What is a Pattern?

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Abstract

The use of the word 'pattern' is ubiquitous in written and spoken discourse. Furthermore, most authors assume the intended reader knows what a pattern is, and hence do not define the term. We present an exploration of the variety of definitions of 'pattern' used in different domains of mathematics, the sciences, the arts, and the humanities, with the goal of searching for an overarching definition of pattern or at least a metapattern in the sea of definitions. Due to space limitations this paper is necessarily an incomplete survey of how the term *pattern* is used in different fields, but it provides a sample from which the interested reader may explore the topic further, and it points a direction in which to search for a practical computational approach to measure the amount of pattern contained in an entity.

Introduction

In 1950 Norbert Wiener [53], the father of cybernetics, wrote: "One of the most interesting aspects of the world is that it can be considered to be made up of patterns." In his book *The Nature of Order*, Christopher Alexander identified pattern as a "fundamentally informative characteristic of life" [1]. Indeed, it may be argued that evolution depends in part on the existence of regularity in the environment [44]. It is thus fair to say that without patterns life would not merely be meaningless, but would probably not exist at all. Not surprisingly the word pattern features prominently in virtually all domains of knowledge, but even so, most books and articles which contain the word assume that the reader must be familiar with its meaning, and hence do not bother to define it. According to the Oxford Dictionary the word pattern originates from the Middle English word *patron* 'something serving as a model,' from the Old French word *PATRON*, which in turn comes from the Latin words *patronus* (*protector*) and from *pater* (*father*). But what exactly is a pattern? Would a mere single point deserve to be called a pattern? The painter Wassily Kandinski affirms that a point can be considered to be a work of art [22], but a work of art need not be a pattern. Would you consider a straight line to be a pattern? That a straight line is an important symbol in our lives is unquestionable. In numerical calculations a short straight horizontal line (the minus sign) has been used to denote subtraction perhaps since the time of Heron and Diophantus of Alexandria around A.D. 300 [3], and in the Chinese written language it is the character for the number one [39]. Defending the importance of the straight line in the discipline of pattern design, Amor Fenn [10] writes: "The straight line is seldom appreciated as a factor of effect. In early essays in design, it is often regarded as mechanical, and it plays a very second fiddle to curves, which perhaps naturally appeal to the neophyte as more ornamental".

Multi-disciplinary Definitions of Pattern

One of the most general definitions of pattern often encountered in the literature specifies it as "the opposite of chaos" [2]. Since chaos is characterized by complete disorder, a pattern must then exhibit order. A straight line is clearly an exemplar of order, and thus would have to be considered as a pattern according to this definition. On the other hand, Norbert Wiener describes a pattern as "essentially an arrangement. It is characterized by the order of the elements of which it is made rather than by the intrinsic nature of these elements"[53]. Similarly, Frank Papentin [34] defines a pattern as "a certain number of objects connected together by a certain number of relationships," and David Wade [52] argues similarly that a pattern must consist of at least two elements: "two like objects, in no particular relation with each other, are merely similar (since although they may be congruent they are not arranged in any order). The addition of a third object allows a degree of regularity to come into play, creating the basis of a recognizable pattern." These two definitions imply that a pattern is made up of a collection of distinct objects of some sort, rather than just a single component such as a single line. Thus two arrangements, one consisting of parallel lines, and another of concurrent lines constitute bona fide patterns under Wiener's definition. The arrangement of parallel lines is in fact pattern No. 255 (p. 105) in Amor Fenn's book, which he characterizes thus: "This constitutes pattern, but is somewhat monotonous in effect. The interest is enhanced by the omission of every third line." These two patterns also fall in the category of *open* patterns defined by Ulf Grenander [17], [18].

