

Alberti's Window: Projective Geometry as the Geometry of Vision

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Abstract

The participants in this workshop will engage in activities with different physical and virtual variations of an "Alberti's Window." The activities will focus on the realization and analysis of projections at different scales and on diverse media. They will watch and discuss classroom videotapes of undergraduate students in a projective geometry class, as well as examples of paintings created by these students on the basis of projections generated by means of an Alberti's Window.

The Geometry of Vision

Euclid's Optics is the oldest known historical attempt to develop a mathematical account of light and vision. Participants in this workshop will discuss the 7 definitions proposed by Euclid [1]:

1. Let it be assumed that lines drawn directly from the eye pass through a space of great extent;
2. and that the form of the space included within our vision is a cone, with its apex in the eye and its base at the limits of our vision;
3. and that those things upon which the vision falls are seen, and that those things upon which the vision does not fall are not seen;
4. and that those things seen within a larger angle appear larger, and those seen within a smaller angle appear smaller, and those seen within equal angles appear to be of the same size;
5. and that things seen within the higher visual range rays appear higher, while those within the lower range appear lower;
6. and, similarly, that those seen within the visual range on the right appear on the right, while those within that on the left appear on the left;
7. but that things seen within several angles appear to be more clear

A contrast will be introduced between these Euclidean origins and the modern emergence of projective geometry whose entities are points and lines interrelated by projections, which transform pencils into ranges and vice versa. The invention of linear perspective was a crucial historic step in this evolution. Alberti's Window will be introduced as a key tool for the conceptualization of linear perspective.

Physical versions of Alberti's Window

Alberti's Windows are transparent panels onto which, or through which, shapes are projected from a center of projection or "eye point". Figure 1 shows the basic setup that we use for an Alberti's Window:

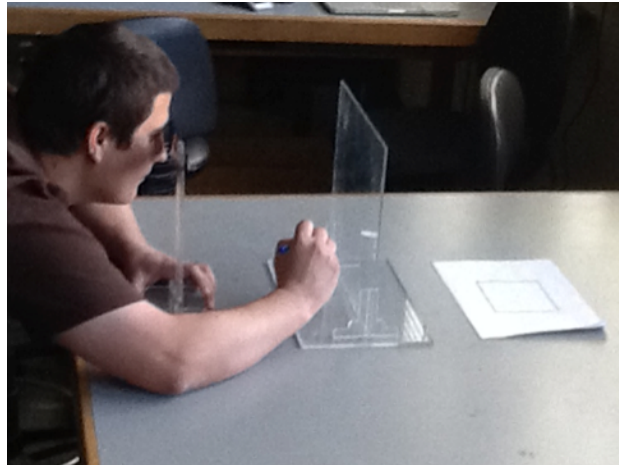


Figure 1: *Using Alberti's Window*

It consists of a moveable "eye piece," with an adjustable height, that serves the purpose of fixing the point of view, and an acrylic window on which the user projects a shape located on the tabletop. The projection is obtained by drawing on the window, with an erasable marker, what can be seen on the tabletop. It can also be used with a source of light, preferably very small, so that images on the window project as sharply defined shadows onto the tabletop. Figure 2 illustrates this setup with a very small LED light. In order to project shapes that are located between the window and the eye piece, or behind the eye piece, we use a piece of string connecting points from the shape to be projected, passing through the eye piece, and imagine how the other end of the string would trace the projection on the window.

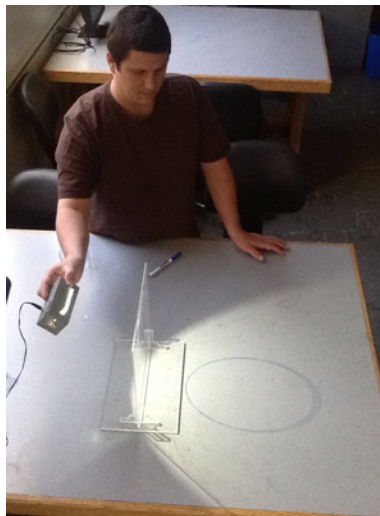


Figure 2: *Projecting with a small source of light*

This setting is amenable for the study of conics and how they project onto each other, so that an ellipse can project onto a hyperbola, or the latter onto a parabola. If the Alberti's Window is used outdoors with large-scale shapes (e.g. in a soccer field on which functional shapes are traced with a long

rope), the projection can vividly illustrate the role of the horizon at the infinite, and make salient differences that are otherwise difficult to notice, such as how parabolas project tangentially on the horizon, whereas hyperbolas cross the horizon. Figure 3 shows students projecting a hyperbola on a soccer field.



Figure 3: Undergraduate students projecting in a soccer field

Virtual versions of Alberti's Window

Virtual versions of an Alberti's Window can be implemented in multiple ways. Figure 4 is a depiction of the cone with its apex at the center of projection. The advantage of this software-based Alberti's Window is that the location of all the components can be changed arbitrarily without being constrained by physical limitations. On the other hand, our experience clearly indicates that students' previous familiarity with physical setups is very important for them to visualize the geometric relations depicted on a computer screen.

