Retrograde Rotation Illusions in Turntable Animations of Concentric Icosahedral Domains

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Abstract

I have constructed for comparison two sets of computer models of the icosahedron, dodecahedron and icosidodecahedron using 3D modeling software. One set uses coordinates attributed to Hess [1] and the other fits within a unit radius circumsphere. Each polyhedrons' sets of vertices, edges, faces, bounding boxes, polysurface and circumsphere are constructed upon named layers that can be turned on or off within a tree of layers. These trees of data constitute the *Taublock* and the *Uniblock.* Three copies of each block are scaled proportional to a golden ratio geometric series: τau, 1, 1/τau. These triads are used to populate turntable animations with varying display parameters for visual effect. With apparent motion and programmatic selection of layers and display attributes these objects, alone and in combination, provide for many visual surprises such as retrograde rotation illusions and the appearance of phantom faceted polyhedra.

Introduction. I think of polyhedra as the elements of design. For years I have used polyhedra to hone my skills with computer aided design and drafting programs (CAD). In this project I produce turntable animations with unusual visual effects using unit radius circumsphere models (Figure 1) and models nuanced by their dependance on the golden ratio (Figure 2). I have also included for continuity with Bridges past, a comparison (Figure 8) of the spline curves, *cyclons* [2] that these figures produce.

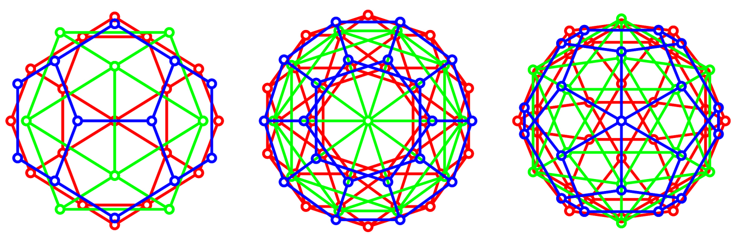

Figure 1 *Uniblock in 3 views aligned to an icosahedron's: mid-edge, vertex and mid-face (left to right).*

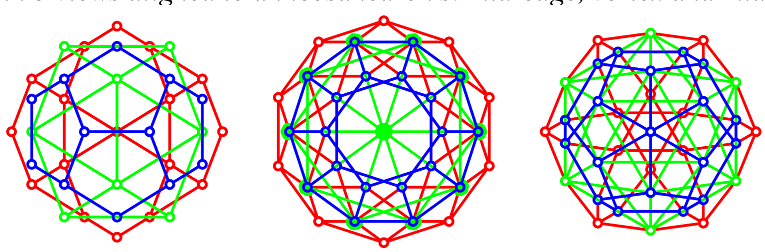

Figure 2: *Taublock in 3 views.*

Block Construction. Using Rhinoceros software I have built a model consisting of three polyhedra, the icosahedron, dodecahedron and icosidodecahedron using coordinates reported by Coxeter [1] as (x,y,z) triples of easy-to-remember values: 0, ±1, ±τ, ±1/τ (c.f. Table 1). The golden ratio, τau = (1+√5)/2 or 1.618... is elsewhere reported as phi φ (the great Tau-Phi pull!). By calculating the coordinate triples it was easy to convert the coordinates to a table of comma-separated string values that could be copied and pasted into Rhinoceros' point command. Then it was a simple matter to connect the dots with polylines and surfaces to complete very accurate models. In previous models I used techniques that required many translations and rotations which introduced rounding errors that would regularly prevent surfaces from combining into solids. This new method of constructing these polyhedra is clearly superior.

Uniblock / Taublock. The main difference between these two assemblies is that within the *Uniblock* each polyhedron interpenetrates the others. In the *Taublock* the dodecahedron is wholly enclosed by the icosidodecahedron. Though each block contains the same polyhedra, the variant circumsphere radii in the *Taublock* aligns certain features; imparting a visual simplicity to 2D representations of the Hess coordinates. By forcing the polyhedra into a unit sphere the *Uniblock* is unnecessarily complicated.

Triad Construction. To add depth to the subject of these animations I copied and scaled both the *Taublock* and *Uniblock* recursively around their respective origins using a geometric series of the golden ratio: τ^1, τ^0, τ^{-1}. If it were not for transparency only the outer layer of these assemblies would be visible. With transparency an infinite regress is implied (Figures 3 and 4).

