From Mathematical Diagrams to Knotted Textiles

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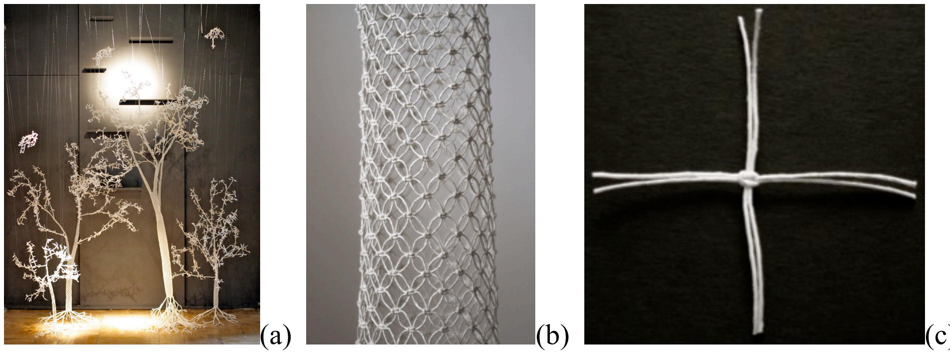
Abstract

This paper describes an ongoing collaboration between a textile practitioner-mathematician and a textile practitioner-researcher, and their investigation of certain knotted textiles from a mathematical viewpoint. The paper examines (1) how mathematical diagrams that characterize an individual craft knot may facilitate the design and production of a new knotted structure and (2) how other connected mathematical concepts, such as braid theory, may be used to characterize knotted textiles. The results of this collaboration are expected to shed light on the role of mathematics in making the experiential knowledge of a knotting process more explicit and the significance of collaborative projects on textile design development.

Introduction

The research focusing on the relationship between mathematical knot theory and knotted textiles is carried out collaboratively between a textile practitioner-researcher (Nimkulrat) and a textile practitioner with a first degree in mathematics (Matthews). In the first phase, the mathematical characterization of craft knots revealed significant differences between craft knots and knots defined in mathematical knot theory. The difference in ends was immediately obvious, as a knot may have loose ends in textile practice, but it is defined as closed continuous curve with no loose ends in mathematical knot theory [1]. A single craft knot used in Nimkulrat's textile work (Figure 1) was analyzed and described by diagrammatic representations commonly utilized in knot theory (Figure 2). The coloring of the diagram reveals a property that is not otherwise obvious. The position of strands finishes in the same place that they start, i.e., Strand a starts on the left in Position 1 and ends in Position 1, likewise Strand b remains in Position 2, c in 3, and d in 4. Figures 2a and 2b diagrammatically represent the craft knot in Figure 1c. Figure 2c shows the ends of the strands joined. What may be seen here is not one knot with many crossings, but a link with four tangled components, each of which is a ring. In knot theory, rings such as these are called the trivial knot or the *unknot,* the simplest form of knot.

This paper reports on the second phase of this ongoing study and examines (1) how mathematical knot theory diagrams that characterize an individual craft knot may facilitate the design and production of a new knotted structure and (2) how other connected mathematical concepts, such as braid theory, may be used to characterize knotted textiles.



Figures 1: *The White Forest (2012) installation (a), the knotted structure of an individual piece taken for mathematical characterization (b), and a single knot (c).*

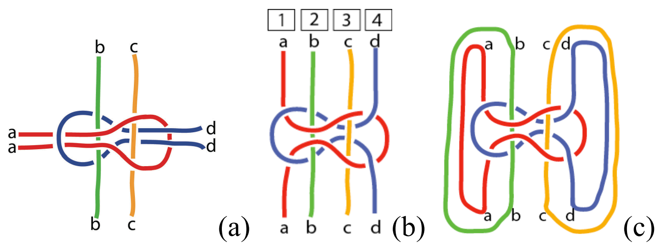


Figure 2: *Diagrammatic representation of Figure 1c (a), its knot diagram showing positions of strands a, b, c and d (b), and a link with four tangled components, each of which is a trivial knot (c).*