The Oxford dictionary offers two slightly less general definitions of a pattern. The first is "an arrangement or sequence regularly found in comparable objects or events." The second is "a regular and intelligible form or sequence discernible in certain actions or situations." These definitions are still general enough to include patterns perceivable with any of our five senses, but they narrow the scope by characterizing patterns in terms of intelligible sequences. Needless to say, the two arrangements defined above are intelligible (ordered) sequences of elements: lines ordered by height and by slope.

Psychologists use the term pattern in more restricted senses. Richard L. Gregory [16] defines a pattern as "some set of inputs, in space or time, at the receptor." The Online Psychology Dictionary [55] defines a pattern as "a temporal or spatial arrangement of independent components to make an involved whole." In these two definitions patterns are confined to the domains of time and space. Musical rhythms produced by acoustic stimuli are prototypical examples of temporal patterns, although notated music typically represents the desired acoustic signals as visual two-dimensional spatial patterns [46]. The two arrangements discussed above are examples of spatial patterns. However, the use of the words 'independent components' and 'involved whole' in the latter definition is problematic. If by 'independent components' it is meant that the components may not touch each other, then the arrangement of concurrent lines is not a pattern. If by 'independent components' it is meant that the components are generated independently of each other, then neither arrangement is a pattern since both are generated according to a precise rule, each line completely dependent on the previously generated line. Furthermore, the term 'involved' suggests that the whole must be complex in some way. However, most people would consider the two arrangements of lines to be exceedingly simple.

Similar terms often used for the word pattern are *shape*, *form* and *Gestalt*. The psychologist Garner used the term pattern to mean gestalt [13], and writes "In general terms, a gestalt is a form, a figure, a configuration, or a pattern." The Oxford dictionary defines form as "the visible shape or configuration of something." The psychologist Gibson argues in his paper titled *What is a Form?* that much more precision is needed in the definition of such terms if they are going to be useful. He laments the fact that "the term form is used by different people to mean different things and by the same person to mean different things on different occasions." According to Gibson "shape, figure, structure, pattern, order, arrangement, configuration, plan, outline, contour are similar terms without distinct meanings. This indefinite terminology is a source of confusion and obscurity for philosophers, artists, critics, and writers. It is an even more serious difficulty for scientists and psychologists." He goes on to propose a number of explicit definitions of visual forms and to discuss their relevance to the study of the perception of form [14] (see also the discussion by Nagy [31]).

In the designer worlds of interior decoration, textiles [29], and fashion, the words pattern and design are often paired together, and here pattern is defined in rather specific ways [7], [12], [19]. The Random House College Dictionary lists the principal definition of pattern as: "a decorative design, as for wall paper, china, or textile fabrics, composed of elements in a regular arrangement." To pattern designer Lewis Day [6], the word "pattern" denotes an ornament, and "especially ornament in repetition. Pattern is, in fact, the natural outgrowth of repetition." Designer, Amor Fenn [10] (p. 104) writes: "The essence of pattern is repetition, and in many forms of decorative work this is ensured by the process of production, as for instance in the case of wall-paper and textile designs. These, whether printed or woven, consist of a mechanically repeated unit which is relatively only a small portion of the area." Figure 1 shows an example of a Greek vase border pattern from Amor Fenn's book, consisting of a sequence of pointed heart-shaped ornaments above another sequence of upside-down heart shapes. Gibson would consider ornaments to be patterns, and thus border patterns such as those in Figure 1 are in fact metapatterns, that is patterns of patterns. Indeed, the concept of border itself is a metapattern in the grand scheme of patterns in nature and culture [51], [50]. Border patterns are called *frieze* patterns in the mathematical literature. Furthermore the categorization of patterns of repetition in terms of frieze groups (or wallpaper groups in the case of two dimensions) has been extensively developed [5].

