The Mathematics and Art of Spirals Workshop

Ann Hanson
Science and Mathematics Department, Columbia College
Chicago, 600 S. Michigan, Chicago, IL 606005
ahanson@colum.edu

Abstract

In this workshop, the participants will learn several properties of spirals. Some of the properties are: that all spirals start from a fixed point and move out from that point in a regular manner. Other universal properties of spirals and their mathematical connections will be demonstrated by making an Archimedean spiral, a Golden spiral and a Baravelle spiral. The presenter will supply all equipment needed for the activities.

A spiral is a curve that moves out from the center in a regular manner getting progressively farther away as it moves out. The spiral shape is so prevalent that it is easy not to notice them. They can be seen in natural objects, such as in the shells of mollusks, sunflower heads and in vortices in air and water. Spirals are also seen in the coil of watch balance springs, the grooves of a phonograph record and in the roll of paper towels. In addition, all spirals have some common characteristics. They all have a fixed starting point (center) and move out from the center in a regular manner. Spirals grow by self-accumulation and solve a problem of efficient growth in terms of energy, materials and expansion within space.

The previously mentioned properties as well as other mathematical properties will be examined and demonstrated through hands-on activities. For example, the participants will make a one type of spiral called an Archimedean spiral using polar graph paper. (See figure 1) One of the properties of this spiral is that the angle formed by the spiral and the radius vector gradually increases as the distance from the center increases. Also the distance from the pole along the radius vector forms an arithmetic sequence. In order to construct this spiral, the participants will use the polar graph paper and start by identifying the center and then determine the rule that will be used in order to move out from the center. Examples of Archimedean spirals can be seen in a coil of rope, a roll of paper towels and in the groove of a phonographic record.

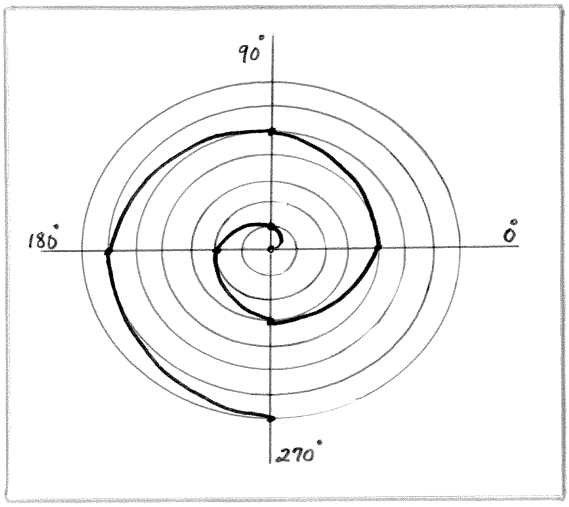

Figure 1: *Archimedean spiral*

A logarithmic spiral has the property that for a given rotation angle, the distance from the pole is multiplied by a fixed amount. When the fixed amount is the golden ratio, then a golden spiral is formed. If you start with a Golden rectangle and divide it into successive squares and rectangles, a logarithmic spiral is formed and this spiral has become known as a Golden spiral. (See figure 2) The logarithmic or equiangular spiral expands its size but never its shape and the distance from the pole along the radius vector corresponds to a geometric sequence. Also, the angle formed by the radius vector and the spiral is constant. Some examples of logarithmic spirals are animal horns, and sunflower and daisy heads.

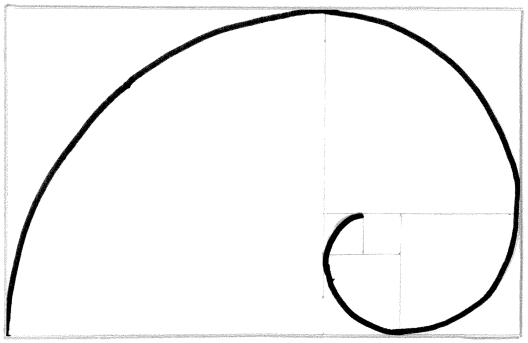

Figure 2: *A Golden spiral.*

One type of Baravelle spiral (see figure 3) will be constructed and its mathematical properties examined. First, start with a square and find the midpoints of the sides. Next connect the midpoints to form a new smaller square inside the original square. The participants will determine the type of right triangles that are formed during this process and the relationship of the area of the new square to the area of the original square. Repeating these steps creates a Baravelle spiral.