New Textile Knot Design

The diagrammatical representation of a link containing four trivial knots (Figure 2c), where the ends of the same strands of a knot are joined, is the main characteristic and inspiration for a new knotted structure presented in this paper. The knot design experimentation began with an attempt to transform Figure 2c into a single knotted component, using neoprene cord (5cm thick) as the material. This material is flexible, and joining its ends is relatively easy through bonding with neoprene adhesive. Nimkulrat made a single knot component (Figure 3b) using four strands of neoprene (Figure 3a) in the same way as she made the knot presented in Figure 2 [3] and then bonded the ends of each strand together. The process was repeated resulting in eight individual components. A closer examination of Figure 2c and the component in Figure 3b suggests that the component may be unraveled. Nimkulrat unraveled the component to test and found that it actually contains four rings or trivial knots (Figure 4). Nimkulrat's knot practice has always utilized thin and stiff paper string as the material. Once tight, paper string knots do not unravel so this observation was facilitated through the use of neoprene, a material that has very different properties. It is also an example of where practice may provide further insights into the mathematical model.

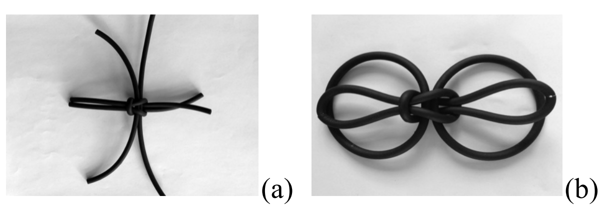


Figure 3: *Four strands of neoprene cord tied into a knot (a) whose ends are joined to create a link containing four trivial knots (b).*

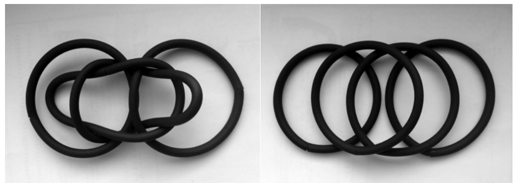
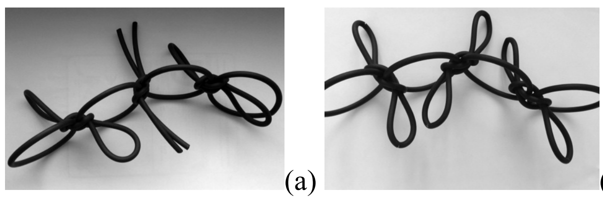


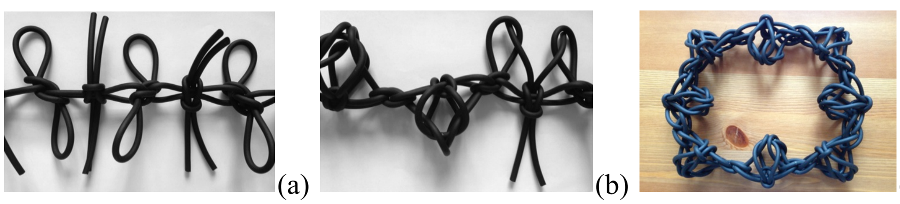
Figure 4: *The untying of the individual knot with no loose ends creates four rings or trivial knots in the mathematics term*.

The knot design experimentation proceeded by joining two individual knot components. To join the components, a ring from each of the two components functioned as the middle or passive strands, while two new neoprene strands were active in the tying of the knot (Figure 5a). The ends of these strands were then bonded (Figure 5b). After this, every ring or trivial knot in the joined components that had not played a direct role in the tying of any knot was squeezed together to form two middle strands, around which two new neoprene strands were directly tied, forming a knot (Figure 6a). To join the ends of each tied strand, both ends were put through the nearest trivial knots on the left and the right (Figure 6b). The joining of links continued until the structure in Figure 6c was achieved.



(a) (b)

Figure 5: *Two new strands of neoprene tied into a knot using trivial knots of the existing links as the middle strands (a). The ends of each strand are joined (b).*



(a) (b) (c)

Figure 6: *Two new strands of neoprene cord tied into a knot using squeezed existing trivial knots as the middle strands (a). Ends of each tied knot are put through the nearest trivial knots on the left and right, and joined (b). Finally, a complex set of links of trivial knots with no loose ends is achieved (c).*

The diagrammatical representation inspired Nimkulrat's making of knots to contain no loose ends and enabled new design through reflection on active and passive strands. It also importantly suggested specific qualities of materials suitable for the knotted structure. The thickness required for bonding the ends of strands and flexibility/manipulability generally needed for tying knots directed the textile practitioner to select the neoprene cord, which she would not have normally selected for her textile knot practice. She was led by the concept of joining all ends to construct a complex knotted structure that she would not have ventured otherwise. The unraveling of the component into four trivial knots shows the possibility of making craft knots from flexible materials that are originally in the ring form. This aspect would expand choices of materials for textile knot practice (e.g., use flexible rings instead of lengths of string/cord) and may lead to a possibility of creating spherical or tubular forms from several rings joined together.

