The Planar Space Groups of Mamluk Patterns

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Abstract

In the region extending from Arabia to Southern Anatolia, elaborate geometric tilings and patterns still cover the walls, floors and ceilings of extant monuments from the Mamluk period (1250 – 1517). These planar Islamic patterns may be classified as belonging to the various two-dimensional space groups based on the distance-preserving transformations or isometries of the plane (translations, rotations, reflections and glide-reflections) that they possess. Classifying the ornamental patterns of these Mamluk monuments allows us to determine which symmetries were preferred and intuitively recognized as being "right" by the Mamluk culture. This paper will discuss and illustrate examples of the different wallpaper groups represented in the Mamluk art found in present day Egypt and Syria.

Introduction

The study of elaborate geometric tilings that cover the walls, ceilings and floors of Islamic monuments holds a particular appeal to those interested in classifying patterns based on the symmetries they possess. Ever since E. Müller reported in her 1944 Ph.D. thesis that she had found 11 of the 17 planar space groups represented at the Alhambra [8], many researchers, including this author [3], have studied the symmetry groups represented by patterns found throughout the Western Islamic world. However, less attention appears to have been paid to similar analyses of geometric tilings in the Eastern Islamic world, notably those of the Mamluks of Egypt and Syria. Many patterns dating to the Mamluk period (1250 – 1517) still exist or have been restored, thus offering a plethora of examples available for analysis. In addition, the author finds Mamluk patterns as a whole to be the most aesthetically pleasing of all the geometric Islamic patterns she has analyzed.

The Mamluks

The mamluks were slaves of Turkish, Mongol or Circassian origin who were bought by the ruling sultans of Egypt to serve as soldiers in their armies. Those that were freed and promoted through the ranks often held positions of great wealth and power in society, as long as their sultan ruled. As a result, the mamluks were intensely loyal to their sultan. "The sultan and his mamluks formed a tightly-knit association, whose members were united by strong bonds of solidarity (thus forming) a sort of double bond: They were in power only so long as he ruled, and he ruled only so long as his power was based on them [1]."

There were two successive dynasties of Mamluks that ruled the region extending from Arabia to Southern Anatolia. The first dynasty was the Bahri, descendents of the Turkish slaves of the last Ayyubid ruler, Sultan Salih. They came to power in 1250 and ruled until 1382 when they were succeeded by the second Mamluk dynasty, the Burji, descendents of the Circassian slaves of the Bahri Mamluk Sultan

Qala'un (1279 – 90). The Burji ruled Egypt and Syria until their defeat by the Ottoman Turks in 1517. During their reign, it was the Mamluks who expelled the Crusaders from the Middle Eastern territories.

According to David Ayalon, "The Mamluks Sultanate was the most important Muslim empire in the latter Middle Ages… Without the Mamluk slave institution and its offshoots, Islam would have never reached its present boundaries, and certainly could not have preserved them for long. Consequently, it would not have become a world religion on the scale and magnitude which it actually attained [1]."

The Mamluks poured considerable sums of money into their building projects many for the benefit of the indigenous population, with numerous fine examples still existing in Egypt and Syria. "The art of the Mamluk period is closely related to the arts and culture of the Ayyubid period. In fact, there is only a very slow and gradual change in the artistic traditions of Western Islam in the Mamluk period [6]."

The elaborate geometric tilings and patterns that cover the walls, ceilings and floors of these Mamluk buildings are of interest to those concerned with classifying the patterns based on the symmetries they possess. Certain pattern types are preferred and intuitively recognized as being "right" by various cultures. So, which of the 17 possible symmetry groups are represented by patterns created in the Mamluk style?" This paper will offer one possible answer by providing the criteria used by the author in classifying the planar space groups, as well as illustrating with examples each of the symmetry groups discovered. Most of the images, that came from online photo archives, such as those of the *Thesaurus Islamicus Foundation Islamic Art Network Photo Archive* [11], the Mamluk Collection of the *Archnet Digital Library* [7], the Islamic Art and Architecture Collection of the *ARTstor Digital Library* [5] and the *Pattern in Islamic Art: The Wade Photo Archive* [9], were cropped for this paper.

