Geometry Ascending a Staircase

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Abstract

A large metal sculpture, consisting of four orbs, commissioned for the stairway atrium of the Fitzpatrick CIEMAS Engineering Building at Duke University embodies the important idea that Science, Technology, Engineering, Mathematics and Art are closely connected. Made from hundreds of pounds of powder-coated, laser-cut aluminum, with an underlying geometric design, it was assembled at a four-hour "sculpture barn raising" open to the entire academic community. Titled "Geometry Ascending a Staircase," this project illustrates how STEM education efforts can be extended to the growing movement of STEAM, which puts the Arts into STEM education.

Introduction

Figure 1 shows the four orbs that make up my sculpture called *Geometry Ascending a Staircase*, hanging in the atrium of the Fitzpatrick CIEMAS Engineering building at Duke University. The low red one is 4 feet in diameter. The higher orange ones are 5, then 6 feet in diameter, up to the highest, yellow one, which is 7 feet in diameter. It is hard to get a photo of all four orbs because of the way they fill the 75-foot tall atrium, but the rendering in Figure 2 gives a sense of the overall design. The low red orb catches your eye when you walk in the ground floor of the building. Then as you walk up and around the stairs you get many views from below, around, and above the individual orbs.

Figure 1: *Geometry Ascending a Staircase.*

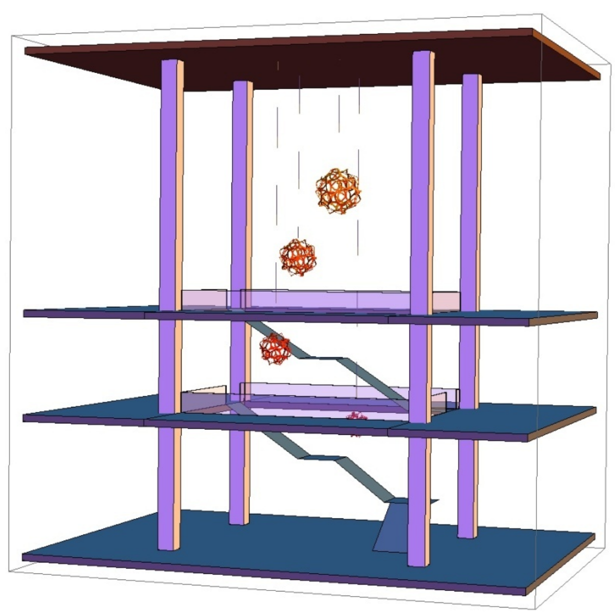

Figure 2: R*endering of the four orbs in the atrium.*

Prof. Ingrid Daubechies invited me to visit Duke University and commissioned the sculpture. About a year was required for planning, design, parts fabrication, test fitting, powder coating, event planning, and other preparations. The four orbs were assembled at a "sculpture barn raising" on Sunday afternoon, October 20, 2013. Many students, faculty, and staff from the Duke University campus community participated in the assembly.

While the final sculpture can be viewed on its own as an artwork, I see the overall project in a larger context as representative of a growing education movement called STEAM, which seeks to integrate the Arts into Science, Technology, Engineering, and Mathematics education [1]. The importance of STEM education is now widely recognized throughout the kindergarten-to-college education system. The past few years have also brought a growing awareness of the value of incorporating the arts into this education. The creativity associated with the arts is vital for creating an atmosphere of innovation in STEM fields. This philosophy is made clear to everyone when an interdisciplinary group focuses on building a large mathematical sculpture in an engineering center.

Design

The rendering in Figure 3 shows how the design develops. In addition to the color varying from red through two shades of orange to yellow, the parts become lighter, and the central opening (absent in the first, red orb) grows larger with height. I think of it something like a flower blossoming. Each orb consists of thirty identical planar parts. Figure 4 shows how the component shape develops and opens. Note the reversal of handedness, with the last two being mirror images relative to the first two.