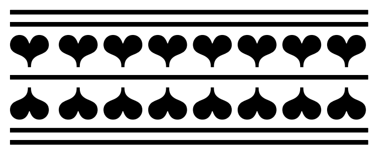

Figure 1 : *Greek vase border pattern listed in the book on abstract pattern design by Amor Fenn.*

A property often ascribed to a pattern is a distinguishable regularity, in which elements of the pattern repeat in a predictable manner. Nikos Salingaros for example considers regularity to be a key property of a pattern [38]. According to Richard Proctor [36], "Virtually any shape or line repeated often enough will produce pattern of some sort because pattern, by definition, results from the repetition of an element or motif. The system of distribution and the relative detail of the motif determine the apparent complexity of a given pattern. Complexity, however, is no guarantee of quality." Almost all the definitions from the field of pattern design, like those above, characterize patterns by employing the property of repetition: the elements of the pattern (such as the heart-shaped motifs in Figure 1) must be repeated to produce an overall pattern.

The visual patterns described so far may be considered as concrete in the sense that they are readily observed with our sense of sight. There are in addition to these, abstract patterns, particularly in the field of mathematics that may be discovered and understood only by means of analysis [43]. Many scholars have emphasized the importance of patterns in the field of mathematics [37]. Doctor Peterson from *Ask Dr. Math* at *The Math Forum*, writes that from the mathematical point of view, the term "pattern is a vague word without any clear definition" [56]. On the other hand, the Internet Web page *Math is Fun* defines pattern as: "things that are arranged following a rule or rules" [57]. In the words of Keith Devlin [9], "mathematics is the science of patterns, and those patterns can be found anywhere you care to look for them, in the physical universe, in the living world, or even in our own minds." The physicist Richard P. Feynman [11] offered the following more succinct definition: "Mathematics is looking for patterns." Mathematician Godfrey H. Hardy [20] wrote: "a mathematician, like a painter or a poet, is a maker of patterns."

One type of pattern searching (or making) that mathematicians explore in depth concerns patterns contained in infinite sequences of elements, in terms of the rules (or formulas) that generate these sequences [42]. Consider the infinite sequence of elements (motifs, or patterns) in Figure 2. Can you, by means of analysis, deduce the missing motif that should be inserted in the place of the question mark (?), or for that matter, all the motifs that should come after the triangle? Ulf Grenander [17] (p. 4) suggests that a pattern is a structure "generated from rules to produce a regular appearance or behavior." So the problem here becomes that of discovering the rules that produce the sequence.

Figure 2 : *An infinite sequence of geometric motifs.*

One line of reasoning and organizing principle might be a repeated 3-pattern of motifs composed of: straight lines, straight and curved lines, and only curved lines. Since the seventh motif consists of only straight lines, the sixth motif should consist of only curves, and thus could look something like the upside-down heart in Figure 3. The discovery of a pattern in this sequence entails the identification of a rule that explains or predicts the sequence. Nolan's definition of pattern [33] lists two necessary and sufficient conditions for such a specification: "*p* is a pattern if, and only if: (1) *p* has an organizing principle, and (2) the organizing principle entails repetition." One plausible organizing principle evident in the sequence in Figure 3 consists of the order and type of lines (straight or curved) present in a segment of three consecutive elements. Furthermore, this first 3-element segment is repeated. Hence this is one possible candidate for the pattern contained in this sequence, valid under Nolan's definition. However, the upside-down heart could just as aptly be replaced with a circle, without violating either the organizing principle or the property of repetition. Moreover, other organizing principles may also 'explain' these two sequences. For example, the groups of three adjacent motifs may be described as having diagonal lines (first, fourth, and seventh motifs), horizontal lines (second and fifth motifs) and neither (third and sixth motifs), respectively. Even more unsettling is the fact that these two organizing principles fail to predict with exactitude all the infinite motifs that should follow the triangle. A good pattern (rule) should be able to predict uniquely the entire sequence of motifs, but the rules posited above admit too much freedom of choice. In other words, these organizing principles foster pattern creation or generation, more than they promote pattern discovery.