It should be noted that there is no indication that the Mamluk artisans made any conscious efforts to classify their patterns according to the symmetries that are present. So why would we be interested in doing this now? Many feel strongly that this is a legitimate avenue to explore, including MIT art historian W. K. Chorbachi who wrote:

> The importance of group theory and its notational system for Islamic art lies in the fact that it provides a tool for exact cataloging of the infinite number of geometric designs used in Islamic art. It is also helpful as an analytical tool in recognizing the symmetry used within a design. … Moreover, it provides a precise language and terminology by which those who are interested in these patterns can communicate precisely with each other about these patterns. All this might seem redundant to the scientists who have been involved in the study of symmetry, yet, for the art historians, it is still an unacknowledged tool [4].

The Criteria Used to Classify the Patterns

All of the Mamluk patterns (some restorations) were analyzed as they exist today, ignoring small imperfections such as minor inaccuracies in the cut tiles, or variations due to human error in laying out the pattern, or damage due to weathering, and so on. Any existing interlacings or medallions containing Arabic inscriptions were taken into account, thus resulting in limited reflection or rotational symmetries of some ornamental patterns. To keep the analysis straightforward and uncomplicated, all of the patterns were considered to be uncolored, thus allowing the focus to be solely on the underlying skeletal structure.

Each pattern had to appear to cover an area of at least three feet by three feet (approximately a square meter) and be *meant* to be a planar wall covering, and not just a "wide" frieze pattern or a small portion within a larger design. It is rare to find large planar patterns in Mamluk decoration. Instead, many show only a few repeat units within a rectangular space surrounded by a plain border, which, in turn, may be surrounded by a frieze pattern. Hence, there are few instances of planar patterns where many copies of the repeat units exist. This leads to the challenge of determining whether a pattern may be considered a planar pattern or not. For this paper, the author was able to find examples that did include at least two repeats in each of two linearly independent directions, except for the two patterns shown in Figures 7 and 9. Each pattern was also analyzed as if the motif repeats indefinitely in all directions, and in tilings, the grout lines were considered to be infinitesimally thin.

A Quick Review of Symmetry Terminology

There are four *isometries* (distance-preserving transformations) of the plane: translations, rotations, reflections, and glide-reflections. A translation is determined by a vector that indicates the direction and distance through which all points are moved. A rotation is determined by a center and an angle; all points are rotated about that center through the given angle. A reflection is determined by a line (called a mirror line) across which all points are reflected. A glide-reflection is a composite transformation determined by a line (called the glide line) and a translation whose vector (called a glide vector) is parallel to the glide line: all points are reflected in the glide line and then translated by the glide vector. A figure in the plane is *symmetric* if there are isometries that move some points of the object to new positions, but leave the appearance of the figure unchanged (*invariant*). The isometries that leave a figure invariant are called *symmetries* of the figure. The *symmetry group* of a figure is the collection of all symmetries of the figure. A repeating pattern in the plane is called *periodic* if it has translation symmetries in two linearly independent directions, and there is a minimum-length translation vector in each of these directions. There are only 17 different space groups of periodic planar patterns, also known as *wallpaper* (or crystallographic) groups. A standard notation for symmetry groups of periodic planar patterns was established by the International Union of Crystallography, and we will identify symmetry groups by the short form of this notation. Explanations of this notation can be found in [10] and [12].

Questions Posed and Answered for Each Pattern

To determine which symmetry groups are represented by the geometric Mamluk patterns, the author posed some of the following questions as patterns were examined. Is there reflection symmetry? If so, are there mirror lines in more than one direction? If so, what is the angle between intersecting mirror lines? Is there rotational symmetry? If so, what is the smallest angle of rotation? Are there rotation centers not on mirror lines? Is there glide reflection symmetry? Do the glide lines coincide with mirror lines? Are there glide lines off the mirrors? Charts and algorithms for identifying the symmetry group of a periodic planar pattern using questions such as these can be found in [10] and [12].

Patterns with No Rotational Symmetry

The four types of patterns with no rotational symmetry are denoted *p*1, *pm, cm*, and *pg*.

The first pattern (Figure 1) is classified as *p*1 since there are no rotational nor mirror symmetries present due to the ‘over-under’ interlacings, and the Arabic calligraphy visible at the top, middle portion of the image (encircled). Thus, the pattern has only translation symmetry. The image (from [11]) is of a carved stucco relief on the upper *qibla* wall of the Ribat-Mausoleum of Shaykh Yusuf al-‘Ajami al-‘Adawi and of Shaykh Muhibb al-Din Abu al-Faraj (also known as the Mausoleum of Mustafa Pasha) [2].

