2-B). Finally, I slightly turned the surface of the main board at a 30° angle on the 'problem solved' template to emphasize Dr Porteus' recommendation to approach the problem in terms of arc rather than straight line to calculate the shortest route (Fig. 2-C).

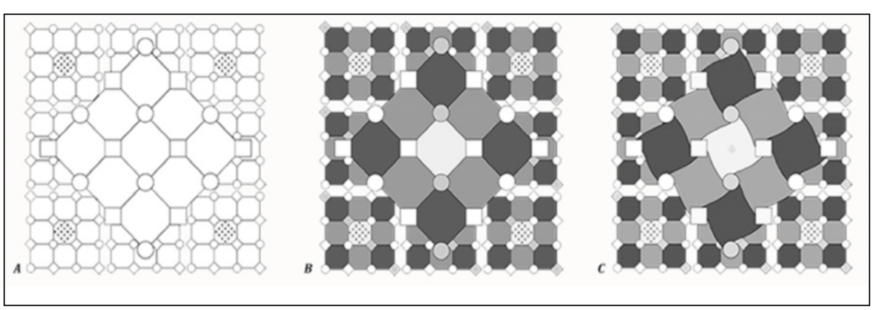

Figure 2: *A) B&W outline. B) Separating the surfaces. C) Tilting the central board.*

I transferred the design into a pixel-based environment to add color to the composition. For that purpose, I selected a specific palette based on Korean traditional color and used the finding of a Shin, Westland survey conducted for the School of Design of the University of Leeds by the Changwon National University [5] to determine the spatial positioning and dynamic interaction of each color in the overall composition (Fig. 3).

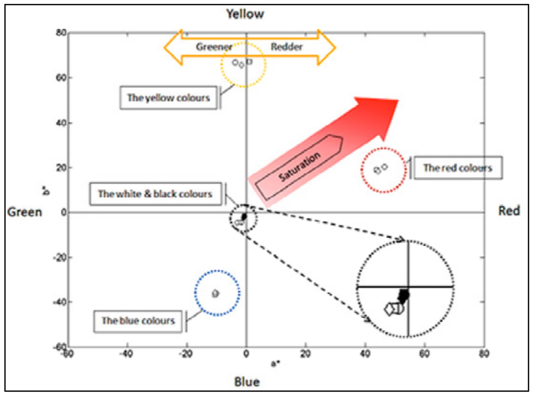

Figure 3. *Shin-Westland spatial color perception survey: preferred (circles), traditional (squares) and trend (diamond) colors in CIELAB space.*

Randomness and visual perception.

To insert an additional element of randomness into the visual statement, I used a variation of an after-image effect first discovered by German scientist L. Hermann in 1870 that highlights the inhibition that neighboring neurons in brain pathways have on each other [6]. (Fig. 4-A).

I increased the size of square #1 (left-center line) of the final model “shortest route problem solved” by 5 points and slightly changed the opacity of color to prompt the eye to subjectively assert what is the minimum distance solution of the stochastic problem. The ensuing design induces the eye to fill black space with white dots and cover white dots black. This effect varies depending on various parameters having to do with visual perception, alertness of the nervous system as well as in this specific case, inference of color size and shape elements (Fig. 4-B).

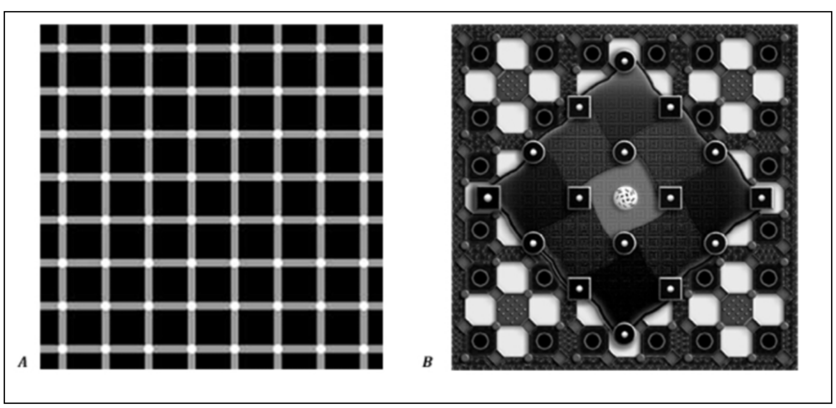

Figure 4. *A) The Hermann grid. Dark patches appear in the section crossings, except the ones that the viewer is directly looking at. B) The shortest route problem – volumes & visual cues.*

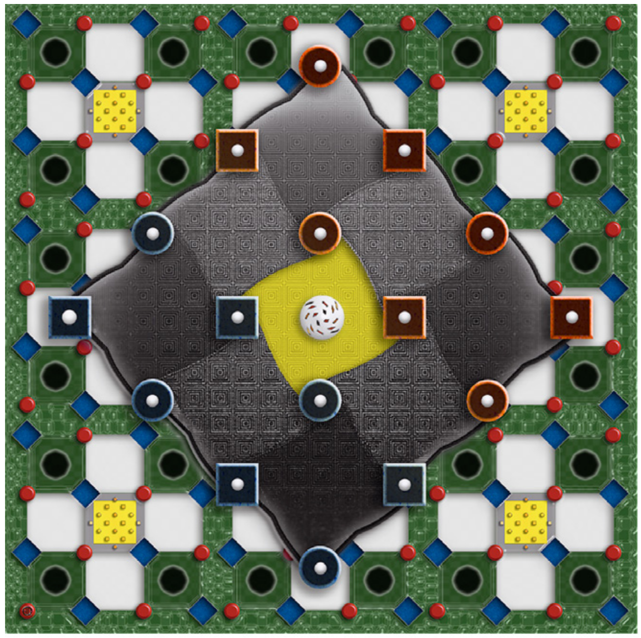

Figure 5. *The 'Shortest route problem solved'. Final composition, color scheme.*

Finally I conducted an informal test with several colleagues, mathematicians, architects and art historians. The consensus was that, once the purpose was explained, it made the problem solving challenging, intriguing but easier and fun to conduct. As an artwork, several mentioned a similarity to kinetic and op art - which also deal with issues of light, color and random after-images effect.

Conclusion

Applications of mathematics in the field of random processes have emerged across the landscape of natural, behavioral, and social sciences, from medical technology to economic planning (input/output models of economic behavior) to genetics and geology (locating oil reserves). More academic and scientific studies will insure a better comprehension of the process. It will also help artists get a better handle on the tools they use to convey their concepts [7] and provide the viewer with a uniquely distinct esthetic experience.

References

[1] Richard Courant and Herbert Robbins, *What Is Mathematics?,* Oxford University Press. 1996.
[2] Evan L. Porteus, *Foundations of Stochastic Inventory Theory,* Stanford University Press. 2002.
[3] M. Scott, *Applied Stochastic Processes in Science and Engineering* Department, of Applied Mathematics, University of Waterloo. ON. 2013.
[4] J. L. Doob, *Stochastic Processes,* John Wiley & Sons. 1953.
[5] Meong Jin Shin, Stephen Westland, Edel M Moore and Vien Cheung, *Colour preferences for traditional Korean colours,* Journal of the International Colour Association. 2012.
[6] John H. Byrne, *Introduction to Neurons and Neuronal Networks*, Department of Neurobiology and Anatomy, The UT Medical School at Houston. 2013.
[7] Jean Constant, *Stochastic Art*, http://www.hermay.org/jconstant/stochasticart/ (as of March 15, 2014)