Math into Metaphor

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Abstract

Metaphor is not merely decoration but central to creative thinking. This paper reminds readers how metaphor is frequently used to generate mathematical terminology, and about metaphor's value in probing potential relationships among different fields of thought and between abstract ideas and the physical world. It walks through an exercise to develop some "muscles" in metaphorical thinking.

Shall I compare thee…?

"… to a summer's day?" asks the poet. Or shall I compare thee to a fruit bat or perhaps to πr^2?

Metaphor is the poet's stock in trade, an exercise in finding interesting things to lay one on top of the other. It may seem like a fancy and somewhat frivolous decoration, but metaphor is actually a process that underlies much—if not all—of our thinking. [1] It can be thought of as a process of tying abstract thoughts back to a physical world so that we more easily recognize patterns and relationships that apply across boundaries.

Thinking metaphorically is not a talent that some folks (like poets) have and others don't. It's characteristic of cognitive processes from birth – for instance, infants respond to entirely abstract configurations of dark and light squares as they do to human faces. They look longer at the ones lined up in a face-like pattern than they do at the other possible arrangements. A little older, and they will play at using a toy block as a car – mapping one set of ideas onto another to play with the common denominators. We are always endeavouring to map one pattern on another, because the results often produce new knowledge and insight.

Metaphorical thinking is like a muscle that poets may tend to use more consciously than mathematicians do, but we've all got the same muscles and we can develop their strength. As with other kinds of skill, the development process involves a back-and-forth between conscious awareness/analysis of the activity and the implicit memory systems that eventually make its use automatic and almost unnoticed. A star athlete takes her implicit knowledge of running developed as a child, consciously analyses the movements involved and practices the best ones to the point where *they* become implicit and automatic. Poets go through the same back-and-forth with metaphor. We can all follow the same process to become better runners or wordsmiths, even if we don't aspire to win Olympic wreathes. The benefit of developing 'metaphor muscles' is a potential improvement in how well we notice patterns and make connections in all fields of thought.

Metaphors under math

Math is also built with metaphors, though many of them are so deeply engrained by now that we hardly notice them. Take the number line, a concept so fundamental that we can hardly imagine thinking of the progression of 1, 2, 3 …. in any other way. Surely it is one of the most abstract notions we have, a infinite series of integers. And yet, when researchers look at our brains thinking about numbers, we act as if we're visualizing a 'real' line in space. In brief, we respond more efficiently to smaller numbers with the left side of our bodies and larger ones with the right; and we look to the left when thinking about smaller numbers and right when we're thinking about larger ones. [2]

There is no absolute necessity for us to think of the progression of integers as a straight line. Why not have a wheel as our underlying metaphor? It would have made modular arithmetic much more intuitive. Why not an expanding array of squares that builds up area instead of extension? But the number line it is and always will be, and we walk that line like a chain gang.

Such basic math metaphors are deeply rooted in our culture and continually reinforced by education systems. However, mathematicians are always dipping into the physical world to name (and describe) newer abstractions. Consider "wavelets" or "cardioids," a "building" or "moonshine" – all terms that have been adopted from ordinary experience as labels for complex abstractions.

"Well, we've got to call them *something*," you may think. Is this kind of label any more significant than calling something a Lagrangian or a Julia set? Many a math concept is simply tagged with the name of a mathematician who developed it. What's the difference?

The difference relates to the way in which metaphor is a fundamental process of thinking, the brain's way of noticing relationships between different things – not necessarily one-to-one correspondences, but patterns that are looser, more suggestive and ultimately more creative. Labelling something a "Cantor set" or a "Hamiltonian" ties that idea to a specific individual and the history of math. This provides valuable context, no doubt. But I would argue that a label like "wavelet" opens up possibilities for working with that idea in a different way. It involves a kind of playfulness that is not mere frivolity – it provides a way of keeping connections open-ended, connections that might not be made in the step-by-step process of working out a mathematical proof.

The term "moonshine" is a case in point. It came about after John McKay noticed that the same large number pops up in two widely separated areas of mathematics: group theory and number theory. John Thompson and John Conway went on to explore the connection and Conway dubbed it "moonshine," a word suggesting daftness, illicit stills up in the hills and A Midsummer Night's Dream, but also suggesting a source of illumination in darkness, a mysterious world that beckons explorers. [3]

The moonshine conjecture was subsequently confirmed. It has since extended its light into other parts of the mathematical forest, including theoretical physics, where it connects with string theory – a connection that potentially brings highly abstract math back to the physical world. (And note that 'string' is itself a metaphor from our everyday world adopted into the realm of abstract thought in an attempt to understand something buried even deeper in that 'ordinary' realm.)