The second pattern (Figure 2 on the following page) is classified as *pg,* taking into account the 'over-under' relationships of the bands. This pattern (*EGY1629* from [9]) has parallel glide lines (two adjacent ones of which are shown in a recreation of the pattern on the right side of Figure 2) but no mirror lines due to the overlapping bands. This image is of a polychrome marble inlay panel from within the *mihrab* (or prayer niche) of the Masjid al-Aqsunqur in Cairo, built in 1346 – 1347 by Amir Aqsunqur, a son-in-law of Sultan al-Nasir Muhammad and former governor of Tripoli, Syria [7]. This mosque (known as the Blue Mosque due to the color of tiles installed there in 1652 – 1654 by an Ottoman amir) is considered to be an exceptional example of early Mamluk religious architecture.

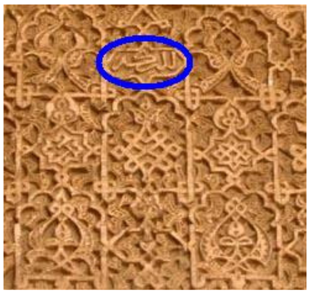

Figure 1. *A* ***p*1** *pattern*

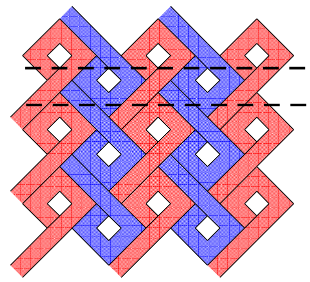

Figure 2. *A* ***pg*** *pattern and a recreation of it on the right*

Two-fold Rotational Symmetry

The five types of patterns with two-fold rotational symmetry are denoted *p*2, *pmm*, *cmm*, *pgg*, and *pmg*.

The third pattern (Figure 3 on the next page) is classified as a *p*2 pattern since it has two-fold rotation centers (as marked) and no reflection or glide reflection symmetry due to the 'over-under' interlacing. This image (*IHC0672* from [7]) shows a polychrome marble inlay panel containing 10-pointed star polygons from the *mihrab* (or prayer niche) of the Masjid Altinbugha al-Maridani in Cairo. Amir Altinbugha was the *saqi* (or cup-bearer) from 1339 – 1340, and a son-in-law of Sultan al-Nasir Muhammad.

The fourth pattern (Figure 4) is classified as *cmm* since it has horizontal and vertical mirror lines through the center of the 14-stars along the perimeter of the figure and two-fold rotation centers between the pairs of two small, adjacent hexagons separating the 14-stars (pairwise), as marked. These centers are not on mirror lines. This figure, containing 14-pointed star polygons, is found as part of a carved and inlaid wooden *minbar* (or pulpit) of the Masjid al-Mu'ayyad, built between 1415 and 1421 by the Mamluk sultan, al-Mu'ayyad Sayf ad-Din Shaykh, one of the great patrons of architecture in Cairo. The madrasa within the mosque became one of the most prominent academic institutions in 15th century Cairo, while the sanctuary was one of the most richly decorated of its time. The image is *EGY 1217* from [9].

The fifth pattern (Figure 5) is classified as *pmg* pattern since it has parallel (vertical) mirror lines in one direction (marked as solid lines), glide lines perpendicular to the mirror lines (dashed lines), and all two-fold rotation centers on glide lines (as marked). This image (*EGY1213* from [9]) shows a polychrome marble inlay panel from within the *mihrab* (or prayer niche) of Masjid al-Mu'ayyad, built in Cairo between 1415 and 1421 by Sultan al-Mu'ayyad (discussed previously in conjunction with Figure 4).

The sixth pattern (Figure 6) is classified as *pgg* since it has glide lines in two perpendicular directions, but no mirror lines. Glide lines make 45^{o} angles with the edges of the bricks (adjacent ones are shown in a recreation of the pattern on the right side of Figure 6). The two-fold rotation centers are not on the glide lines; they are at the centers of rectangles formed by the grid of glide lines (one is marked). This image (from [2]) can be found as a decorative brick wall tiling on the minaret of the Mausoleum of al-Sultaniyya, constructed in the 1350's – 1360's in Cairo, possibly for the mother of Sultan al-Nasir Husan [7]. Sultan

Husan ascended the throne in 1347 at the age of thirteen and was eventually assassinated by the commander-in-chief of his army.