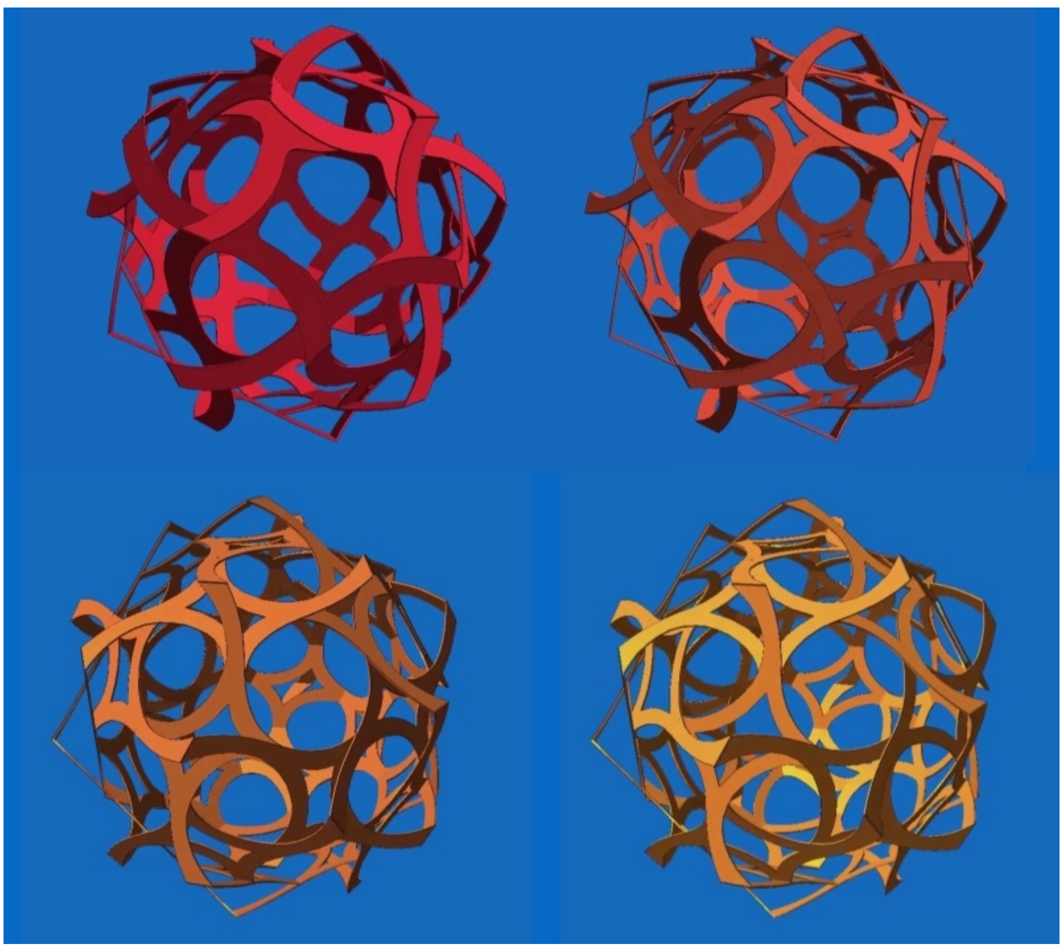

Figure 3: R*endering of the four orbs, showing the change in the design from one to the next.*

