A Workshop on N-regular Polygon Torus using 4D Frame

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Abstract

In this workshop we will show an n-regular polygon torus and its applications where n=4, 8 using 4D Frame. We will bring the 4D Frame for participants to use. As n approaches infinity, an n-regular polygon torus approaches to a circle–torus. Participants can make an 8-regular polygon torus (i.e. octagon-torus) by the Pythagoras theorem using 4D Frame[i], consisting of 16 regular octagons. Therefore people will understand the simple mathematical structure and its spatial beauty in their work.

Figure 1: *8-regular polygon torus using 4D Frame*

1. Introduction

A donut is an example of torus from daily life. (**Figure 2**) As you can see from **Figure 3**, we can find geometrically mathematical concept to inspire students.

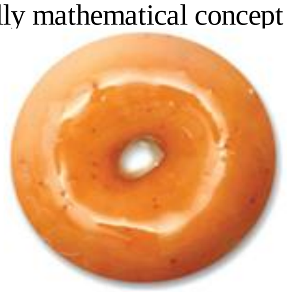

Figure 2: *a donut which is circle-torus*

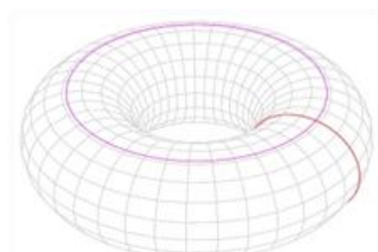

Figure 3: *a donut structured*

2. General Mathematical Definition of Torus

In mathematics a torus is a surface of revolution generated by revolving a circle in three-dimensional space about an axis coplanar with the circle. If the axis of revolution does not touch the circle, the surface has a ring shape and we call it a circle-torus.

Figure 4: *a circle torus with a coplanar axis*

A torus can be defined parametrically as

$$(x, y, z) = ((R+r\cos\varphi)\cos\theta, (R+r\cos\varphi)\sin\theta, r\sin\theta)$$

where φ and θ are angles which make a full circle, starting at 0 and ending at 2π, so that their values start and end at the same point, R is the distance from the center of the tube to the center of the torus, r is the radius of the tube. Its surface area and interior volume are easily computed using the Pappus' Centroid Theorem[ii] giving Area as $A = 4\pi^2 rR$ and Volume as $V = 2\pi^2 R r^2$.

3. N-regular Polygon Torus

Combining an n-regular polygon with a number of unit sets, we can make a donut shape (We call it an n-regular polygon torus). When n=8, the process is suggested simply in Figure 5-a and 5-b. An 8-regular polygon torus consists of 16 regular octagons with quadropod connectors and tubes by 4D Frame (We also call it octagon-torus). (Figure 6)

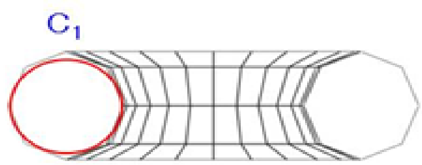

Figure 5-a: *8-regular polygon (C_1)*

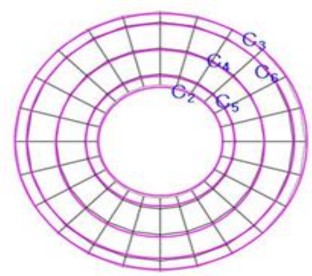

Figure 5-b: *8-regular polygon torus (C_2 ~ C_6)*

* principal curve : C_1, C_2, C_3
* asymptotic curve : C_4
* geodesic curve : C_1, C_2, C_3, C_4

Let's say principal curve C_1 and C_2 have 3 cm frames as you can see below.
U_1=3 (cm), U_2=3 (cm)

Circumference of C_1 = 24cm …… [Figure 6-a]
Circumference of C_2 = 48cm …… [Figure 6-b]

