Salvador Dalí and the Fourth Dimension

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Abstract

Salvador Dalí often used mathematical objects and ideas in his paintings and he enjoyed sharing ideas with mathematicians and scientists. This article describes a series of yearly meetings with Dalí starting in 1975, including discussions of "Corpus Hypercubus" and of his works in progress, especially his paintings featuring images from the Theory of Catastrophes of his friend René Thom.

Introduction

It was the Fourth Dimension that brought me into contact with Salvador Dalí in 1975 and the challenges of trying to visualize objects in the fourth dimension kept us in contact for a decade (Figure 1). During that decade, we met at least once a year, mostly in New York City in the early spring. He usually booked two suites in the St. Regis Hotel, one where he and his wife Gala resided and one that he used as a working studio. Each time he arrived in New York, he called us in Providence, Rhode Island and asked us to bring down whatever new things we were working on, and he wanted us to see what he was doing. Those meetings provided me with a unique opportunity to learn where this remarkable artist found the inspirations for some of his most mathematical paintings, and how he actually constructed them.

Figure 1: *Thomas Banchoff with Salvador Dalí at the St. Regis Hotel in New York City, 1975.*

As my colleague Charles Strauss said when we were first invited, “The worst we can get out of it is a good story” and indeed we ended up with a number of good stories. Dalí is the most unusual person I have ever met (including mathematicians!)

Dalí painted the masterpiece, “Corpus Hypercubus”, in 1954 (Figure 2). It is rated as one of his most popular and recognizable paintings, right after the melted clocks. While the deformed clocks are often considered statements about space-time, “The Crucifixion”, as it was originally called, makes a statement about four-dimensional space and soon it became known by its geometric name.

One year after, in 1955, I visited the Metropolitan Museum of Art in New York City for the first time; I was a seventeen-year-old high school student. In my junior-year daybook, I made a note on a date in August: “Salvador Dalí’s ‘Crucifixion’ is impressive.” I could not have imagined what effect that encounter with the artist would have on my own career.

Figure 2: *“The Crucifixion—Corpus Hypercubus”, Dalí, Metropolitan Museum of Art, 1954.*

Background

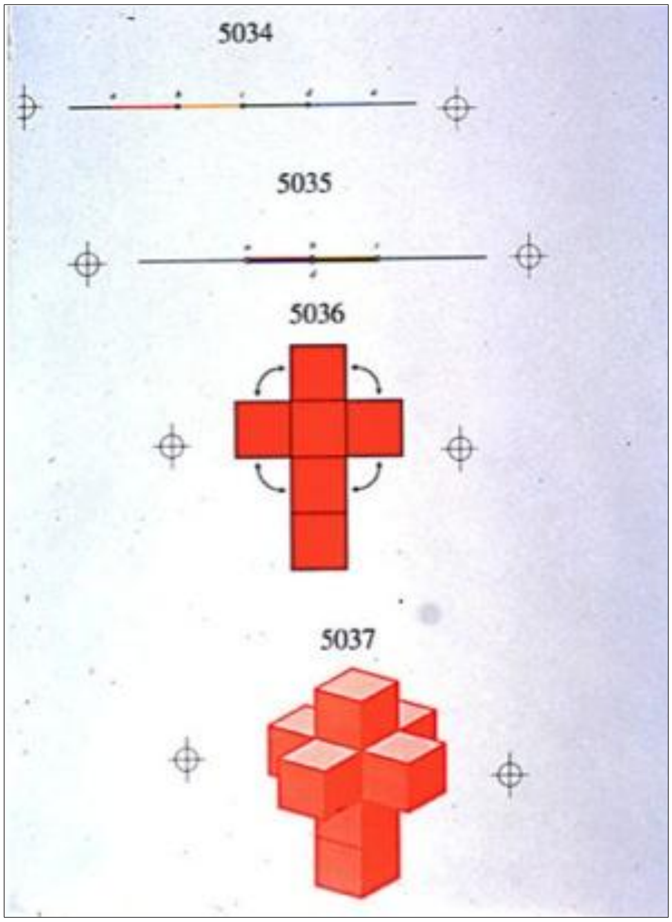
I first read about the fourth dimension in 1948 when I was about ten years old. There was a story in a Captain Marvel comic book (now known as “Shazam”) about a visit to a futuristic laboratory where scientists “were working on the seventh, eighth, and ninth dimensions”. A thought balloon went up from the boy reporter: “I wonder whatever happened to the fourth, fifth, and sixth dimensions?” That idea about higher dimensions captured my imagination and I decided to keep at it. I’m still wondering.