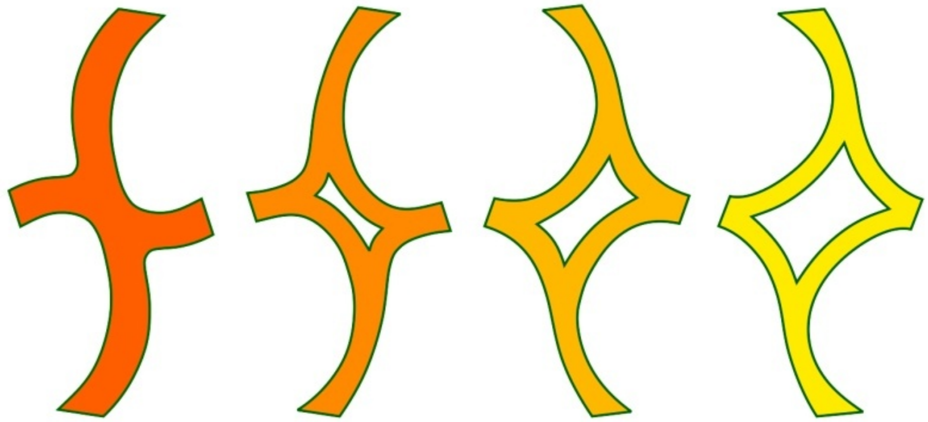

Figure 4: *Component design for the four orbs (not to scale) showing the change from one to the next.*

The derivation of the starting shape at the left in Figure 4 can best be understood by considering the well known icosahedrally interwoven arrangement of six pentagons shown in Figure 5, which is one of the "orderly tangles" of Alan Holden [3]. When made of thirty flat rectangles in this way, the six pentagons do not touch each other, so would not make a connected sculpture. To solve that problem, I added a short bar crossing each rectangle, which connects each component to its neighbors, as seen in Figure 6. The original rectangles lie in the thirty planes of a rhombic triacontahedron and the added bars remain in the same planes. The part shape of the lowest orb is basically this barred rectangle, but curved to have a more organic form. Then the change from left to right in Figure 4 is just a gradual, gentle modification of the form, thinning the struts and opening the center.

The pattern of two 3-sided and two 5-sided openings around each part corresponds exactly to the pattern of faces in an icosidodecahedron, which is dual to the underlying rhombic triacontahedron. From the geometry of these polyhedra and with the aid of special-purpose software [4], I could work out the necessary lengths and angles and prepare the parts drawings so the components meet precisely at the lines in space where these planes intersect. The design includes small brackets for connecting the parts and many bolt holes that must align accurately.