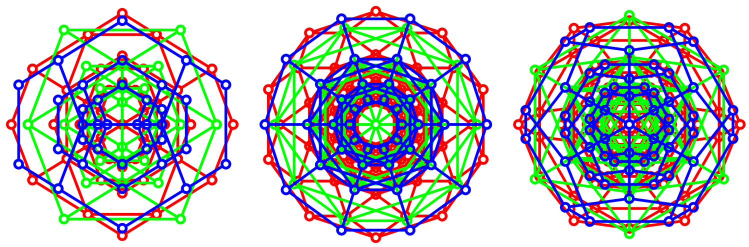

Figure 3: *Uniblock triads.*

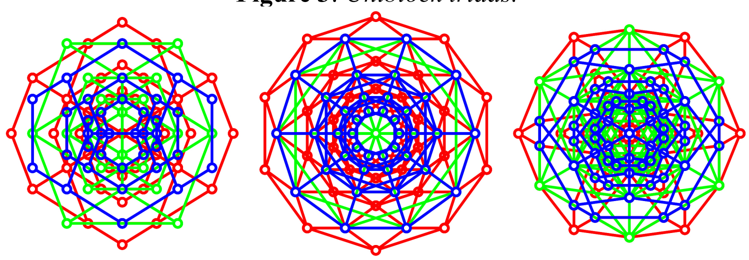

Figure 4: *Taublock triads.*

Animations. Working with symmetrical 3D objects on 2D screens is fraught with visual difficulties. Despite years of experience, I often think I am working with elements in the foreground only to discover I've attached objects to the backside. During model construction I am forever nudging and rotating the view to get a better angle on the data displayed. For a CAD operator movement of the scene provides extra clues as to foreground and background, enabling more accurate manipulation of one's model. When presenting one's CAD work, turntable animations are effective, quick to render. Sometimes with surprising results

For reasons explained below in the Rendering section, the animations I have created for this project use default lighting schemes with display modes that permit real time or near realtime results at a rate of over 4000 high definition (1920x1080) images an hour. Each animation is created in one of 3 named views corresponding to the axes of symmetry of the icosahedron: *IcosaMidEdgeNormal*, *IcosaMidFaceNormal* and *IcosaVertexNormal*. Each animation records in 2D, a representation of a 3D assemblage in rotation about the y-axis of the selected view.

Rendering. Rhinoceros is supplied with several display modes including: wireframe, shaded, ghosted, and rendered. Also Rhinoceros supports a sophisticated ray trace plug-in, Flamingo, which can produce very realistic images, if you are willing to take the time to set up a scene with light sources and go have coffee (or a vacation) while your machine crunches the numbers. Placing light sources in one's model with Flamingo is problematic. I have had limited success with it because of the temporal disconnect from setting the lights in the scene and waiting for the rendered results. My use of transparency as a material property obfuscates lighting the scene as well. Perhaps next generation 3D displays and physics engines will allow more realism in future work.

Illusions. Of course, every pattern of light and shadow that we interpret while watching our screens is an illusion, dependent upon 'persistence of vision'. Painters and other visual artists exploit other visual cues in their work to deceive the eye (trompe l'oeil): perspective, atmospheric bluing, occultation, and scale (larger in the foreground than the background). What seems to be happening in my project's context is akin to the 'face mask illusion' wherein a rotating face mask with both concave and convex aspects is interpreted only as convex (interestingly schizophrenics are not fooled). In the face mask illusion the play of light and shadow confuse the brain and we default to the 'I see a face like the ones I'm used to!' mode of seeing.

Another link to the face mask illusion is in the word polyhedron itself. *Hedron* derives from the Greek for face. With transparency, both concave and convex views of the subject polyhedron are visible at the same time, although with differing luminosities. An amusing aside, I use iPhoto to catalog my geometric images. When I turned on iPhoto's face recognition feature, many of my pictures were flagged as having faces to be identified.

Without witnessing my animations you will just have to believe me that these illusions exist. Some people with whom I have shared these movies do not acknowledge having seen them. The motion illusion I am reporting may be entirely a figment of visual cortexes like mine that seem to give precedence in the scene to regions of higher luminosity.

The default lighting routines of Rhinoceros and its interpretation of transparency result in back faces being brighter than front (Figure 5). Despite these faces being in the background and appropriately smaller, my eyes fixate on them when interpreting the scene making them pop to the foreground.