Later on, another science fiction comic book called “Strange Adventures” also influenced me. In it, “The Captured Cross-Section” told about an amorphous blob that appeared out of nowhere, grew to a maximum size and then shrank down and disappeared. The scientist character explained that it must have come from the fourth dimension, and he demonstrated the dimensional analogy by poking his finger through a paper napkin. A square living in the napkin, he said, could watch as the circle slice grew, then receded and disappeared as he withdrew his finger.

By the time I was sixteen, I had even formulated a theory of the Trinity based on that dimensional analogy, and I would tell it to any teacher or classmate who would listen. By then, I had also read “Flatland” and I knew how to mirror the visit of the Sphere through the plane by inflating and deflating a balloon to simulate a visitation of a Hypersphere through our space.

That slicing sequence from the "Strange Adventures" comic was at the heart of my theory of the Trinity and the Fourth Dimension. I developed it further when I wrote a major term paper in an honors theology class as a sophomore in college. My bibliography included twenty items, a very large number for the old days when finding references required searching the Periodical Index of Literature. Just a few years later, I found the number of references to the fourth dimension increased very quickly. It accelerated even more once computer searches became common.

Higher-Dimensional Polyhedral Surfaces



When I was working on my PhD thesis under Prof. Shiing-Shen Chern at the University of California, Berkeley, I remembered Dalí's unfolded hypercube (Figure 3). I needed to understand analogues of convexity for objects like the torus, and that led me to investigate polyhedral surfaces in three-space with the property that any plane would separate them into at most two pieces.

Smooth surfaces with this "two-piece property" shared rigidity characteristics with smooth convex surfaces in that any two isometric surfaces of this type had to be congruent. But my first results exhibited a pair of polyhedral objects with the two-piece property that were *not* congruent. I recognized that polyhedral tori with sixteen quadrilateral faces were projections into three-space of a torus in four-dimensional space built from square faces of a hypercube. It was only later that I made the connection between this polyhedral surface and the fold-out model of the hypercube, the central figure in Dalí's "Corpus Hypercubus" (Figure 2).

Figure 3: *Unfolding Hypercube Sequence, "Beyond the Third Dimension" [1].*

Back in graduate school, Prof. Chern introduced me to Nicolaas Kuiper, a visiting professor from Amsterdam. Kuiper had recently proved results for smooth surfaces in four- and five-dimensional space satisfying a condition equivalent to the TPP. His results showed there were no smooth surfaces with this property in any dimension greater than five. When we talked, Kuiper suggested that I prove analogous results for polyhedral surfaces, but after two weeks, I discovered that it was possible to construct TPP surfaces with square faces in cubes of any dimension. The polyhedral theory was totally different from the smooth theory, a surprising situation. Analyzing these examples proved to be the basis for my PhD thesis in 1964.

When I started giving talks about my thesis results, I showed a polyhedral model that could be assembled in six-space to form a surface with the TPP, and I learned how to fold it flat into a plane. Folding a related polyhedron, I constructed the hypercube, the central figure in Dalí's painting. In 1975, it was a picture of that unfolded model which brought me in contact with Dalí.

In 1967, I began teaching at Brown University. A colleague found out about my interest in four-dimensional geometry, and introduced me to the work of computer scientist Andy van Dam. It was Andy who directed me to his recent PhD student Charles Strauss who had constructed a "three-dimensional blackboard" for interactive industrial design. Charles was looking for new applications. Since I had a great many visualization problems that needed new techniques, it was a natural collaboration and Charles and I worked together for the next twelve years.