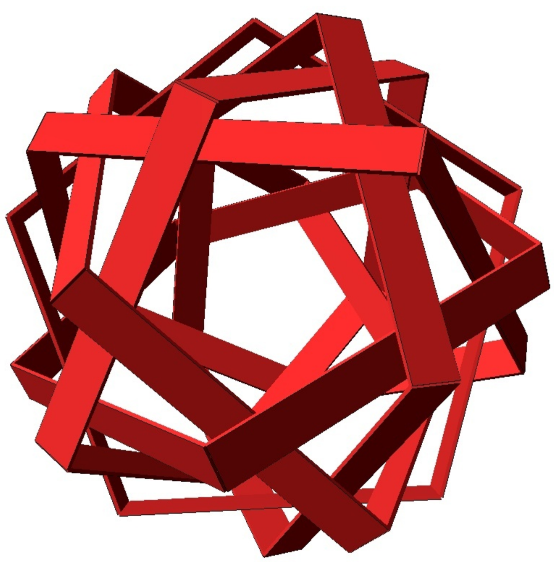

Figure 5: *Six disconnected interwoven pentagons.*

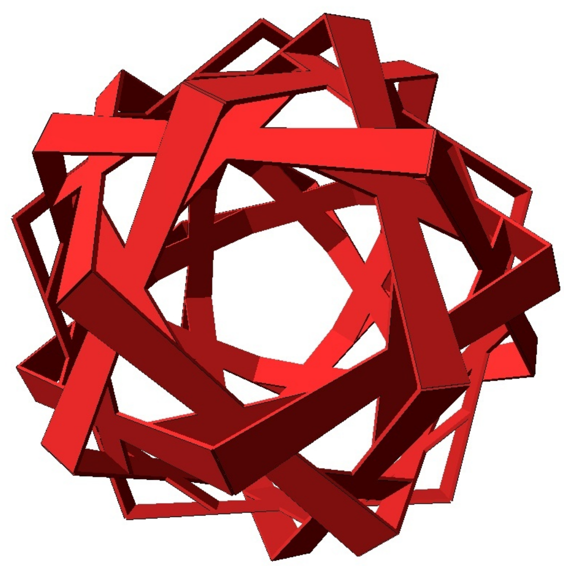

Figure 6: *Six pentagons with connecting bars.*

Fabrication and Test Fitting

I had the parts laser-cut from one eighth inch thick aluminum by an industrial laser-cutting firm, then test-fit them at my studio in New York to verify that everything fit well. To reduce the production cost, I designed the parts of the larger two orbs to be fabricated as two halves (which lay out more efficiently on a sheet of stock) then bolted together to make the full part shape. So the smaller orbs consist of thirty components while the larger ones consist of sixty. Each orb also requires ninety custom-made laser-cut angle-brackets plus 360 stainless steel nuts and bolts and 720 stainless steel washers.

I could assemble the components of the first three orbs by myself without much trouble, so I built one a day for three days. But for the largest orb I invited a bunch of friends over for a sculpture assembly party. See Figure 7. This was also a way for me to practice teaching the assembly process to people who were unfamiliar with the design, to see what aspects of the construction might need careful explaining. Later, after disassembling the test-fit construction, I sent the parts and brackets out to be powder coated in the four colors and had them shipped to Duke University for the actual assembly.

Figure 7: *Assembling the metal parts at a test-fitting party, before powder coating.*

Sculpture Barn Raising

On the assembly day, we set up four tables on the ground floor of the atrium. Each table held the parts and wrenches for making one sculpture. The first steps are to connect the angle-brackets to the parts in precisely the right way. In addition to finding the correct set of holes, the relative orientation of the parts, the brackets, and the nuts and bolts all matter. And don't forget to include the two washers! Before we started, I trained a "build leader" for each table and assembled one module as a model to copy. Participants could come and go throughout the afternoon, but the build leaders stayed and provided continuity as I ran around from table to table. Figures 8 shows these initial steps for one of the smaller orbs, which are relatively easy to put together because their parts are one-piece. But for the larger orbs, as in Figure 9, the parts are each assembled from two half-pieces, and the unders and overs had to be just right for the result to have a 2-fold symmetry axis, so there was more potential for confusion, but everything worked out fine.

Figure 8: *Building modules for the small orb.*

Figure 9: *Building modules for the largest orb.*

After making the thirty modules, we started assembling them together on a blanket on the ground. The first step is to make an equator of five units in the shape of a regular pentagon, as seen in Figure 10. These join using a second type of bracket with a different dihedral angle and there is a handedness issue to check, so again there is plenty of room for possible confusion. Using the equator as a base, parts are added above it to create one hemisphere as shown in Figure 11. At this stage, the shape of the parts guides you, so it comes together largely automatically and it's fun to watch the growing structure. It's also fun to crawl within the alien igloo architecture and work from the inside to complete the hemisphere. As Figure 12 shows, after turning the hemisphere over, the second half can be assembled just like the first.

Figure 10: *Assembling the modules to form one pentagon equator on the ground.*

Figure 11: *Completing a hemisphere.*

Figure 12: *Going higher after turning hemisphere over.*

At a certain point in this type of construction, we reach a stage where the participants understand the structure and are capable of finding mistakes and helping each other if any question arises. So I thought I could relax and watch the participants finish everything on their own. But then we discovered that despite careful planning, we were somehow missing a couple of boxes of washers (Figure 13). So we sent folks to buy some at area hardware stores. Happily, we got the washers and finished assembling everything.

Figure 13: *Almost done, but we ran out of washers!*

In each orb, three of the bolts were replaced with eye-bolts as a set of redundant attachment points where the chain connects. At this point, the participant's work is complete, except for carrying each orb to the point from which it will be lifted. Then the riggers came (Figure 14) to hoist the orbs up to the four hanging chains, which had been installed to the proper heights a few days earlier. It takes a high-tech lift to reach up to the 75-foot ceiling. This is the tensest moment because there is always a possibility for an orb to be dropped and damaged. You replay all the load-safety calculations in your head and are reassured that all the bolts, chain, and shackles are rated to carry a load several times the heaviest orb. Its an odd goodbye when your babies are about to float off into the sky like a balloon. Although they remain within sight, they will likely be out of reach for the rest of my life.