Characterization through Braid Theory

This section considers whether a single craft knot in Nimkulrat's work in Figure 1c can be characterized through braid theory. Both braid theory and knot theory come under the branch of mathematics known as topology. Braid theory however allows for loose ends. As Figure 1c contains loose ends, it was thought that an analysis of her work in this way may be productive.

A braid may be imagined as a number of threads "attached 'above' (to horizontally aligned nails) and hanging 'down', crossing each other without ever going back up; at the bottom, the same threads are also attached to nails, but not necessarily in the same order" [4]. Two knots are considered mathematically equivalent if one may be transformed into another without cutting and re-joining [3]. Similarly, two braids may be considered equivalent if their strands can be rearranged without detaching at the top and the bottom or without cutting. A knot or link containing several components may be obtained from a braid by joining the top ends to the lower ends; the resulting knot is called a closed braid. According to Alexander's theorem, every knot can be represented as a closed braid [2]. Sossinsky's algebraic notation [4] is introduced for describing the process of braiding a simple plait and Nimkulrat's craft knot. Considering the three strands as the group of a braid shown in Figures 7 and 8, a strand must always occupy a position and may only move to an adjacent space.

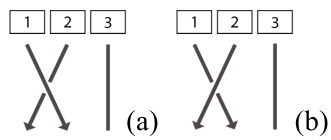


Figure 7: *Representations of moves using Strands 1 and 2: Move 'a' (a) and Move 'A' (b).*

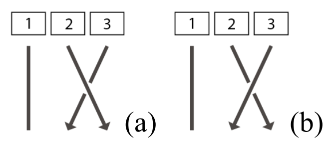


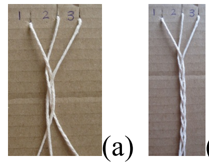
Figure 8: *Representations of moves using Strands 2 and 3: Move 'b' (a) and Move 'B' (b).*

The possible moves of the three strands are given a notation to describe their braiding processes (Table 1). Sossinsky refers to two different notations [4]. The first notation uses (1) subscripts to label the strands, e.g., b_1 for Strands 1 and 2, b_2 for Strands 2 and 3, and (2) a superscript to identify the inverse move of 'right over left' move, e.g., b_1^{-1} indicating that Strands 1 and 2 are involved in the move and that Strand 2 is twisted over Strand 1 from right to left (Figure 8b). The second notation uses letters a, A, b, and B to represent b_1, b_1^{-1}, b_2, and b_2^{-1}. As it is more legible, the second notation was adopted here.

Move	Notation 1	Notation 2	Figure
Strand 1 over Strand 2	b_1	a	Figure 8a
Strand 2 over Strand 1	b_1^{-1}	A	Figure 8b
Strand 2 over Strand 3	b_2	b	Figure 9a
Strand 3 over Strand 2	b_2^{-1}	B	Figure 9b

Table 1: *Possible moves and notation for the group of a three-strand braid.*

Using this notation, the process of plaiting three strands was examined step by step (Figure 9). Table 2 shows process steps that are repeated, resulting a long simple plait (Figure 9b). Figure 10 illustrates the diagrammatic representation of the plaiting process, revealing the notation "aBaBaBaBaBaBaB" for Figure 9b.

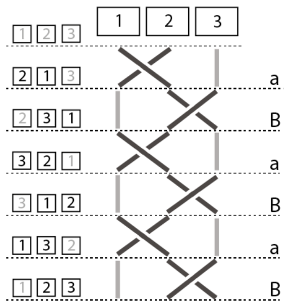


(a) (b)

Figure 9: *Plaiting, the moves described in the first five steps of Figure 10 (a) and a longer plait (b).*

Move	Notation
Twist Strand 1 over Strand 2	a
Twist Strand 3 over Strand 2	B

Table 2: *The two different moves used in plaiting.*



(a) (b) (c)

Figure 10: *A diagrammatic representation of a simple plait shows the positions of strands at the end of the moves (a), the moves (b), and the notation (c). The algebraic notation for these steps is aBaBaBaB.*

This method was then applied to the craft knot in Figure 1c. Figure 11 illustrates the knotting steps that are required to produce this knot. Figure 12 shows the translation of these steps using braid notation.