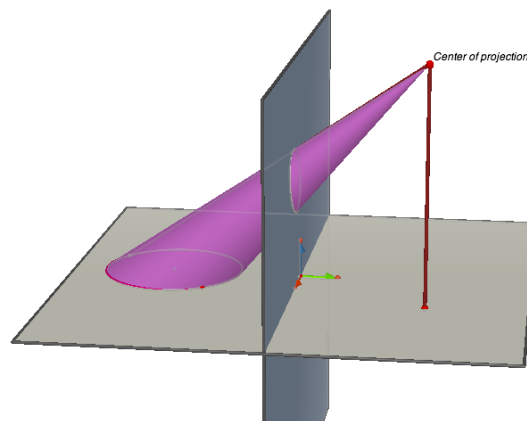


Figure 4: Alberti's Window created in Cabri 3D

Another virtual version of Alberti's Window was developed with Geometer's Sketchpad (GSP). In this version, the two planes, the one with the original shape and the one with its projected shape, are overlaid. A single plane depicts a superposition of both. In Figure 5 we illustrate this virtual version, green entities (e.g. a circle in this Figure 5) are on the original plane, and orange ones are the projected ones. "Eye height" is the height of the center of projection, "base line" is the intersection between the two planes, and "eye distance" is the distance between the window and the center of projection. Participants will use

this Sketch and work to visualize the relationships between this 2D diagram and a physical setup of Alberti's Window.

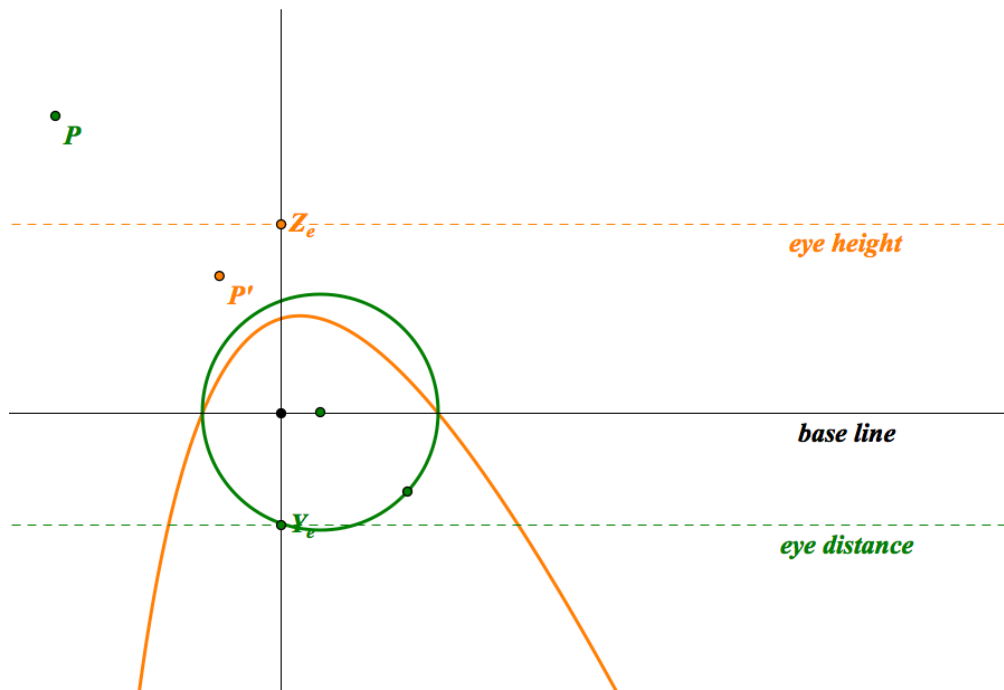


Figure 5: *Alberti's Window with Geometer's Sketchpad*

Classrooms Videos and Examples of Students' Art

The final segment of this 75-minute workshop will center on the analysis of videotaped classroom episodes and of students' reflections on their experiences creating pieces of art in this class. The workshop participants will familiarize themselves with diverse versions of Alberti's Windows, and with ways in which they can be used for the learning of projective geometry and for art projects. In particular, workshop participants will work in teams with the physical Alberti's Window, markers, string, and a light source, all of which the workshop leaders will provide, to develop an understanding of projections. There will be less emphasis on the virtual versions of Alberti's Window. However, workshop participants will be given access to the Geometer's Sketchpad version of Alberti's Window that they can use on their personal computers or iPads. After participants develop an understanding of the Alberti's Window, they will engage in at least one art activity using the window.

References

- [1] Burton, H. E. (1945). "The Optics of Euclid." *Journal of the Optical Society of America*, 35(5): 357-372.