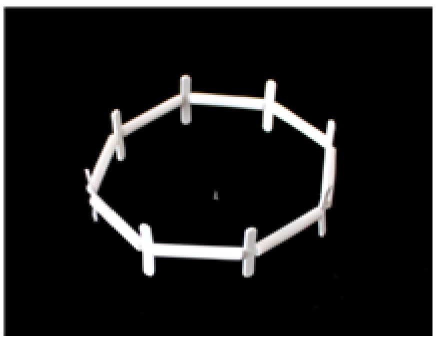

Figure 6-a: *C_1 using 4D Frame*

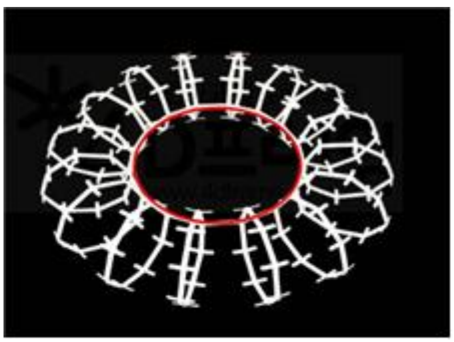

Figure 6-b: *C_2 using 4D Frame*

Considering the length of the quadro pod(5cm), the circumference of C_1 and C_2 will be 28cm and 56cm.

∴ Diameter of C_1 : $D_1 = 28 \div 3.14 \fallingdotseq 9$ (cm)
∴ Diameter of C_2 : $D_2 = 56 \div 3.14 \fallingdotseq 18$ (cm)
∴ Diameter of C_3 : $D_3 \fallingdotseq 36$ (cm) ……………… [Figure 7-a]
∴ Circumference of C_3 : $D_3 \times 3.14 \fallingdotseq 113$ (cm) ….. [Figure 6-c]

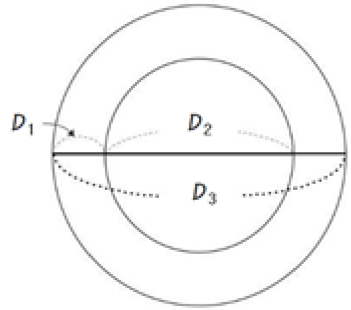

Figure 7-a: *Diameter of C_1, C_2, C_3*

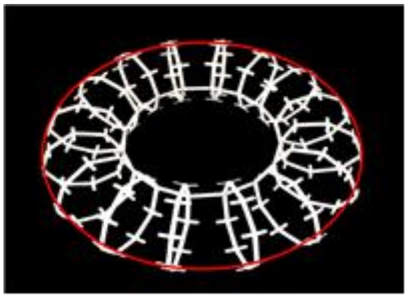

Figure 6-c: *C_3 using 4D Frame*

With the same method, we can get C_3' unit frame $U_3 = (D_3 \times 3.14 - 8) \div 16 \fallingdotseq 6.5$ (cm) and C_4' unit frame $U_4 = (D_4 \times 3.14 - 8) \div 16 \fallingdotseq 6.5$ (cm), $U_4 \fallingdotseq 4.8$ (cm) (Figure 7-b, 6-d).

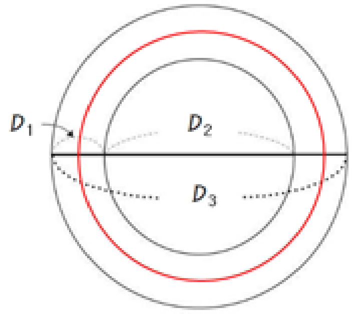

Figure 7-b: *Diameter of C_4*

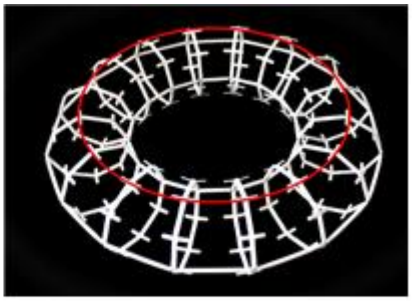

Figure 6-d: *C_4 using 4D Frame*

Let's calculate the length of C_5 and C_6 by calculating D_5 and D_6 by the Pythagoras theorem.
By the Pythagoras theorem, △OAB is a right-angled triangle.

