Cardioidal Variations

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Abstract

The cardioid is a simple and elegant curve. It can be seen in a pan or in a child's milk bowl. For many people, the cardioid is perhaps the first encounter with sophisticated mathematics. A cardioid can be obtained by different methods. Implementing those methods is a good programming exercise, which rewards one with a pleasant design. It is also the basis by which one can modify code to obtain a rich family of three dimensional objects whose diversity is still being explored.

Introduction

A cardioid is a plane curve. Its name comes from its heart-like form. It can be defined in different ways. I used two of those constructions to define three-dimensional objects easy to describe, easy to 3D-print, which offer a infinite variety of attractive simple shapes. The paper is organized as follows: we first describe the methods, then algorithms that give rise to three-dimensional variations, and finally some artistic applications.

One Curve, Two Methods

Pedoe [3] describes a method for constructing a cardioid as the envelope of a set of circles:

- Draw a circle and choose a point on its circumference.
- Draw circles with centers lying on the initial circle, and passing through the chosen point.
- The envelop of this set of circles is a cardioid (see Figure 1).

Another way to draw a cardioid is given by the following algorithm, which we can call the *String method*, whose result is shown in Figure 2 :

- Draw a circle.
- Around the circumference of this circle, draw n equidistant points.
- Numbering those points $1 \dots n$, draw a line between points i and $(2 \times i) \, mod(n)$.

For both methods, the resulting figure lies in a plane, but one may prefer to obtain three dimensional objects, in order, for example, to add some diversity in the shape, hoping to find some surprising points of view.
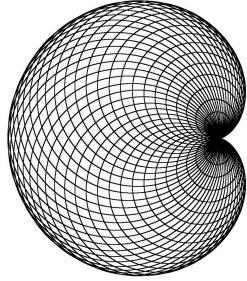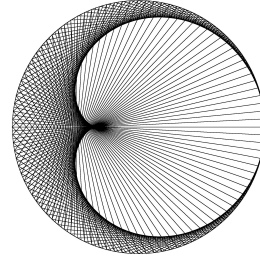
Figure 1 : *Pedoe method for cardioid*

Figure 2 : *String method for cardioid*

Variations

In the case of the string method, one can replace the line segments with circles, orthogonal to the plane of the design, with diameter equal to the segment length; the resulting object is shown on Figure 3. This extension of the original construction method is still not satisfactory, since it only gives a single new object. One would prefer an algorithm generalization that leads to an infinite family of new shapes. This kind of generalization is possible with the algorithm deduced from the Pedoe method: instead of simply drawing the circles in the same plane as the generative circle, apply some rotation to those circles. One can define this rotation as a function of the position of its center on the original circle. Each definition of such a function creates a new shape. Moreover, the simplicity of the definition makes it easy to describe shapes in a format suitable for sending to a 3D printer. Figures 4 to 6 show examples of real world achievements of this process. Since it is easier, cheaper and quicker to render than to print three-dimensional objects, one can generate an infinity of variations in terms of two-dimensional rendering, and experiment with this new universe of shapes. Figure 7 can be seen as a catalog of three-dimensional candidates.
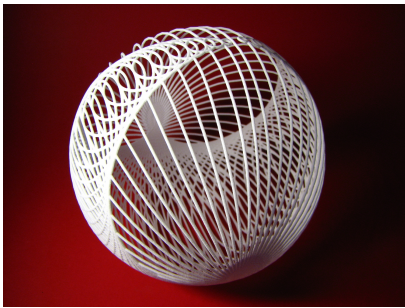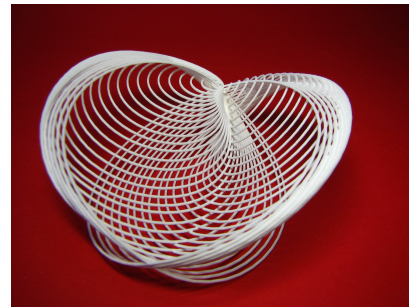