One of our first projects was a study of a hypercube formed by translating a three-dimensional cube in a fourth direction and then projecting the resulting polyhedron back into three-space. With relatively primitive equipment, we photographed individual images and put them together to create a 16 mm film of this object as it rotated in four-dimensional space. Using the analogue of the mapmaking technique of stereographic projection we produced a remarkably beautiful sequence of images of a torus turning inside out, called "The Flat Torus in the Three-Sphere". We were hooked.

By 1973, when I went to UCLA on my first sabbatical, we had already produced early versions of films on "The Hypercube: Projections and Slicing" as well as a film we called "Complex Function Graphs.

Meeting Dalí

We had produced a number of computer-animated films by 1975, and we had begun to investigate stereoscopic slides. In January of that year when I was on my way to the Joint Math Meetings in Washington DC, one of the public relations persons at Brown University introduced me to his associate Tom Zito, a staff writer for the Washington Post newspaper. He interviewed me for a couple of hours in a restaurant, stopping whenever I mentioned a name or a technical term to make sure he had the correct spelling. He had a photographer take a picture of me holding an unfolded hypercube. The very next morning his article appeared in the Style Section of the Post, with a picture of me showing the unfolded surface and with an inset of the Dalí "Corpus Hypercubus" in the background. There was also a picture of the rotating hypercube, with the rotational effect indicated by afterimaging. When I spoke to the writer the next day, he asked what I thought of the article (Figure 4). I liked it, I said, but he was abashed when I pointed out that he had misspelled my name throughout the article!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Visual Images And Shadows From The Fourth Dimension

Shadows From the Fourth Dimension

Figure 4: (Left) *Washington Post Style Section, January 22, 1975, page 1,* (Right) *Washington Post Style Section, January 22, 1975, page 2.*

When I got back to Brown, Charles said he was surprised that the Post had printed the Dalí painting since it was clear that they had not had time to go through proper procedures to acquire the rights to reproduce the image. It was well known that Dalí liked publicity, but on his own terms. I was slightly alarmed when I returned from teaching class one day to find a note from the secretary asking me to contact a person in New York City "representing Salvador Dalí". Charles said, "It's either a hoax or a lawsuit". When I called back, a woman who identified herself as Senor Dalí's appointments secretary said that he was in New York City and he wanted to meet us. Could we come down? I asked Charles what he thought, and he came up with his good line: "Well, it's either a hoax or a lawsuit, but the worst we can get out of it is a good story."

We came down to New York City by train and met with Dalí in the cocktail lounge of the St. Regis Hotel where he more or less held court. A number of visitors arrived and were received by Dalí, and we were seated at his right and left side as "the ambassadors from mathematics land". He had just finished a project on holographic images and he was embarking on a series of stereoscopic oil paintings so he wanted to discuss viewing techniques. I was impressed by his background knowledge and his questions about our stereo slides of surfaces in four-space. When I showed him my folding hypercube, he was quite taken with it and he said, "I may have this." It was not exactly a question. When I asked him to say more, he said that he had just established a museum in Figueres, near his birthplace in Catalonia, and that he wanted to put a version of this moving model as a display. I agreed.

Figure 5: (Left) *Thomas Banchoff and Charles Strauss, 1980,* (Right) *Veronese Surface Sequence.*

He was very interested in our stereo films. He asked for a return visit two weeks later to show him our initial version of the Veronese surface, a nineteenth century representation of the real projective plane (the unique smooth surface with the TPP in five-space, as proved by Kuiper) (Figure 5). He wanted me to meet two of his main collectors, A. Reynolds Morse and Eleanor Morse who had stayed in New York an extra day at his insistence. We brought our elementary "beam-splitting" apparatus and glasses with polarized lenses and discussed the value of the stereo effect, which was not as dramatic as Dalí had hoped. He did become excited by the appearance of the patterns of folds and cusps in the changing views of the surface, in particular the three-cusped hypocycloid that he recognized as an "elliptic umbilic catastrophe". I was impressed that he was familiar with the images from the catastrophe theory of Rene Thom. Only later did I realize that he was already interacting with Thom and that those same images would appear in his final paintings.