Calling a concept "moonshine" instead of something like "the Thompson-Conway conjecture" is an invitation to explore it through non-deductive methods such as intuition, analogy, and speculation—approaches that have long been associated with creativity in mathematics. [4]

An exercise in metaphor

This section of the paper outlines an exercise to help practice thinking in metaphor. The approach works well across a wide range of ages, and it can also be an excellent way of helping students 'get' an overall concept in mathematics by looking beyond the formula to the relationships it embodies.

As an example, we will start with that abstract concept of the number line and bring it back to the world of ordinary experience – and in the process, play around with the ideas that underpin it in the mathematical realm.

First step: think of the number line and make a list of the mathematical features it has. For instance, integers are arranged along the number line in a specific order and can't be moved out of it. This has implications. For example, it is the basis for the commutative property—$(a + b) = (b + a)$—which wouldn't hold true if numbers could straggle around any old way. In between the integers, there are all sorts of other numbers: rationals, irrationals, transcendentals. There are infinitely many such numbers between any two integers. There are positive and negative numbers, with the strange ambiguity of zero in between. There are many other characteristics of the number line, but let's start with these.

Step two is to make a list of lines we experience in our ordinary world. A clothesline, with pegs on it. The queue at a supermarket checkout. Boot laces. A flight of steps. A tape measure. The aim is to come up with as many stringy things as you can. Choose one of these examples, and make a list of words associated with it. The familiar experience of the supermarket queue, for instance, might prompt "shopping cart" and its contents like lettuce, ground beef, bananas, plus the tabloid papers in the racks, the shuffle, the sounds of the cash register, and the clerk's automated "Did you find everything you were looking for?"

Step three is to think about how the abstract mathematical relationships of the 'number line' concept might map onto relationships in the ordinary world. That clothesline, for instance – its pegs might become the integers, with the sagging line of the towels hung between them like an extra dimension where we fit the rational numbers. Boot laces: they have to go through the holes in a particular order or it screws everything up. Maybe you think of the positive integers going down one side, the negative ones down the other.

Step four is to play with the connections. Write down phrases, sentences, a paragraph using them playfully. You may be surprised that it turns serious or leads in directions you did not expect. This is not a time for proof, it's an exercise in open-endedness and the pursuit of moonshine. So at the checkout counter, there's a woman with her cart piled high, a man with a newspaper and two bananas. It doesn't matter what order they go through the line, $a + b$ will still equal $b + a$. You still have to wait until every sub-item is counted and it will all add up to the same amount of time to wait. Your lettuce is wilting, the milk is going sour, the tabloid headlines are about people having much more interesting lives. The commutative property becomes a metaphor for the whole idea of *tempus fugit,* and many a poem has been written on *that.*

Through this exercise, you can take just about any idea from mathematics, whether simple or complex, and translate it back into experiences from the physical world. It works most easily with those terms that have a resonance in ordinary language, like moonshine. But it can be done with any term. Try it with πr^2 – what kind of person might be the area of a circle? Or what kind of person might be an irrational number?

To conclude, here is a poem that grew out of thinking about the number line and observing an everyday experience: a little girl who was just learning her numbers.

The number line

Little girl drags the cat across the living room,
like a resigned slinky toy.

"How old are you, Marie?"
"Five *and a quarter."*

She already knows her real numbers
and the gaps between the integers –
how they stretch enormously long
like coils on the spiral toy
that swings its elastic length down stairs
one step at a time.

And she understands the paradox – although
every segment of the number line
can be slivered into days and hours and long, slow
minutes of waiting,
the series converges to a limit.
Six more sleeps 'til Christmas.

The cat waits to escape. [5]

References

[1] Turner, Mark, *The Literary Mind: The Origin of Thought and Language,* (Oxford, Oxford University Press, 1996) See, in particular, Chapter 2.

[2] Fias, W. & Fischer, Spatial representation of numbers. In *Handbook of mathematical cognition*, (ed. J. Campbell) 2005, pp. 43-54

[3] Carlton, V.L., An analysis of the educational concepts of fourteen outstanding mathematicians, 1790-1940, in the areas of mental growth and development, creative thinking, and symbolism and meaning. 1959. Cited in Mann, Eric Louis, "Mathematical creativity and school mathematics: Indicators of mathematical creativity in middle school students" (2005). *Doctoral Dissertations*. Paper AAI3205573. http://digitalcommons.uconn.edu/dissertations/AAI3205573

[4] Ronan, M. *Symmetry and the Monster: The Story of One of the Greatest Quests of Mathematics,* 2007, pp. 216-220

[5] Major, Alice, *The Occupied World*, 2006, p. 81