Figure 3 : *Variation from string method*

Figure 4 : *Variation from Pedoe method*

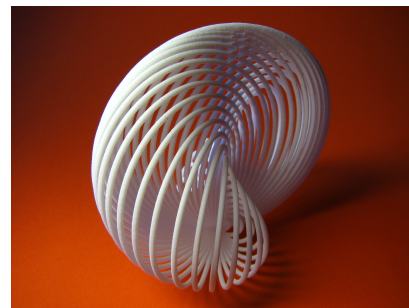**Figure 5** : *Variation from Pedoe method*
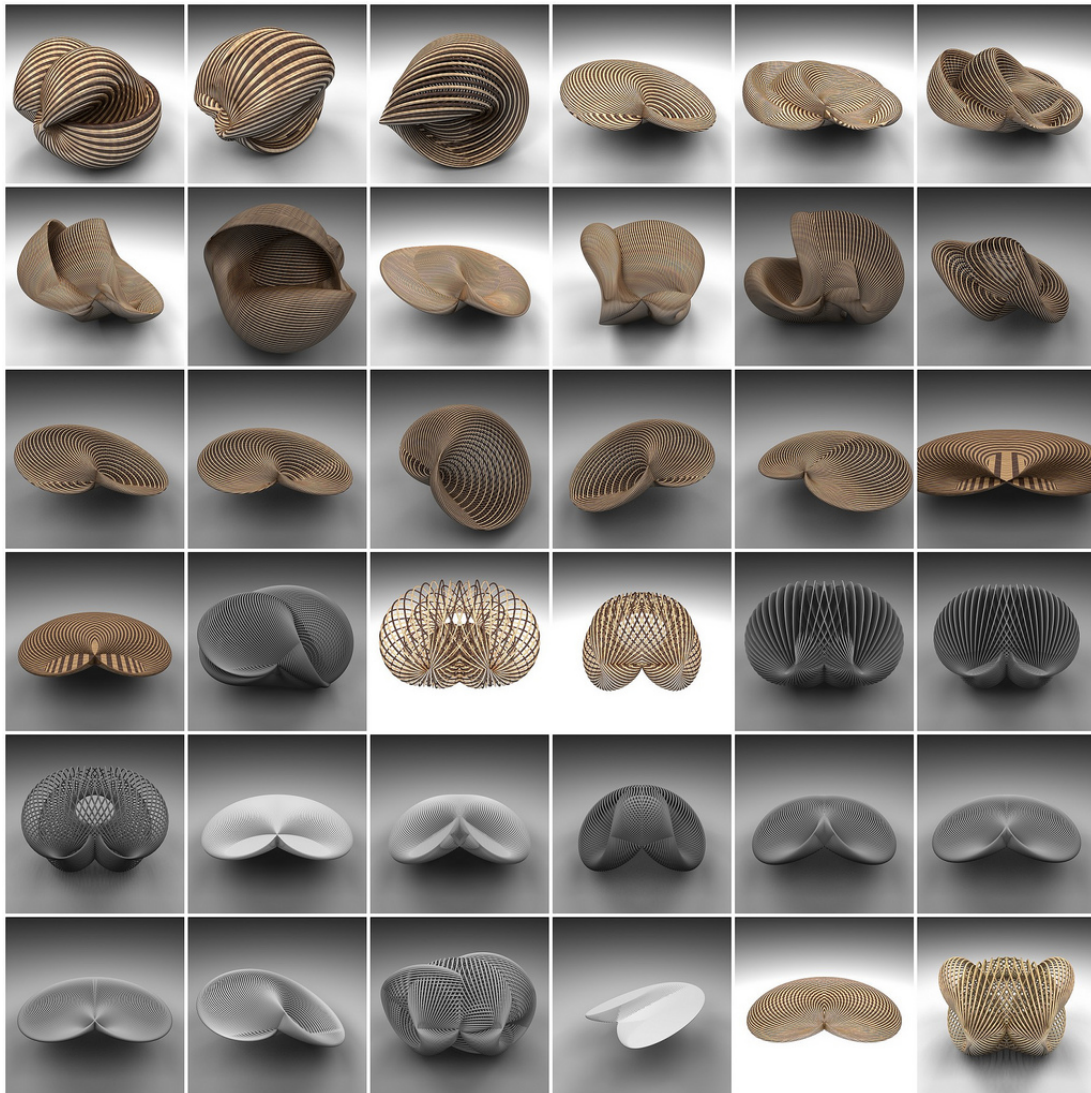