Projects from Yearly Visits

In our visit the following year in 1967, we saw the "Dalí Lincoln" or "Lincoln in DalíVision" when it was still in process (Figure 6). A black-and-white study from Bell Laboratories had reduced a portrait of Abraham Lincoln to a collection of grey squares, and Dalí saw the opportunity to incorporate it into a colored painting that would look entirely different when viewed from various distances, a phenomenon he had already achieved in earlier canvases. We saw him looking through the small end of a pair of opera glasses to see what the view would be like from twenty meters away, after which he would go up to modify the color in one square, then repeat the process. Seeing him at work was enlightening.

Figure 6: "*Gala Contemplating the Mediterranean Sea which at Twenty Meters Becomes the Portrait of Abraham Lincoln (Homage to Rothko)" by Salvador Dalí, 1976.*

In another visit the following year, we saw Dalí working on two separate pictures by Raphael from the Vatican Apostolic Palace,"The School of Athens" and "The Fire in the Borgo". Both are set into the same half-elliptical frames so if they are viewed stereoscopically, the mind wants them to fuse the images in the left and right eyes but the set-up causes "retinal rivalry". The contents switched back and forth from the right eye to the left. When I saw the piece in progress, there were no additional elements present, but ultimately Dalí inserted a stereo pair of a collection of overlapping colored rectangles to force a binocular interpretation (Figure 7).

Figure 7: *"The School of Athens" and "The Fire in the Borgo", Dalí, 1979.*

In 1980, Dalí was excited about a new project involving distorted perspective, the "hundred meter horse". The idea was that you would come out of a museum and find yourself looking up at the front view of a large statue of a horse in perfect proportions. Only as you moved to the side would it become clear that in fact the horse was greatly distended, with shoulders several meters away and the rump far away, the length of a football field (Figure 8, Left).

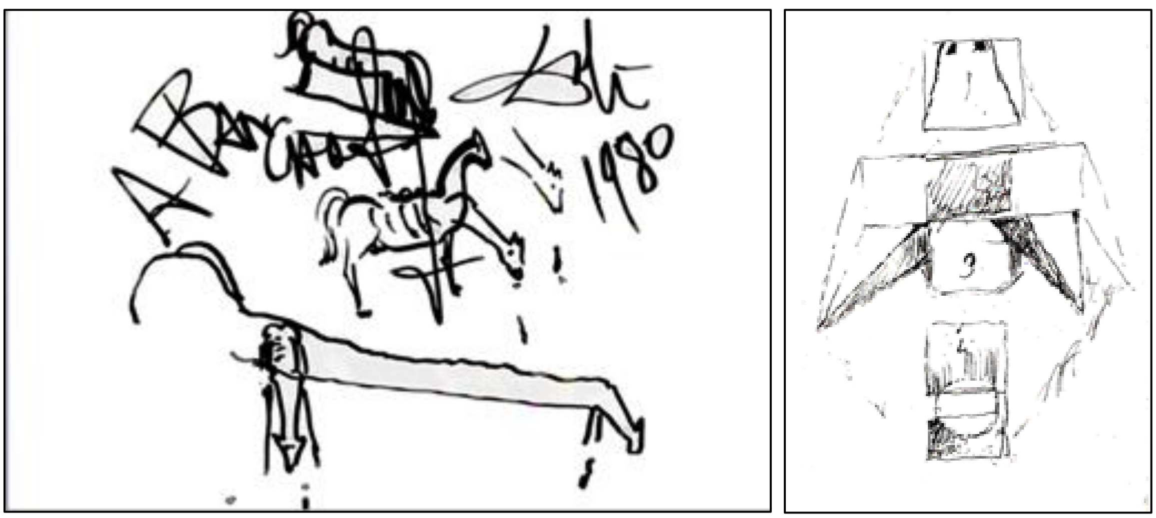


Figure 8: (Left) *The Hundred-Meter Horse, signed sketch by Dalí, 1980,* (Right), *Horse from the Earth to the Moon, Sketch by Dalí, 1982.*

Dalí had a plaster model of a horse sent to us and Charles set up a scanner to produce a set of horizontal curves. We then applied various geometric transformations to show perspective projections from different viewpoints (one was the view from inside the horse, the "Trojan Soldier's View of the World"). I brought our working images to the opening of Dalí's exhibit of stereoscopic paintings at the Guggenheim Museum in New York City, and curious onlookers watched from a distance as I sat with Dalí in New York City on a side bench as he looked at the preliminary slides. The invitation to that show included the picture of the woman with the distended upper leg, similar to "The Ghost of Vermeer that Can Also Be Used as a Table" from the A. Reynolds Morse collection at the Dalí Museum in St. Petersburg FL.