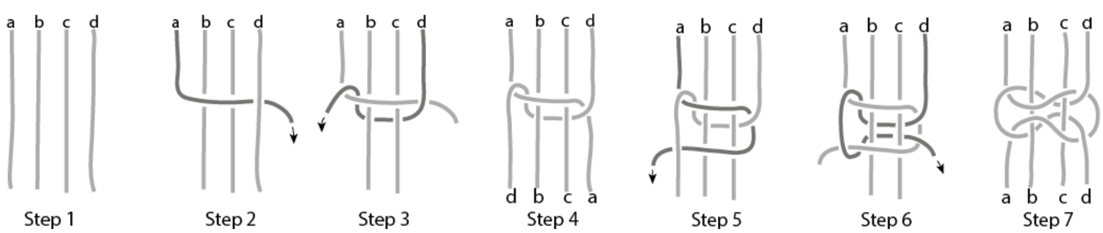


Figure 11: *Steps involved in tying a knot in Figure 2.*

In formulating the braid notation, a problem was encountered at the end of Step 3 and again at the end of Step 6. To complete these steps, the strand must pass through a loop and, in order to do this, the strand must first pass up the braid. By definition, all moves in braid theory must be in a downward direction. This would indicate that pure braid theory cannot be used to characterize the craft knot in question. In order to continue the analysis of this knot using braid theory, the theory was therefore modified to allow the upward move. The notation given for this move was *. It refers to not only the upward move of the strand that is last twisted over in the previous move, but also the passing of it through a loop created by the strand twisting over it. Figure 12 shows the * move in Steps 4 and 7 and the abCba*CBaBc* notation that describes the Figure 1c knot. The * move may also be seen in Figure 11 at the end of Steps 3 and 6. In Step 3, the * move takes place when Strand d moves up the braid and passes through the loop created by Strand a as it passes over Strand b in Step 2. Similarly the * move is used in Step 6 when strand d passes up through the loop left by Strand a in Step 5.

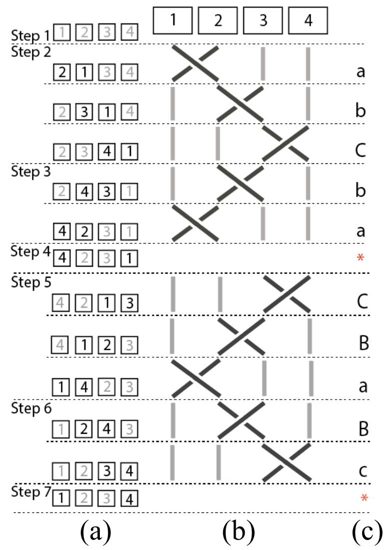


Figure 12: *A diagram indicating the process steps to derive the braid notation for Nimkulrat's knot shows the positions of strands at the end of the moves (a), the moves (b), and the notation (c).*

Conclusion

This paper demonstrates firstly that the mathematical characterization of a single craft knot (1) can lead to a new design and more intricate way of making knotted textiles with no loose ends and (2) can inspire a new choice of material for knot practice. Secondly, modified braid theory can be used to characterize the same craft knot and to derive a braid notation for it. The modified braid theory permits upward moves and defines a corresponding new notation. Although mathematical braid theory in its pure form is invalid as no upward moves are allowed by definition, the characterization of the craft knot in question using a modified theory is useful for three reasons: (1) it explicitly shows the upward knotting move which is not otherwise obvious; (2) the notation contains all the instructions for textile practitioners in a concise way; and (3) it may inspire practitioners to make connections between textile knot practice and braid theory and in turn to question whether craft design and practice may be stimulated by mathematics. Questions arise now as to whether it would be possible to design from the notation and how multiple rows of knots would be analyzed and described. These will be a focus of future work.

This collaborative research illustrates the use of science (i.e., mathematics including the methods of analysis and the use of diagrams to articulate properties) in facilitating the practitioner's reflection on and communication of creative process in a more objective and detailed way. In this case, knot theory and braid theory were used as a method to investigate, transfer, and replicate the procedural knowledge of the knotting process, so that the content becomes partly propositional.

References

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