Mathematics Education and Early Abstract Art

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As a group, the artists educated near the turn of the 19th and 20th centuries possessed greater mathematical knowledge than expected of artists today, especially regarding constructive skills in Euclidean geometry. Educational theory of the time stressed such skills for students in general, who needed these to enter the workplace of the time. Mathematics teaching then emphasized the use of manipulatives, i.e., visual and interactive aids thought to better fix the student's acquisition of mathematical skills. This visual training in mathematics significantly affected the early development of abstraction in art. This paper presents examples of this visual mathematics education and samples its effects on the development of abstract art in the first decades of the 20th century.

Introduction

Today's art student can train with nary a nod to mathematics. Although art education remains affected by the innovations of mathematically, especially geometrically adept artists from nearly 100 years ago, its application of mathematical elements requires no substantive experience with mathematics. Instead the fledgling artists handle these elements procedurally. Perspective, for example, was part of the genesis of a sophisticated new geometry of projection during the Renaissance, but students learn to apply it by a set of procedures involving the determination of a horizon line and vanishing points and the construction of converging lines. Executing this set of procedures needs no knowledge of the underlying how and why of 3D objects mapped onto a 2D surface.

This was not always the case, as practical geometry was once a more important technical skill than it is today and consequently received greater stress in education. In the development of abstract art geometric objects and patterning offered a ready-made and familiar category of abstract objects to which artists could refer. It helped, too, that the mathematical instruction of the pioneering abstract artist accentuated visual comprehension of principles.

Models of Abstraction

When the Russian sculptor Naum Gabo arrived in Munich in 1911 to study engineering, Germany was the ideal place to see physical models of algebraic surfaces, as they would appear when graphed into 3D coordinates. These were the products of model-making firms in Munich that marketed to universities worldwide. In the latter 19th century the use of visual learning tools dominated education there, where the philosophy of *anschaulich* held sway. *Anschaulich* can variously be described as "accessible to insight" or "imaginable", but the term has no direct English translation [13]. It carries the connotation of thinking by developing mental pictures of abstract relationships and then making these visible to the mind's eye.

One famous outcome of this educational philosophy was Albert Einstein, who learned under this system and credited his discovery of relativity to such mental visualizing. Equally famous was the influence of Froebel blocks (Figure 1), designed in the 1830s by German educator and founder of kindergarten, Friedrich Froebel, on the architect Frank Lloyd Wright (Figure 2.) Late in life Wright wrote of that influence in his autobiography:

> *That early kindergarten experiences with the straight line; the flat plane; the square; the triangle; the circle!*

> *…the square became the cube, the triangle the tetrahedron, the circle the sphere. These primary forms and figures were the secret of all effects . . . which were ever got into the architecture of the world.* [6]

Froebel placed primacy on the child constructing a conceptual and visual architectonics of space. Wright's quote is an echo of Froebel's theories:

> *The importance of the vertical, the horizontal, and the rectangular is the first experience, which the child gathers from building; then follow equilibrium and symmetry. Thus the child ascends from the construction of the simplest wall with or without cement to the more complex and even to the invention of every architectural structure…* [5]

Anna Wright purchased the blocks after seeing them at the Centennial Exposition of 1876 in Philadelphia [18]. At that time in the United States German educational tools were in demand and marketed here. The fame of these tools had spread worldwide. A show of surface models collected by Felix Klein for instruction at the University of Göttingen, for example, crossed the Atlantic in 1893 to be featured at the World Columbia Exposition in Chicago [13]. Klein and fellow mathematician Alexander Brill had earlier founded the best known of the model publishing firms in Munich, where they produced plaster, string and cardboard models for shipment throughout the Western world.

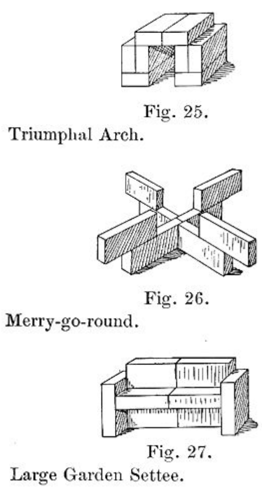


Figure 1. *Friederich Froebel, Gift #4 Forms of Life, included in Edward Wiebe's "Paradise of Childhood", 1869. A how-to book on the use of Froebel's blocks still in print today [19].* **Figure 2**. *Frank Lloyd Wright, Unity Temple, 1904, Oak Park, lL. Library of Congress, Historic American Buildings Survey Philip Turner, Photographer June 1967*

Nowhere in the U.S. were the educational innovations of Germany more sought after than in the mathematics departments of a then burgeoning university system. By 1893 Klein was no longer in the model publishing business, but had gained a reputation as the world's foremost mathematics educator. His travels in the U.S. on the occasion of the Chicago exhibition brought him to New York where he met with professors of mathematics from throughout the country. From this meeting emerged the American Mathematical Society.