In 1981, Dalí wanted to propose a change of scale. Instead of a hundred meters, the horse would be one hundred kilometers long, with the shoulders several kilometers away and the rump on the crest of a mountain. The sketch showed Toledo in the distance. No additional calculations were necessary since the only change was one of scale. The same mathematics would work for all sizes, although it was becoming clearer that it was unlikely that the project would actually be built.

The third time we discussed the project was the following year in Paris in 1982. Dalí was staying with Gala in the Hotel Meurice and he was not feeling well. According to the person who was then handling his affairs, Dalí was not receiving visitors at this time, but he wanted to talk to me. He wanted to change the scale again, so that this time the shoulders would be on a mountaintop and the rump would be on the moon (Figure 8, Right). In order to view the statue from the proper perspective, you would have to be at the right place and at the right time! It was even clearer that the project was not likely to be created in fact, but the mathematics was still all in place.

Dalí and Catastrophe Theory

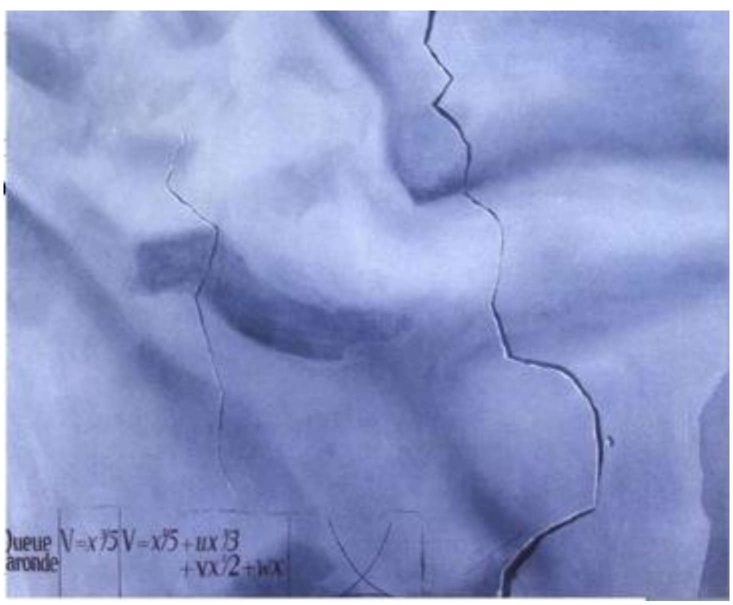
Our last visit where we showed films and slides was in Pubol near Barcelona, where Dalí retired after the death of Gala. He was working on his final paintings, inspired by images from the catastrophe theory of his friend Rene Thom [3], and we could see next to the easel a cello with the elongated S-shape reminiscent of the integral sign and a cubic curve with an inflection point. The characteristic form of a swallow tail catastrophe represented a chalice. As we watched the latest version of our film on the Veronese surface, Dalí excitedly commented about the appearance of the visual catastrophes, somewhat to the consternation of the attending nurses but I explained that he was not speaking nonsense but rather describing exactly what he was seeing.

Dalí was very interested in being accepted by scientists and mathematicians, and René Thom was one of his favorite people (Figure 9, Left). Dalí very much enjoyed the images that appeared in the theory of catastrophes, as developed by Thom, the Fields medalist who was a permanent member of the Institut des Hautes Etudes Scientifiques outside of Paris. One of the important manifestations of that theory was the study of singularities of projections of objects into planes and spheres. Usually shapes appear to change continuously, but occasionally there is a sudden change in form, some new phenomenon that indicates a qualitative change that takes place in surprising ways. Examples are the changes that occur in the contours in a landscape as floodwaters gradually submerge a geographical feature. More dramatic are the drastic effects of major seismic events such as earth tremors and displacement of fault lines during earthquakes. Thom identified a small set of phenomena that are characteristic of such qualitative changes and he classified them using algebraic formulas that could be viewed using computer graphics. Many of these phenomena are familiar to structural engineers who study the effects of forces on bridges or roads or buildings. Buckling of beams or stretching of springs beyond their elastic limits are examples of extreme behavior brought about by gradual loading which for a time produces very small effects and then abruptly causes an irreversible change in form.