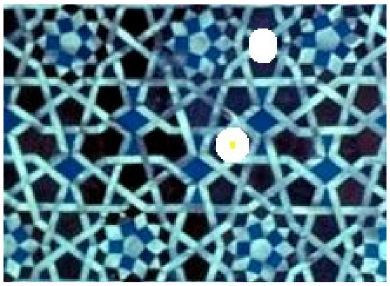
Figure 3. *A **p2** pattern*

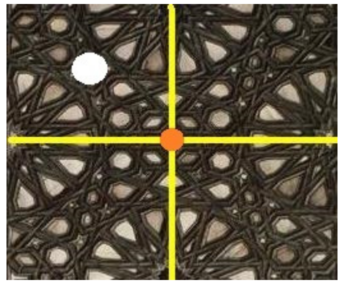
Figure 4. *A **cmm** pattern*

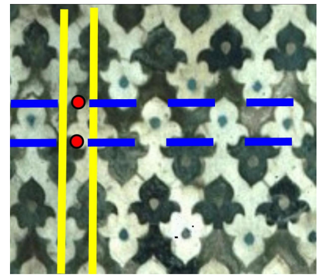
Figure 5. *A **pmg** pattern*

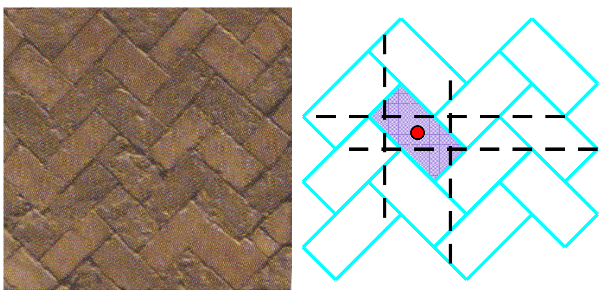
Figure 6. *A **pgg** pattern and a recreation of it on the right*


Figure 7. *A **p3** pattern*

Three-fold Rotational Symmetry

The three types of patterns with three-fold rotational symmetry are denoted *p*3, *p*31*m,* and *p*3*m*1.

The seventh pattern (Figure 7 above) is classified as *p*3 pattern since it has a three-fold rotation center (as marked at the center of the regular hexagon towards the center of the photograph) and no reflection or glide reflection symmetry. This image (*EGY0831* from [9]) shows a carved stone panel under an arch on the exterior of the masjid built in Cairo by Amir Sayf al-Din Qijmas al-Ishaqi. Qijmas was a Circassian Mamluk, who held several positions until he became “Master of the Horse” (the head of the stables) for Sultan Qaytbay in 1479.

The eighth pattern (Figure 8) is classified as *p*31*m* since it has mirror lines inclined at 60 degrees to one another (in three distinct directions), and has three-fold rotation centers *not* on mirror lines (as marked). This image (*SYR0212* from [9]) shows a polychrome marble inlay panel of the al-Madrassa (or religious school) al-Zahiriyya in Damascus, Syria. Baybars I (1260 – 1277), who was famous for the conquest of most of the Middle East and for expelling the Crusaders, was buried in the richly decorated building.

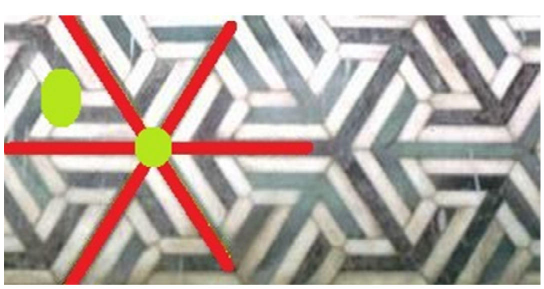
Figure 8. *A **p31m** pattern*

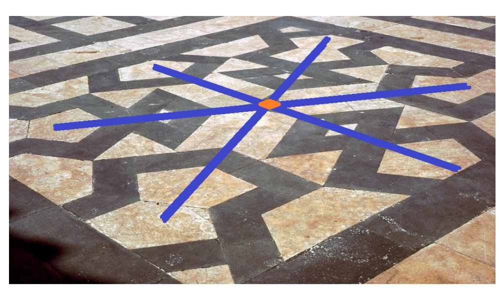
Figure 9. *A **p3m1** pattern*

The ninth pattern (Figure 9 on the preceding page) is classified as *p*3*m*1 since it has mirror lines inclined at 60 degrees to one another (in three distinct directions), and has three-fold rotation centers *only* on mirror lines (one is marked). This image (*SYR0517* from [9]) shows a floor tiling in the courtyard of the Great Mosque of Aleppo, Syria. This mosque is the largest, and one of the oldest, mosques in Aleppo, dating back to the 8th century. However, it was rebuilt by the Mamluks after it was razed by the Mongols in 1260.