Practical Geometry

The mathematics education of the early 20th century innovators of abstraction in art varied in degree, but not in the overall emphasis on practical geometry and on visual instruction. Since the late Middle Ages practical geometry, in the form of compass and straightedge constructions of Euclidean geometry, was considered a necessary component of the fine artisan's training.

Albrecht Dürer addresses this in the introduction to his 1526 geometry text "Unterweisung der Messung mit dem Zirkel und Richtscheit":

> *It is this skill, which is the foundation of all painting. For this reason, I have decided to provide to all those who are eager to become artists a starting point and a source for learning about measurement with rulers and compass. From this they will recognize truth as it meets their eyes, not only in the realm of art but also in their proper and general understanding...* [3]

A set of geometric drawing tools was part of the stock in trade of the established artist into the 19th century. Gilbert Stuart, internationally acclaimed as a portraitist and best known for his paintings of George Washington, acquired a fine set of 30 brass tools after making his name during study in London. Stuart's set was a magazine crafted by the firm of George Adams, the scientific instrument makers to the court of King George III. A magazine was the most extensive kit offered by the firm and its use required a correspondingly extensive knowledge of geometry.

By the late 19th century an industrializing society reinforced the teaching of practical geometry in public schools, as the need was promulgated for machinists, engineers and other industrial craftsmen. Those inclined toward visual careers often began their higher education in technical and design institutes before switching to art. This was the case for a number of pioneering artists in the early decades of the 20th century. In such schools students learned the geometry of curves with structural applications, such as the catenary and parabola, and curves with mechanical applications, such as cycloids and pursuit curves [4]. Students learned to construct these using compass and straight edge.

Visual Manipulatives

The field of geometry had in the meantime advanced well past its Euclidean origin and mathematics educators began to lobby for a more analytic approach to geometry that emphasized algebraic over visual study. They sought to provide a base for later study in higher geometry. Resistance was strong and visual manipulatives remained the standard in education. Joshua Holbrook introduced one popular set of visual models used in the United States for elementary education in 1833. By 1870 its use was mandated by law in over 2000 schools [10].

One set of teaching models that particularly stirred critics as being too objective and distracting from formulation were those made by W. W. Ross, superintendent of public schools in Fremont, Ohio from 1864 until his death in 1906. Ross's models addressed geometry in higher grades and used dissection to demonstrate the origin of curves and surfaces normally studied analytically. Ross' set was extensive, comprising 18 plane figures and 23 solids, half of which were dissected. In the introduction of his manual, "Mensuration Taught Objectively, with Lessons on Form," he avers:

> *... every ordinary operation in the mensuration of surfaces and solids with possibly one exception can be taught objectively and illustratively so that the pupils shall perceive the reasons of the steps from the first, and the operations themselves shall become the permanent property of the reason rather than the uncertain possession of the memory.* [16]

Arguably, the first mathematical objects to appear in 20th century abstraction were the geometric solids represented in sets like those of Holbrook and Ross. Giorgio DeChirico, for example, populated many of his paintings with manikins, whose body parts referenced these models. De Chirico described his imagery as metaphysical, an effect augmented by his allusions to mathematics. DeChirico's geometric shapes even bore inscribed lines reminiscent of those appearing on the instructional models used in primary and secondary schools (Figure 3). Shapes like cones and spheres often featured curves to define important sections: a cone, for example, might include the engravings of circles, ellipses, parabolas and hyperbolas to demonstrate the conic sections.