Dalí and Thom were delighted to discover that they were both fascinated by a geological feature involving three tectonic plates coming together in the Pyrenees not far from the border of France and Catalonia, in Perpignan. Dalí's painting of the railway station in that town is an extreme study of central perspective, contrasted with the routine cycle of daily prayer symbolized by figures from Millet's "Angelus" (Figure 9, Right). Dalí used that geological feature as the inspiration of one of his last paintings, the only one that explicitly includes an algebraic formula (Figure 10).

Figure 9: (Left) *René Thom, 1923-2002,* (Right) *"La Gare de Perpignan", Dalí, 1965.*

Such changes are also quite familiar to differential geometers studying pictures of transparent surfaces as they rotate or deform in space. As an example, a torus of revolution viewed from a point on its axis has apparent contour consisting of a concentric pair of circles, but as the torus rotates about a horizontal line, the contour suddenly develops four cusps, an example of a "swallowtail catastrophe". Dalí especially appreciated such qualitative changes in form that take place during continuous deformations.



One phenomenon he found particularly appealing was the appearance of a three-fold cuspidal form in the projections of a famous nineteenth century model, the Veronese surface, named for the Italian mathematician who studied its algebraic and geometric properties. One view of this surface has a special central point that unfolds into a "hypocycloid with three cusps".

Figure 10: *'Topological Abduction of Europe - Homage to René Thom" 1983.*

One curious coincidence led to my only joint paper with René Thom, in 1980 when I arrived for a visit at the IHES. Thom mentioned that he had just published a paper including the statement that a deformation of a cusp of a Gauss mapping of a surface would always have an even number of cusps. I said that this wasn't necessarily true and when he asked for an example, I said I would show one to him. We found a 16 mm projector and we watched how a fold curve appeared that had exactly three cusps. We worked out the formulas to show that what we saw was really a stable phenomenon, so that small perturbations preserved this odd number of cusps. He then suggested that we write a joint note to this effect for Comptes Rendus of the Paris Academy of Science.

Final Comments

It was the same example that Dalí had seen in Pubol, the castle near Barcelona that he had given to his wife, Gala, and where he retired after her death. I was able to visit him there where his final painting, "The Swallow's Tail" featured the pair of cusps arising from projections of a surface into the plane, for example the folds and cusps in a torus of revolution or one of its so-called conformal deformations into a cyclidal surface. The easel supporting this nearly completed picture had next to it several texts featuring geometric diagrams, including the monograph "Cusps of Gauss Mappings" written together with my colleagues Clint McCrory and Terence Gaffney [2]. That painting is listed as Dalí's last major work, once again uniting mathematical and religious symbols and forms (Figure 11, Left).

Figure 11: (Left) *"The Swallow's Tail", Dalí, 1983,* (Right) *Banchoff at Dalí Museum, Figueres, 2004.*

In the Salvador Dalí Museum in Figures, in Catalonia near the coastal town where Dalí was born, there is a display in a large glass case of an unfolded hypercube fashioned in gleaming metal, next to the wooden model that Dalí used when he designed and painted "Corpus Hypercubus". A photograph of that painting is visible on a side wall. Inserting a 20 centavo coin in a slot next to the display causes the polyhedron to gyrate, exhibiting its folding structure. When Dalí had said, "I may have this" in our first meeting nearly thirty years earlier I could not have had any idea that my construction would have a permanent part of his museum (Figure 11, Right). It can be a memorable encounter when mathematicians and artists meet.

References

[1] Banchoff, T., *Beyond the Third Dimension*, 1990, 1996, Scientific American Library
[2] Banchoff, T., and McCrory , G.C., *Cusps of Gauss Mappings*, 1882, Pitman Publishing Co.
[3] Thom, R., *Structural Stability and Morphogenesis*, 1975, W. A. Benjamin