Figure 3 : *One feasible solution for the sequence problem.*

On the other hand, consider the sequence shown in Figure 4. The organizing principle of this sequence predicts unequivocally all the infinite motifs that follow the triangle. Furthermore, in the sequence generated by this pattern there is no repetition at all, every motif if different from every other motif. Therefore the second condition listed by Nolan in his definition of pattern is superfluous in this context. The sequence in Figure 4 is determined by one of the most compelling and fundamental of all mathematical rules. Thus an organizing principle is sufficient to define pattern in this sequence of motifs. It is left as an exercise for the reader to discover the correct rule. (Hint: use number theory.)

Figure 4 : *The correct solution for the sequence problem.*

One issue that invariably comes up in matters of perception, particularly that of either visual or auditory patterns, is whether the pattern is the external stimulus itself or some other percept residing in the mind of the perceiver. Is the pattern objectively observable and measurable or is it a subjective experience. That human beings sometimes do not see a stimulus that is present, and at other times see patterns that do not exist outside of the perceiver are well known occurrences. A given stimulus pattern may be perceived as different patterns depending on the context in which the stimulus pattern ocurrs [47]. The phenomenon of finding meaningful patterns in meaningless noise is called *patternicity*, and conversely, not perceiving patterns that are present in the visual stimulus is called *apatternicity* [40]. However, the phenomenon of perceiving patterns where there are none in the stimulus need not be restricted to meaningless noise. Even structured patterns may elicit such perceptions. Such *subjective contours*, as they are called, are created by the perceptual mechanisms in our brains [54]. They are in fact hallucinations normally experienced by everyone. William Gibson has written a novel that revolves around the human natural inclination to perceive patterns, and the resulting risks and consequences [15].

Much discussion on patterns focuses on geometric patterns, and therefore a word is in order concerning what is meant by this term. Concerning geometry the ancient Greek philosopher Plato [58] considered that "God ever geometrizes." Two millennia later the astronomer Johannes Kepler [41] wrote: "Where there is matter, there is geometry." Since matter is ubiquitous, one could argue that all things are patterns, and that all patterns are geometric. Using more specific terms Terry W. Knight [24] offers a definition of pattern based on regularity and transformations of geometric structures: "A pattern is a set of spatial elements: points, lines, planes, or volumes, in two or three dimensions."

Randomness, Complexity, and Pattern Measures

At present there is an on-going debate among philosophers about whether patterns such as illusory contours are real or imagined [32]. However, this is a thorny issue best left to the philosophers to untangle. A more down to earth issue is whether a random configuration of points deserves to be called a pattern. Are lack of pattern and randomness the same concept? Is perceived non-randomness as opposed to true non-randomness enough to make something a pattern? According to philosopher Daniel Dennett [8] "Where utter patternlessness or randomness prevails, nothing is predictable." In a discussion on randomness it helps to differentiate between the *process* used to generate the patterns and the resulting *product*. A completely random process may in fact quite easily generate patterns that are highly structured. Consider flipping a coin sixteen times to obtain a pattern of heads (H) and tails (T). A pattern such as HTHTHTHTHTHTHTHT, which is very highly structured, is just as likely as a pattern such as HTHHHTHTTTHHHHHT, which is very irregular. Furthermore, in the latter irregular pattern clusters of consecutive heads or tails are easily perceived, which also contribute to the perception of pattern. Similar behavior is apparent with dot patterns in two dimensions. The human visual system is easily able to extract structural information from random sets of dots [49]. In this context experiments have demonstrated that the dots that lie on the periphery (convex hull) of the set play a prominent role in providing shape information about the entire collection [21].

