Braids: A Mathematics Documentary

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Abstract

We present a mathematical documentary about braid theory realized with computer generated animations. Braid theory is an example of a deep mathematical subject that involves different areas (mainly topology and group theory), has various applications and is a very active research subject. Many notions can be introduced in an informal and visual way: this makes braids significant also from the viewpoint of popularization since artistic and aesthetic issues are crucial for the communication with the general public. Our documentary is available under a Creative Commons licence at http://matematita.science.unitn.it/braids.

1 Introduction

Braids are ubiquitous in everyday life: hairdressing, knitting, ropes, wicker baskets and bread are only a few objects where braiding is used. Also art offers a wide range of examples: celtic knots and braids, jewelry and many decorative motifs in textiles, buildings, monuments and mosaics. On the other hand, braids are the objects of a very active research field in mathematics. The importance of braids in mathematics and their everywhere presence in art and in common objects make them a suitable and interesting them for popularization.

We realized a mathematics documentary in computer generated animations to give an introduction to braids as mathematical objects. Computer graphics can be considered a form of art: animations are used to fascinate the public and to draw attention. Also the soundtrack is crucial for these purposes.

In this paper we sketch the motivations, goals and contents of our documentary and describe how we realized it.

2 Motivation and goal

Braids were introduced in topology by Emil Artin in the 1920s [3] and have received increasing attention since then. Their study involves mainly topology and group theory but has applications in many areas, ranging from knot theory to theoretical physics, from biology to robotics. Research is very active nowadays in all the mentioned areas.

As three-dimensional topological objects, braids are very suitable to be visualized and described in an informal way: it is easy to draw beautiful images of them, that can be understood and enjoyed also by non-specialists. Moreover, continuous movements that play a crucial role in the definition of braids are very well visualized through computer animations.

On the other hand computer graphics fascinates the public and can put it in contact with science and research in a pleasant way. This is why artistic and aesthetic issues are crucial in our documentary. The success and diffusion of mathematics documentaries such as *Dimensions* [2] and *Chaos* [1] show that there is a wide request for such realizations, coming from both the scientific world and the general public.

We chose to address a non-specialist audience: all the themes are presented in an informal way. We focused on an audience made of students in last years of high school and in the first years of university and teachers interested in new ways of teaching and motivating students. The documentary can be an occasion for them to approach a field of mathematics that is usually not present in school curricola. However, we hope to give the possibility to see, understand and enjoy braid theory at many different levels, from young students to mathematicians expert in related fields. We would like to invite a wide audience to approach mathematics without fear and prejudices, giving an idea of a current research topic and showing connections among different fields of mathematics.

Although the narration is mainly informal, many results and ideas presented in the documentary are very deep. We chose to insert some mathematical notations to invite the audience to get in touch with mathematical formalization. In order not to intimidate a non-specialist audience, formal notations are always accompanied by animations, to make the sight enjoyable even if the contents may result not always fully understandable without careful consideration or some further explanations.

3 Content

Our documentary is divided into four chapters, presenting different perspectives on braids. Each chapter is about 15 minutes long. We present a brief summary of the contents together with some stills from the documentary, one from each chapter. Most of the topics are described formally in [4] or in the references given there.

Chapter 1 is an introduction to braids: here they are presented as topological objects. Continuous deformations without cuts (called isotopies) are allowed: braids that can be transformed into each other by such deformations are considered the same. Braids having the same number of strands form a group where the operation is composition, which consists in putting a braid after the other and connecting the strands. The documentary shows generators and relations to give the Artin's presentation of the braid group.

Chapter 2 presents an algorithmic problem, the word problem in braid groups: given any braid, can isotopies deform it into the identity braid? Two algorithms are presented to solve it: the first one was discovered by Artin, but its complexity is very high. The second one is a recent result and offers the opportunity to ask an open question about its complexity.