Figure 6 : *Variation from Pedoe method*

Figure 7 : *A catalog of cardioidal variations*

Art Appeal and Applications

I worked on cardioidal variations as I usually do with other mathematical definitions and objects: first I tried to program the algorithm and obtain some renderings, then alter the original code to generalize it and obtain some interesting variations [1]. Quite surprisingly for me, the cardioidal variations seemed to have attracted several people, and among them some artists from different fields of art, who contacted me to speak about some possible collaboration. At this time (March 2014), two of those contacts led to concrete achievements:

- The rock group Marillion asked me for permission to use the cardioidal variation depicted on figure 6 as the symbol of their last record (Sounds That Can't Be Made) (see Figure 9)
- London's milliner Gabriela Ligenza [2] asked me to collaborate with her on the definition of two hat prototypes. We choose two models, one of them being the variation on Figure 5, the other one an original model. Both models have been adapted for millinery (Figures 8 and 10, pictures by Josh Shiner).
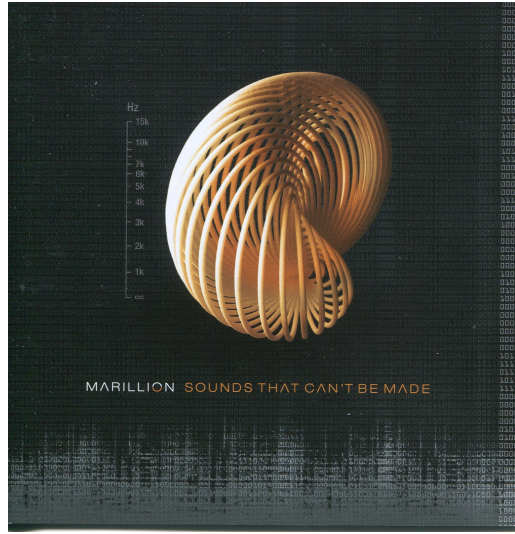
Figure 8 : *G. Ligenza*   Figure 9 : *Marillion record*   Figure 10 : *G. Ligenza*

Conclusion

The computational power of modern personal computers, the performance of graphic cards, the resolution of computer screens, the availability of sophisticated programs (ray-tracing software, 3D renderers), innovative programmable machines (laser cutters, 3D printers) allow the dreams of mathematicians to become real: imagine objects, build their representation, manipulate them. Moreover, they can go forward and imagine new objects derived from the 'standard' ones. Ousting those objects from their labs, exhibiting them to the public, create new connections between mathematicians and artists, strengthening their mutual collaboration.

It is often difficult to define what beauty or aesthetics is. In the light of this example, an answer might be that the underlying simplicity of the definition, together with the complexity of the generated universe of shapes, could be an important part of what makes artistic appeal. Occam's razor principle and Kolmogorov complexity might be called to the rescue. . .

References

[1] Francesco De Comité. Circle Packing Explorations. In George W. Hart and Reza Sarhangi, editors, *Proceedings of Bridges 2013: Mathematics, Music, Art, Architecture, Culture*, pages 399–402, Phoenix, Arizona, 2013. Tessellations Publishing.

[2] Gabriela Ligenza. Gabriela Ligenza's website. `http:www.gabrielaligenza.com` (accessed 15/04/2014).

[3] D. Pedoe. *Geometry and the Visual Arts*. Dover Books on Mathematics Series. Dover Publications, 1976.