There have been some attempts to equate the amount of pattern possessed by an entity, with its complexity. Daniel Dennett [8] suggests that patternlessness and randomness are equivalent. In traditional information theory complete randomness is considered to be equivalent to maximum complexity [4], [25], where the complexity of a pattern is measured by its shortest description in some language (minimum description length). Some researchers have tried to define mathematical measures of *aesthetics*, which may be considered to be measures of pattern. Klinger and Salingaros [23] propose a numerical measure for evaluating the aesthetic interest of simple patterns. Their measure depends on two parameters of the pattern: the number of different types of elements, and the number of symmetries in their arrangement. These ideas suggest one possible definition of pattern: an entity has pattern if its minimum description length is less than the length of the entity itself. However, an entity with minimum complexity does not necessarily have pattern. A string of identical symbols may be considered to be too regular to have pattern. Some musicologists would say that a string of equally spaced pulses is not a rhythmic pattern but a mere pulsation [46]. Something more is need to measure pattern, and Papentin's measure of complexity [34] is relevant here. Papentin starts from the tentative definition that the length of the shortest possible description of a system is a measure of its *disorder*. Then he defines complexity in terms of two differentiated types: *organized* and *unorganized* complexity. Organized complexity, denoted by C_{org}, is measured by the length of the minimal description of the rules that determine the formation of the pattern. Unorganized complexity, denoted by C_{unorg}, is measured by the length of the minimal description of the random aspects of the pattern (the aspects of the pattern that cannot be described by rules). Papentin's measure of complexity is given by the equation $C = C_{org} + C_{unorg}$.

There is a very large number of possible ways to measure organized complexity or disorder. In practice it is impossible to be sure that all methods have been considered in order to get at the minimum description length. To obtain a practical method that yields an upper bound on the minimum description length one must restrict the search to a few easily computable properties of the entity being measured. An approach taken by Krüger [26], and Papentin, and Krüger [35] is to define order in terms of *homogeneity* and *symmetry*. Krüger defines homogeneity as the amount of invariance of a binary sequence under its cyclic permutations, and symmetry as the amount of invariance of a binary sequence under the cyclic permutations of its inversion. Both of these properties are easy to compute, and thus practical. Symmetry may be measured either globally, as Krüger does, or more generally in terns of sybsymmetries [1], [48]. Subsymmetries are more relevant to pattern perception that global symmetries because they are sensitive to a hierarchy of symmetries. Furthermore subsymmetries encompass both global symmetries and local symmetries. However, although either of these measures is able to pinpoint human judgments of simplicity, they are more ambiguous when it comes to complexity [45]. It appears that humans can easily agree when patterns are simple, but less so when patterns are complex.

Conclusion

The very general definitions of pattern offered by some authors may tempt one to conclude that every object, real to imagined, is a pattern. However, a property that emerges from the more specific and useful definitions outlined is that of repetition. A pattern must have something that is repeated either exactly or according to recognizable transformations such as mirror symmetry [27], [28]. In his study of the definitions of shape, George Nagy [31] concludes that “the very notion of shape appears to be amorphous rather than cleanly limned.” We have seen that defining pattern is closely related to defining a quantitative measure of the complexity of a pattern, an area fraught with difficulties. John Maddox [30] summarizes this situation with these words: “The search for means of telling the complexity of numerical data has been urgent but frustrating. Now, there may be some relief in the assertion that there is no single measure.” The road to obtaining a single measure of how patterned an entity is may encounter similar stumbling blocks. Finding the computationally efficient properties of pattern that work well in a variety of contexts is a challenging open problem.

References

[1] C. Alexander, *The Nature of Order: An Essay on the Art of Building and the Nature of the Universe, Book 1 - The Phenomenon of Life*, Berkeley, CA: The Center for Environmental Structure: Routledge, (2004), p. 4.

[2] S. Bell, *Landscape: Pattern, Perception and Process*, New York: Routledge, (1999), p. 85.

[3] F. Cajori, *A History of Mathematical Notations*, Vol. 1, Cosimo, Inc., (2007).

[4] G. J. Chaitin, “On the simplicity and speed of programs for computing infinite sets of natural numbers”, *Journal of the ACM*, 16/3:407-422, (1969).