Four-fold Rotational Symmetry

The three types of patterns with four-fold rotational symmetry are denoted *p*4, *p*4*m,* and *p*4*g*.

The tenth pattern (Figure 10) is classified as *p*4 since it has only four-fold (marked as a large oval) and two-fold rotation centers (marked as a smaller oval). This image (*DB 1039421936* from [5]) shows a portion of a square kufic-style calligraphic inlay panel in the Complex of Qala'un, founded by Sultan al-Mansur Qala'un in 1284 - 1285. Qala'un was father of Sultan al-Nasir Muhammad, whose thirty-two-year third reign was the most prosperous and stable of the Mamluk period. Qala'un was also the grandfather of al-Nasir Hasan (discussed previously in conjunction with Figure 4).

The eleventh pattern (Figure 11) is classified as *p*4*m* since it has mirror lines inclined at 45 degrees to one another (in four distinct directions) and all rotation centers lie on mirror lines (one is marked). This image (from [11]) is found as part of a carved wooden panel containing 5- and 12-pointed curved stars with ivory inlay on the *minbar* (or pulpit) of al-Ghamri in the funerary complex or Khanqah wa-Masjid of Sultan al-Ashraf Barsbay. Presented as a gift in 1453, the *minbar* is considered a Mamluk masterpiece.

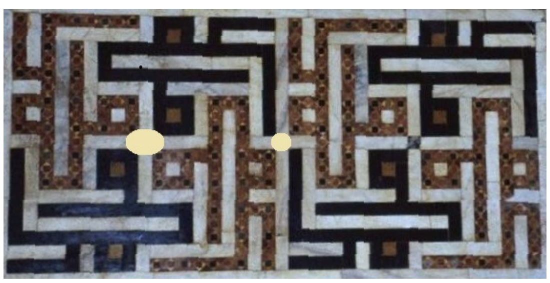

Figure 10. *A* ***p*4** *pattern*

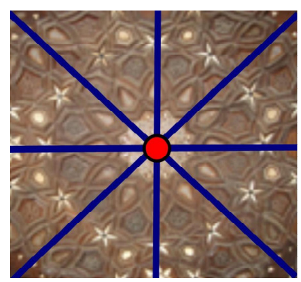

Figure 11. *A* ***p*4*m*** *pattern*

The twelfth pattern (Figure 12 on the following page) is classified as *p*4*g* since it has mirror lines in two perpendicular directions (these cut through the centers of the arrow-like motifs). Two fold rotation centers lie on mirror lines (marked as a small dot) and the four-fold rotation centers do not; they are located at the centers of the small squares (one is marked as a larger dot). This image (*ICR1786* from [7]) is found as a polychrome marble inlay panel of the Haram al-Ibrahimi in Hebron, Palestine. "The Haram al-Ibrahimi (also known as the Sanctuary of Abraham) is considered to be the fourth most important religious site in Islam (after Mecca, Medina and Jerusalem)" [13]. It was renovated by the Mamluks Sultan Zahir Barquq and Amir 'Alam al-Din Sanjar al-Jawali.

Six-fold Rotational Symmetry

The two types of patterns with six-fold rotational symmetry are denoted *p*6 and *p*6*m*.

The thirteenth pattern (Figure 13 on the following page) is classified as *p*6 since it shows six-fold rotational symmetry (a square marks a rotation center) and has no mirror lines. There are also two-fold and three-fold rotation centers. This image (from [11]) shows a decorative panel above a doorway (lintel) of carved masonry at the Mosque of Amir Azbak al-Yusufi, built in1494 - 1495. Yusufi was one of the "great amirs of the sword" and held several high government positions [7].

The fourteenth pattern (Figure 14) is classified as *p*6*m* since it has both six-fold rotation centers and mirror lines. Six mirror lines meet at the six-fold rotation centers and are inclined at 30 degrees to one another (in six distinct directions, as marked). There are also two-fold and three-fold rotation centers; these are located at intersections of mirror lines. This image (from [11]) shows a portion of an inlaid wooden panel containing 12-pointed star polygons on the *minbar* (or pulpit) of the Mosque of Qijmas al-Ishaki (discussed previously in conjunction with Figure 7).