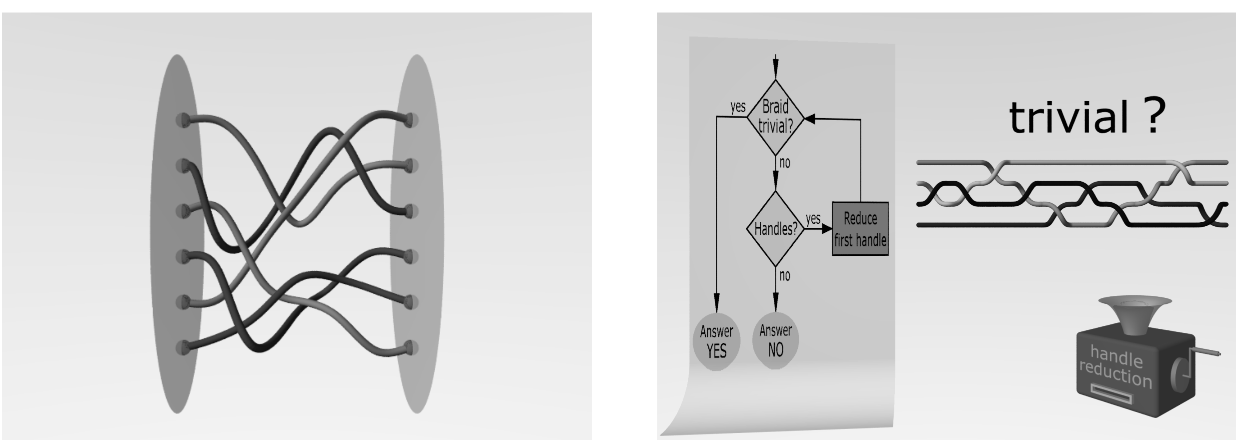

Figure 1: *(left)* A braid is a collection of strands with some additional properties. *(right)* The representation of an algorithm to detect trivial braids.

Chapter 3 introduces other topological objects, knots and links. Some examples illustrate the problem of their classification. Connections between links and braids are drawn and some classical results are presented. Finally, the Jones polynomial, one of the most powerful invariants of links, is presented. Jones' discovery revolutionized the study of knot theory and earned him the Fields Medal in 1990, the most prestigious award in mathematics.

Chapter 4 starts with a traditional dance around the maypole: dancers twist colored ribbons around the pole to create a braid. This suggests to formalize braids as motions of points and this insight yields a characterization of braids as dances. A specific class of dances is considered, corresponding to a subgroup of the braid group. It allows to present new connections between braids and links that recall those displayed in Chapter 3. The chapter ends with the statement of another algorithmic problem, called generalized word problem. The problem presented in Chapter 2 is a particular case of it.

Figure 2: Braid are strongly related to knots *(left)* and dances *(right)*.

4 Developing the documentary

The realization of the documentary was a long process involving many recursive steps: studying topics in braid theory, writing a storyboard, realizing short movies, checking the balance and mutual significance of images and texts, checking the rhythm and the logical thread of the narration, modifying the storyboard, and so on. At the end of this process a draft version of each chapter was shown to mathematicians, artists and students, to collect useful comments. Taking into account those comments many improvements were made with the aim of making the documentary less technical and more understandable to a wide audience, adjusting the rhythm, making beautiful and appealing animations.

The following steps were the recording of the narrative voice, the insertion of opening and end credits, the composition and arrangement of the music and the final editing.

The music for the soundtrack was composed, arranged and synthesized specifically by Raul Masu. Special attention was paid to a variety of aspects, such as the relevance and connection to images and animations. In fact, music should remain in background and not overwhelm the explanation. Moreover, it should highlight the important moments and concepts, and link with the same musical ideas different moments where the same mathematical tools are used. As an example, in Chapter 2 the same sound is played every time the grinder runs. An important aspect was the choice of a leading theme for the opening of each chapter to identify the whole documentary.

For the realization of the documentary we made use of free and open source software only. The main part of the work was realized using POV-Ray [6], a rendering software, that is, a tool to generate images from a three-dimensional virtual model. More precisely, POV-Ray uses a raytracing technique: given the description of a virtual scene and the coordinates of an image window and of a viewpoint, the software traces rays that simulate the paths light would follow from the viewpoint through each pixel of the window in the scene. The color of each pixel in the window is calculated simulating the effects of light on the objects into the scene. The output is the image calculated as the projection of the scene from the viewpoint onto the image plane.

The virtual scene is described using a specific language which has a small set of "primitives", that is, elementary geometric forms such as triangles, spheres, cylinders, prisms. Complex objects are constructed as unions and intersections of the elementary ones. All the objects are described in coordinates. In the rendering process a crucial role is played by the properties of surfaces and of lights, that are specified in the POV-Ray language.

The software produces still images, but it is possible to use a parameter to encode time and describe animations (motions, deformations, etc.) depending on it. Then the stills are encoded into a movie. For this purpose we made use of MEncoder [5].

5 Acknowledgement

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