[5] J. H. Conway, H. Burgiel, and C. Goodman-Strauss, *The Symmetries of Things*, Wellesley, MA: A. K. Peters, Ltd., (2008).

[6] L. Day *Pattern Design*, London: B. T. Batsford. Revised and enlarged by Amor Fenn in 1933. Dover Edition in 1999. (1903), p. 2.

[7] L. F. Day, *The Anatomy of Pattern*, London: B. T. Batsford, (1887).

[8] D. C. Dennett, “Real patterns,” *The Journal of Philosophy*, 88/1, (1991), p. 30.

[9] K. Devlin, *Mathematics: The Science of Patterns*, Scientific American Library, A division of HPHLP, New York. (1997), p. 210.

[10] A. Fenn, *Abstract Design and How to Create it*, Dover Publications, Inc., Mineola, N.Y., (1993), p. 107.

[11] R. P. Feynman, *The Pleasure of Finding Things Out: The Best Short Works of Richard P. Feynman*, Cambridge, MA: Perseus, (1999), p. 175.

[12] P. Flinders, *Decorative Patterns of the Ancient World*, London: Crescent, (1930).

[13] W. R. Garner, "Good patterns have few alternatives: Information theory's concept of redundancy helps in understanding the gestalt concept of goodness," *American Scientist*, 58/1, (1970), p. 34.

[14] J. J. Gibson, "What is a form?" *Psychological Review*, 58, (1951), p. 403.

[15] W. Gibson, *Pattern Recognition*, New York: G. P. Putnam's Sons, (2003).

[16] R. L. Gregory, *The Intelligent Eye*, New York: McGraw Hill, (1970), p. 15.

[17] U. Grenander, *Elements of Pattern Theory*, Johns Hopkins University Press, Baltimore, (1996), p. 33.

[18] U. Grenander, and M. Miller, *Pattern Theory: From Representation to Inference*, Oxford University Press, Oxford, (2007).

[19] K. Hagan, *The Complete Pattern Library*, New York: Harry N. Abrams, Inc., (2005).

[20] G. H. Hardy, *A Mathematician's Apology*, Cambridge: Cambridge University Press, Canto Edition. (1992), p. 13.

[21] S. W. Jewell, *Visual Perception of Shape and Area in Random Dot Patterns*, Ph.D. Thesis, Rice University, Houston, Texas. (2007).

[22] W. Kandinski, *Point and Line to Plane*, New York: Solomon R. Guggenheim Foundation, (1947), p. 35.

[23] A. Klinger, and N. A. Salingaros, "A pattern measure," *Environment and Planning B: Planning and Design*, 27:537-547, (2000).

[24] T. W. Knight, "Infinite patterns and their symmetries," *Leonardo*, 31/4, (1998), p. 306.

[25] A. N. Kolmogorov, "Three approaches to the quantitative definition of information," *Problems in Information Transmission*, 1/1:1-7, (1965).

[26] M. Krüger, "Binary sequences. II. Homogeneity and symmetry," *Information Sciences*, 31:15-31, (1983).

[27] Y. Liu, and G. T. Toussaint, "The marble frieze patterns of the Cathedral of Siena: Geometric structure, multi-stable perception, and types of repetition," *Journal of Mathematics and the Arts*, 5/3:115-127, (2011).

[28] Y. Liu, and G. T. Toussaint, "Categories of repetition in the geometric meander art of Greek and Roman mosaics," *Hyperseeing*, 25-38, (2010).

[29] Y. Liu, and G. T. Toussaint, "A tessellation-transformation method for categorizing and generating geometric textile design patterns," *Design Principles and Practices*, 2:101-111, (2008).

[30] J. Maddox, "Complicated measures of complexity," *Nature*, 344, (1990), p. 705.

[31] G. Nagy, "The dimensions of shape and form," In *Visual Form*, Arcelli, C., *et al.*, (Eds.), New York: Springer Science+Business Media, (1992), p. 416.

