Aesthetic Appeal of Magic Squares

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Abstract

Magic squares are rectangular arrays of numbers whose row, column and diagonal sums are all equal. Magic squares are frequently constructed from underlying square designs with certain symmetries. However, these symmetries are hidden to the viewer who only sees the numbers. Consequently, it superficially appears that the driving appeal of magic squares is their numerical properties. In a recent paper, Fang, Ming and Jianmin provided statistical evidence that magic squares have superior aesthetic appeal over random squares. In this paper, we retest this conjecture on aesthetic appeal using a different more appropriate statistical test. We test the hypotheses that i) the numbers in a magic square or ii) the symmetries in the underlying square design enhance aesthetic appeal. Our conclusion is that while magic and symmetry-based squares superficially appear to have superior aesthetic appeal, this superiority is not statistically significant.

Introduction: Background and Goals

Magic squares are rectangular arrays of numbers whose row, column and diagonal entries are all equal. Several construction techniques for magic squares use squares or pairs of squares with underlying symmetries [4]. However, these underlying squares and their symmetries are hidden from the viewer who only sees the numbers occupying the cells of the magic square. Consequently, it would superficially appear that the primary appeal of magic squares is numerical, not artistic.

Fang, Ming and Jianmin provided statistical evidence for their *aesthetic conjecture*, a conjecture that magic squares do have aesthetic appeal [2]. To prove the aesthetic conjecture, the authors first coded each number to a domino like grid: for example, the number 1 was mapped to a single square, the number 2 was mapped to a 1 x 2 domino, etc. Then, each entire magic square was converted to an image, by coding each number in the magic square. The authors presented 40 randomly ordered squares, 20 with magic images and 20 with random images, to a group of 92 students who rated the images on a scale from 1 to 5. While statistical theory predicts that about 50% of the preferences would be for magic images, a 61% preference was observed, providing evidence for the aesthetic conjecture.

The experiment motivates the question *why*: what underlying feature of magic squares drives their aesthetic appeal? Two possibilities are: i) their numerical properties and ii) the symmetries of the squares used for their construction (which however are hidden from the viewer). In this paper, we re-examine the aesthetic conjecture as well as the conjecture that numerical squares with symmetry-based designs have aesthetic appeal, by using a different statistical test which is more appropriate. The experimental design is provided in the next section and results are presented in the final section. Our basic conclusion is that magic squares as well as squares based on underlying symmetries do not have superior aesthetic appeal.

Experimental Design

We use the Carus method of magic-square generation ([1],[4]) This is illustrated in the left hand square in Figure 1. The numbers 1,4,6,7,10,11,13,16 are written in normal order starting from the upper left corner, proceeding left to right and from top to bottom. The remaining numbers 2,3,5,8,9,12,13,14 are written in sequential order starting at the lower right corner, proceeding right to left and bottom to top.

1	14	13	4
12	6	7	9
8	10	11	5
13	3	2	16

1	15	14	4
5	11	10	8
9	7	6	12
13	3	2	16

Figure 1. *Magic (left) and non-magic but symmetry-based squares (right) based on Carus method.*

The right of Figure 1, presents a symmetry-based (non-magic) square based on a similar construction method. The numbers 1,4,5,8,9,12,13,16 are written in normal order starting from the upper left corner, proceeding left to right and top to bottom. The remaining numbers 2,3,6,7,10,11,14,15 are written starting in the lower right corner, proceeding right to left and bottom to top. In this particular example, the row sums of the square on the right of Figure 1 are identically 34; but the column sums differ.

Many such magic and symmetry-based squares are constructible. Each such square may be coded into an image square by mapping the number 1 to one dot, 2 to two dots, etc.

Raters were asked to rate the aesthetic appeal of square images by using 1 to indicate superior, 3 for inferior and 2 for indifferent. Technically, all we can say with certainty is that the ratings are *ordered*, 1 is superior to 2 and 2 to 3. We cannot however make meaningful statements of distance, for example, we cannot say the superiority of a rating of 1 over a rating of 2 is the same superiority (distance) as a rating of 2 over 3. In other words, we only assume the data is *ordinal*. For this reason, we chose the non-parametric median test of Mood [3, pg 447]. The null hypothesis is that all squares have the same median preference.

Results

There were 12 magic images, 12 symmetry-based images and 48 random images rated. The median of the magic images was 1.5 while the median of the random and symmetry-based images was 2. Thus superficially, one could argue that the raters preferred magic images; (in [1], preference-percentages were calculated). However, the *p*-value for the test was 44%. In other words, despite the superior median of the magic images, we have no reason to reject the null hypothesis that the median of all images is the same. *If* we assume that the ordinal rating reflects a continuous measurement, we can apply the Kruskal-Wallis statistical test. The results are similar: although the average rankings for the magic, symmetry-based and random squares were 30.8, 41.4, 36.7 respectively, suggesting different aesthetic appeal, the *p*-value for the test was 46.2% (41.5% with adjustment for ties) showing no statistically significant difference.

References

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