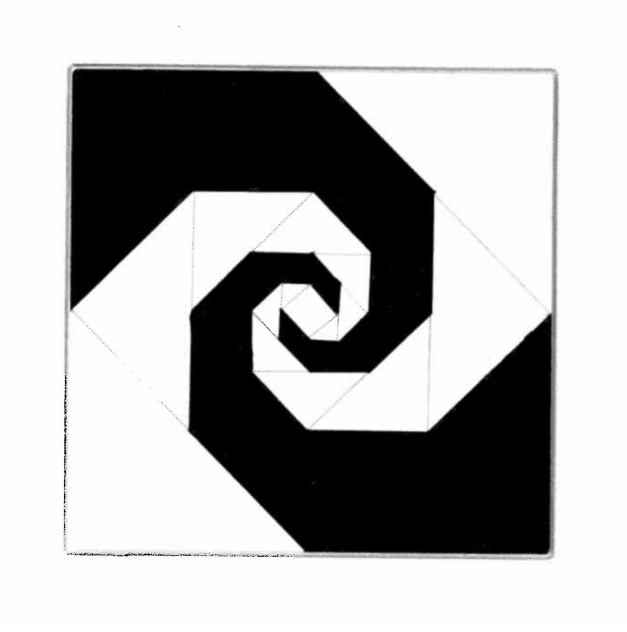

Figure 3: *A Baravelle spiral*

If time permits, the last type of spiral that will be constructed is an angular spiral (see figure 4). This spiral will be constructed with a compass and a straight edge inside a dynamic rectangle. Before constructing the spiral, the mathematics of dynamic rectangles will be examined.

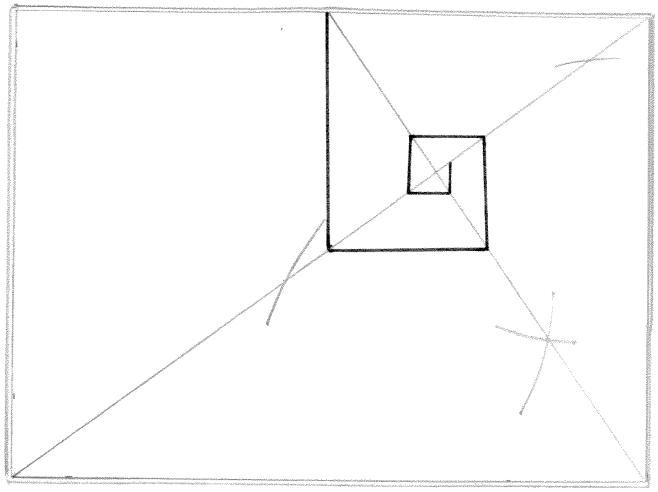

Figure 4: *An angular spiral*

In addition to learning the mathematics related to these spirals, the spiral could be used creatively to make math/art projects. If students learn about these spirals and are given an assignment to create a math/art project using the spiral, they can make some beautiful projects. Below are two samples of artwork that students from Columbia College Chicago created in their Math in Art and Nature class.

Figure 5: *Baravelle spirals by Tammy Karr*

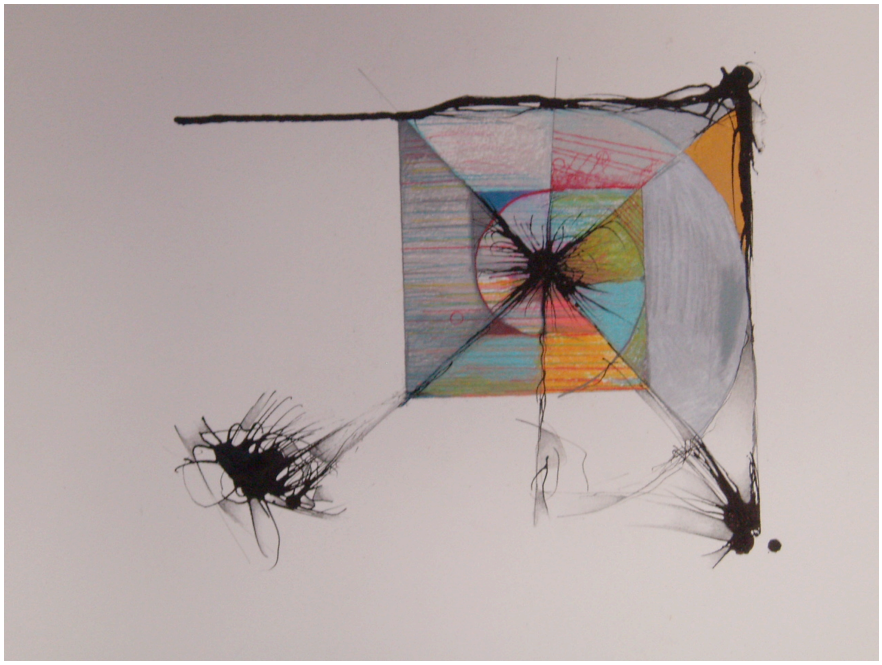

Figure 6: *Creative spiral by Benjamin Borter*

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