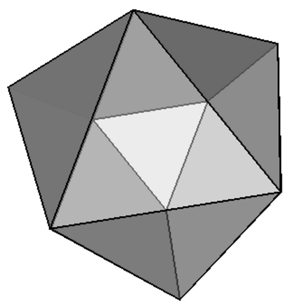

Figure 5: *Icosahedron with transparency. The small bright triangle is the rear face of the icosahedron.*

The phantom faceted polyhedra appear when multiple polyhedra are displayed (Figure 6). Here an icosahedron and dodecahedron with common circumsphere interpenetrate. It reminds me of looking into a geode. This phantom polyhedron is the core of the two parent solids. Here the pentagons are sections of the dodecahedron's faces surrounded by 5 irregular hexagons that are part of the icosahedron's faces. This core is produced by the boolean intersection of the icosahedron and dodecahedron (Figure 7).

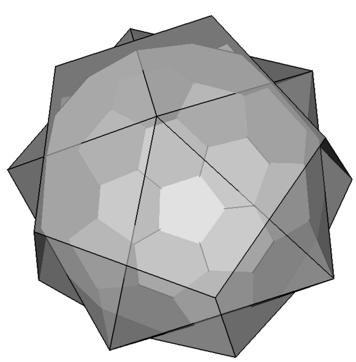

Figure 6: *Icosahedron and Dodecahedron and phantom Hex Pent polyhedron.*

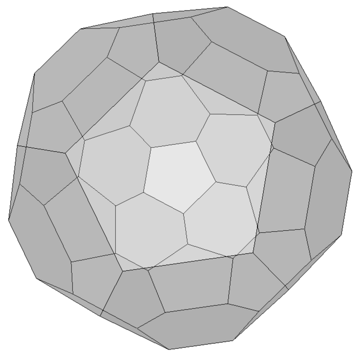

Figure 7: *Boolean intersection of Icosahedron and Dodecahedron.*

Conclusion. I began using computers to model polyhedra in 1973 when I wrote a program to generate geodesic dome coordinates with no graphical output, only numbers. Forty years later with lots of experience with graphical user interfaces I have returned to an appreciation of the simplicity of starting with the numbers. There is a poetry to Hess's coordinates. (Table 1). These visual artifacts and these illusions are eye candy, if you will. They seem to be why I keep clicking away in front of my screens.

X	Y	Z
0	±τ	±1
±1	0	±τ
±τ	±1	0

Table 1: *Icosahedral coordinates using the golden ratio.*

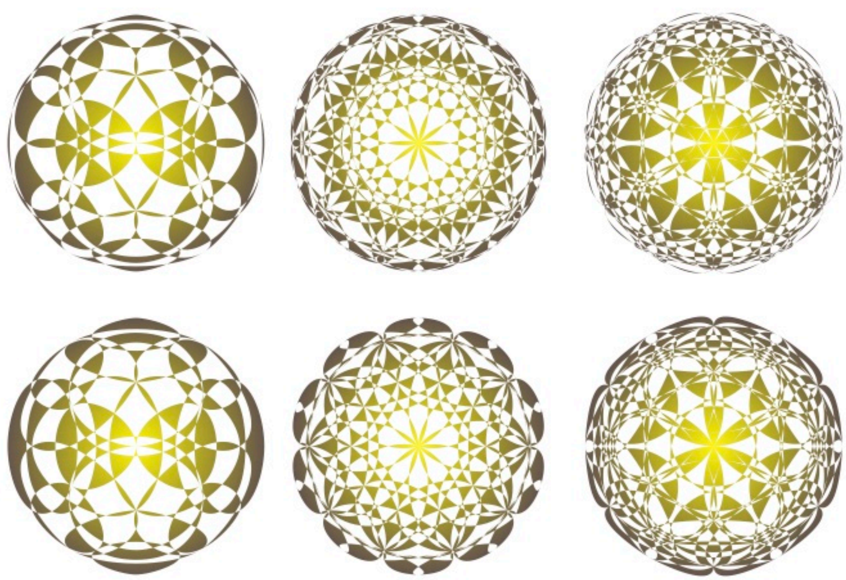

Figure 8. *Cyclons [2] of the Uniblock (top) and Taublock.*

References

[1] H. S. M. Coxeter, *Regular Polytopes,* Dover Publications Ltd., pp. 50-54. 1973.

[2] C. L. Palmer, "Digitally Spelunking the Spline Mine", Renaissance Banff, Mathematics, Music, Art, Culture, Conference Proceedings, pp. 309-312. 2005.