$r^2 = 2(r - x)^2$ U

$r \fallingdotseq 4.5$

$x \fallingdotseq 1.3$ (cm) (Figure 8)

$\therefore D_5 \fallingdotseq (18 + 1.3 \times 2) \fallingdotseq 20.6$ (cm)

$\therefore C_5 \fallingdotseq 20.6 \times 3.14 \fallingdotseq 64.7$ (cm) ……… [Figure 6-e]

$\therefore U_5 \fallingdotseq (64.7 - 8) \div 16 \fallingdotseq 3.5$ (cm)

With the same method,

$D_6 \fallingdotseq 33.4$ (cm)

$C_6 \fallingdotseq 104.9$ (cm)

$U_6 \fallingdotseq 6$ (cm)

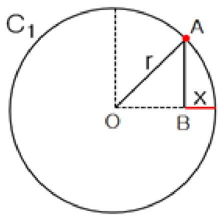

Figure 8: *Calculate D_5 from C_1*

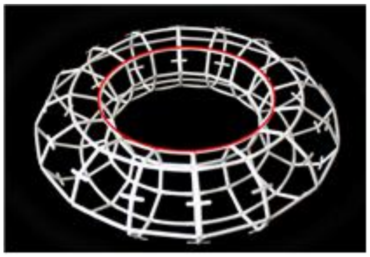

Figure 6-e: *C_5 using 4D Frame*

We can apply the torus by cutting half or quarter as in **Figure 9-a, 9-b.**

Figure 9-a: *a half 8-regular polygon torus (a half regular octagon torus)*

Figure 9-b: *combine two half regular polygon tori with 90 degrees*

Similarly we can make a 4-regular polygon torus (a square torus) as shown in Figure 10 and Figure 11.

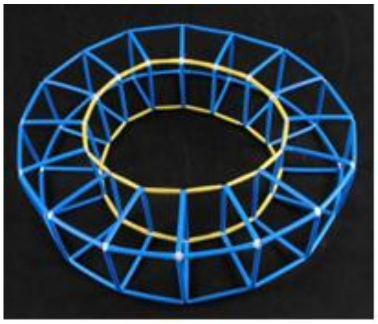

Figure 10: *a square torus*

Figure 11 *: application of a square torus*

4. Conclusion and Perspectives

As n approaches infinity, an n-regular polygon torus approaches a circle–torus. Varying the sizes and the number of polygons we will get different and applicable open or closed torus structures as Figure 12.

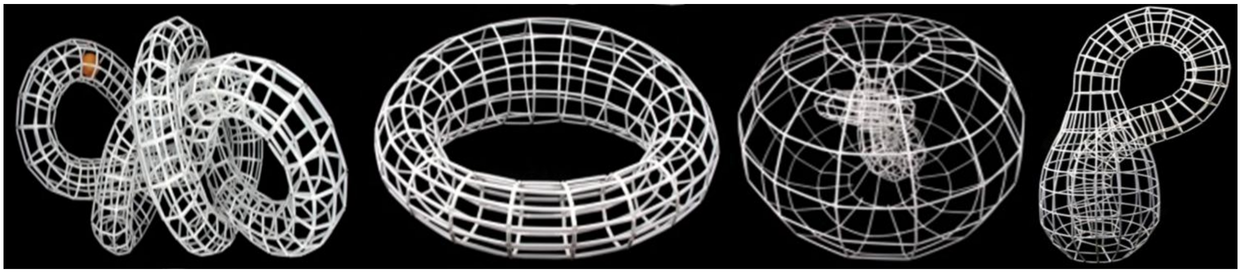

Figure 12: *a knot structure, application of torus, a knot structure in sphere, a Klein's bottle*

References

[1] http://en.Wikipedia.org/wiki/Torus.
[2] Ho-Gul Park, The 3Rd Soil, 4D Frame 1st ed.(2006), 4D land Inc
[3] http://www.4dframe.com
[4] Andrew Pressely, Elementary Differential Geometry (2nd Ed), Springer
[5] James R. Munkres Topology (2nd Ed), Prentice Hall

[i] 4D Frame only means the name of product. It has nothing to do with any four-dimensional geometry.
[ii] In mathematics, Pappus' centroid theorem (also known as the Guldinus theorem, Pappus–Guldinus theorem or Pappus' theorem) is either of two related theorems dealing with the surface areas and volumes of surfaces and solids of revolution.