Additional images and a video of the assembly can be seen online [2, 5]. The result is exactly what I envisioned, both aesthetically as a sculpture and functionally in terms of its educational message. I feel the sculpture fits in beautifully with the architecture and activities of the space and I hope the students and faculty at Duke University enjoy it on many levels.

Figure 14: *Riggers raise the orbs into position and attach them to the chains.*

Conclusions

I believe that involving people in the construction of a mathematical artwork like this hands-on sculpture assembly is a powerful way to demonstrate that mathematics is a creative, living, joyful subject, full of beautiful ideas. When a group of people get together to construct a large mathematical object, it makes clear how math is valuable not just for science and engineering, but also for art and design. This sort of event also helps correct a misconception some people have that in mathematics there is always a single right answer. This sculpture is not the solution to an equation or any sort of unique result or previously defined mathematical object. Instead, it is an artistic creation, designed using mathematical ideas, but with the goal of being a beautiful structure with interesting internal relationships. And I would argue that that's a good metaphor for most of the work done daily by professional mathematicians. Math is very much an art. There is an aesthetic to what makes a beautiful theorem or an elegant proof. Math professors pass this culture on to their students, who discover it requires active mental participation.

Unless you are actively involved in doing mathematics, you can't fully appreciate its art. So with a sculpture assembly event like this, I try to get the public involved in the beauty of seeing patterns and understanding a larger structure in a way somewhat analogous to how an audience listens to a concert or watches a dance performance. But because mathematical thinking must be active, I keep the audience here busy building. They are constantly solving little puzzles by discovering patterns and extending them to position the next pieces. I hope they're also seeing some of the beauty which I enjoy in the structure. Doing abstract mathematics is similar, but the pieces are ideas instead of colored metal components.

A second goal of this construction is to help infuse the spirit of artistry into the culture of an engineering school. This is the STEAM idea. The culture and methods from the world of art and design can provide a spirit of creativity which leads to enhanced innovation and problem solving in all technical endeavors. I believe that when the engineering community participates in a sculpture barn raising, there are positive long-term benefits of this sort. And I hope that some of the participants, having been introduced through this event to the world of mathematical art, will now be inspired to create their own artworks and pass on these ideas to future generations.

Finally, if you are near Durham, NC, please visit *Geometry Ascending a Staircase* in the Fitzpatrick CIEMAS building at Duke University. I hope you enjoy walking up the stairs, discovering something of its patterns and structure, and thinking about the valuable connections between mathematics and art.

Acknowledgments

Prof. Ingrid Daubechies commissioned the sculpture, championed it through various campus committees, took care of many local arrangements during the project's evolution, and served as one of the table build leaders. The other table build leaders were Tingran Gao, Rujie Yin, Andrew Harms, and Meg McNall. Minnie Glymph and Amanda Dixon did the event photography. Many other folks at Duke worked behind the scenes, including Myron Taschuk, George Truskey, Michael Gunter, and Kathy Peterson. And a big thank you to everyone who participated in the assembly!

References

[1] See, e.g., http://stemtosteam.org/, https://www.risd.edu/About/STEM_to_STEAM/, http://www.steamedu.com/, http://steam-notstem.com/ or http://en.wikipedia.org/wiki/STEAM_fields
[2] Duke University video, http://www.youtube.com/watch?v=ghaJEF-VnLs
[3] G. Hart, "Orderly Tangles Revisited", Proceedings of *Bridges 2005: Mathematical Connections in Art, Music, and Science,* Banff, AB, 2005.
[4] G. Hart, "Symmetric Sculpture," *Journal of Mathematics and the Arts*, vol 1, no. 1, pp. 21-28, March, 2007.
[5] G. Hart, http://georgehart.com/sculpture/GAS/Geometry-Ascending-a-Staircase.html