[32] N. Nelkin, "Patterns" *Mind and Language*, 9/1:56-87, (1994).

[33] G. Nolan, "What are patterns?" (2009) http://philosophy-of-pattern.com (accessed July 5, 2013).

[34] F. Papentin, "On order and complexity. I. General considerations," *Journal of Theoretical Biology*, 87:421-456, (1980).

[35] F. Papentin, and M. Krüger, "Binary sequences. III. Complexity versus homogeneity and symmetry," *Information Sciences*, 31:33-39, (1983).

[36] R. M. Proctor, *The Principles of Pattern: For Craftsmen and Designers*, New York: Van Nostrand Reingold Company, (1970), p. 8.

[37] M. D. Resnik, *Mathematics as a Science of Patterns*, Oxford: Clarendon Press, (1997).

[38] N. A. Salingaros, "Architecture, patterns, and mathematics," *Nexus Network Journal*, 1, (1999), p. 75.

[39] J. Schmidt, *New Practical Chinese Reader*, Beijing: Language and Culture University Press, (2002), p. 11.

[40] M. Shermer, "Patternicity: Finding meaningful patterns in meaningless noise," *Scientific American*, 299/5, (2008), p. 48.

[41] A. Sihvola, "Ubi materia, ibi geometria," *Proceedings of the 8th International Conference on Electromagnetics of Complex Media*, Lisbon, Portugal, (2000), p. 187.

[42] N. J. Sloane, and S. Plouffe, *The Encyclopedia of Integer Sequences*, San Diego: Academic Press, (1995).

[43] L. A. Steen, "The science of patterns," *Science*, 240:611-616, (1988).

[44] D. W. Stephens, "Change, regularity, and value in the evolution of animal learning," *Behavioral Ecology*, 2/1:77-89, (1991).

[45] G. T. Toussaint, "The complexity, homogeneity, order, and symmetry of finite binary sequences," *Proceedings of the Conference on Multi-Disciplinary Foundations and Intellectual Perspectives*, San Antonio, Texas, April 17-19, 2014.

[46] G. T. Toussaint, *The Geometry of Musical Rhythm,* Boca Raton, FL: Chapman and Hall/CRC Press, (2013).

[47] G. T. Toussaint, "The use of context in pattern recognition," *Pattern Recognition*, 10:189-204, (1978).

[48] G. T. Toussaint, and J. F. Beltran, "Subsymmetries predict auditory and visual pattern complexity," *Perception*, 42:1095-1100, (2013).

[49] W. R. Uttal, *The Perception of Dotted Forms*, Hillsdale, NJ: Lawrence Earlbaum Associates, (1987).

[50] T. Volk, and J. W. Bloom, and J. Richards, "Toward a science of metapatterns: Building upon Bateson's foundation," *Kybernetes*, 36/7-8:1070-1080, (2007).

[51] T. Volk, *Metapatterns - Across Space, Time, and Mind*, New York: Columbia University Press, (1995).

[52] D. Wade, *Symmetry: The Ordering Principle*, Wooden Books, Walker and Company, New York, (2006), p. 2.

[53] N. Wiener, *The Human Use of Human Beings: Cybernetics and Society*, University of Michigan: Houghton Mifflin, Edition 6, (1950), p. 3.

[54] I. Winckelgren, "How the brain 'sees' borders where there are none," *Science*, 256:1520-21, (1992).

[55] *The Online Psychology Dictionary* http://psychologydictionary.org/pattern/ (accessed July 5, 2013)

[56] *The Math Forum* http://mathforum.org/library/drmath/view/64445.html (accessed July 7, 2013)

[57] *Math is Fun* http://www.mathsisfun.com/definitions/pattern.html (accessed July 4, 2013)

[58] *Platonic Realms-Math Quotes* http://www.mathacademy.com/pr/quotes/ (accessed September 18, 2013)