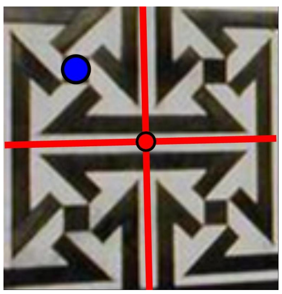

Figure 12. *A* ***p4g*** *pattern*

Figure 13. *A* ***p6*** *pattern*

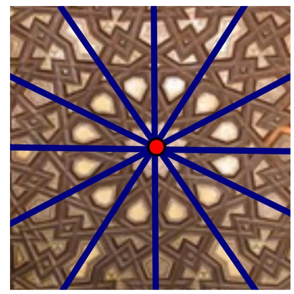

Figure 14. *A* ***p6m*** *pattern*

Discussion

The author was able to locate patterns representing 14 of the 17 groups on Mamluk monuments constructed between 1250 and 1517, if one accepts that the patterns shown in Figures 7 and 9 truly represent planar patterns that fit our inclusion criteria. Recall that in Figure 7, the 3-fold pattern within the 'rounded' triangle consisted of only three copies of an arabesque motif. And, in Figure 9, only a small portion of another 3-fold pattern with mirror symmetry is shown in the floor panel. Hence, it could be argued that these two examples do not fit our inclusion criteria and thus not represent legitimate planar patterns.

Otherwise, the three pattern types not found on Mamluk monuments constructed between 1250 and 1517 include two of the four types with no rotational symmetry (*pm* and *cm*) and one type with two-fold rotational symmetry, *pmm*. Despite searching carefully through many hundreds of images available from books and digital libraries ([11], [7], [5] and [9]), the author could not locate any of these pattern types. Examples of these pattern types might have been missed by the author or might have existed during medieval times but have since been destroyed. However, examples of these three pattern types do exist as ornament considered by art historians as created in the "Mamluk style" on structures built after 1517 by Mamluk amirs living under Ottoman rule. The images and pattern types were officially excluded from this study, but are discussed below.

Figure 15 is classified as *pm* since it has mirror lines and glide lines only in one (vertical) direction, and all glide lines are on mirror lines (as marked). The image (from [11]) of the carved masonry panel over which the water flowed at the *sabil* (or public water fountain) and *kuttab* (Qur'anic school for boys) of Sulayman Bey al-Kharbutli, a Mamluk amir under the Ottomans, built in 1744 [7].

Figure 16 is classified as *cm* since it has (vertical) mirror lines (marked as *solid* lines) and glide lines (marked as a *dashed* line) halfway between adjacent mirror lines and no rotational symmetry. This image (from [5]) shows a stone screen of the window on the south façade of the Sabil-Kuttab (public water fountain and Qur'anic school for boys) of 'Abd al-Rahman Kathkuda, another Mamluk amir under Ottoman rule, built in Cairo in 1744.

Figure 17 is classified as *pmm* since it has mirror lines in vertical and horizontal directions, and all two-fold rotation centers occur at the intersections of mirror lines (as marked). This image (from [11]) shows the door of the mausoleum of Amir Kathkuda (discussed previously in conjunction with **Figure 16**) at the al-Azhar Mosque, built in Cairo in 1753.

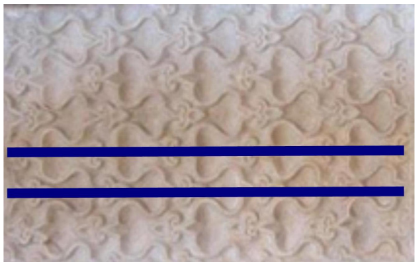

Figure 15. *A* ***pm pattern***

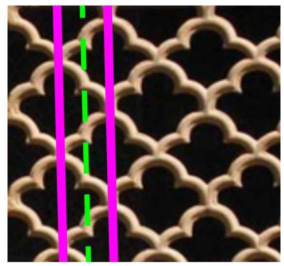

Figure 16. *A* ***cm*** *pattern*

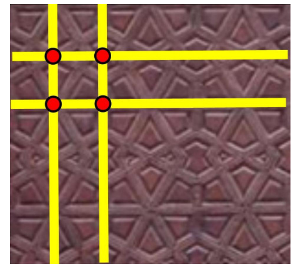

Figure 17. *A* ***pmm*** *pattern*

Acknowledgments

This work was supported by a Monmouth University 2013 Summer Faculty Fellowship grant for which the author is sincerely grateful. The author is also indebted to the reviewers for their useful comments and suggestions for improvement.

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