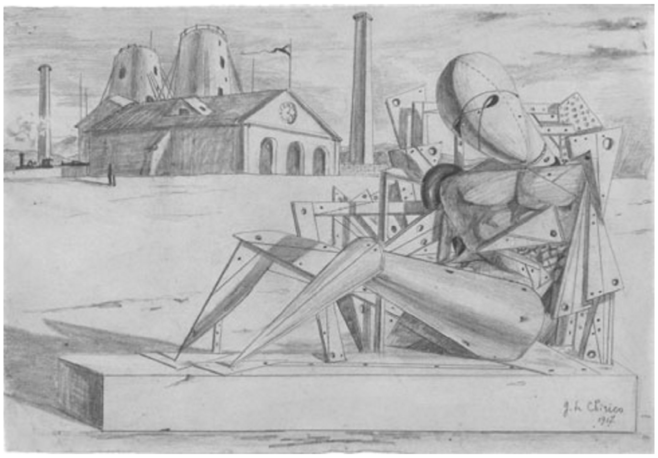


Figure 3. *Giorgio de Chirico, Solitude, 1917, Pencil and wash on paper, 8 1/4 x 12 5/8" Gift of Abby Aldrich Rockefeller (by exchange) and Purchase. © 2008 Artists Rights Society (ARS), New York / SIAE, Rome.* **Figure 4**. *Man Ray, Mr. Woodman, photograph, ca. 1925.*

Strong visuals as mnemonic devices were not just the purview of sculpted models, but appeared in texts as well. Most noted in this regard was Oliver Byrne, whose 1847 adaptation of the first six books of Euclid minimized text and labels in favor of brightly colored visuals (Figure 5). Covering Euclid's exposition of plane geometry and proportion, Byrne restricted his colors to the three artistic primaries, red, yellow and blue, and black and white [1]. Byrne's diagrams looked much like the paintings of Constructivist and De Stijl art of the coming century.

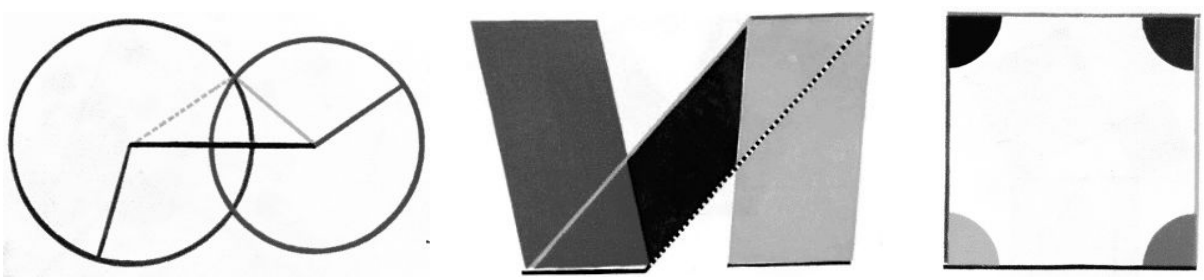


Figure 5. *Oliver Byrne, diagrams from The First Six Books of the Elements of Euclid, William Pickering, London, 1847 [1]. Pages 22, 37 and 47, respectively.*

In England of the late Victorian era a push toward phasing out the teaching of constructive methods in England was thwarted by the impassioned argument that its elimination would damage the moral character of British schoolchildren. The culture at large perceived constructive geometry as a character-building tool. The English language reflects this with more than its share of metaphors for expressing moral behavior in terms of geometry. Trustworthy people are "straight" talkers and "upright" citizens who are "on the level", while those who talk in "circles" lead us to wonder what their "angle" is. To be prepared is to be "squared" away.

Moreover, prevailing philosophy asserted that Euclidean geometry was not just epistomologically valid, but certain. Its truth went deeper than just common sense. Kant's notion of a priori structures required that our minds be hard-wired for Euclidean geometry and that this geometry corresponds with the essential nature of space. At the physical scale at which life is lived the certainty of Euclid's constructs had proven itself over and over again. At the scales – astronomical and atomic – toward which physics was trending, they did not work.

Though Gabo could view plenty of algebraic models during his studies in Munich (where he also met De Chirico) actual production of models had almost ground to a halt at the time of his residency there. By the 1930's such models had fallen out of favor, rarely used for instruction and even more rarely crafted by geometers. Ironically the 1930's were the beginning of the surface models' greatest impact on art. Sculptors' typically responded to these models not as mathematics, but as the reifying of an order embedded in nature. As such, artists saw in these models parallels to the other natural objects sharing shelf space in museums.

Even those artists, who are categorized as among the most subjective and alogical of modern sculptors, procured inspiration from these models. In commenting on the collection of models that Max Ernst suggested he view at the Institut Henri Poincaré in Paris, the Surrealist Man Ray succinctly stated the prevailing attitudes of artists toward the models:

> *The formulas accompanying them meant nothing to me, but the forms themselves were as varied and authentic as any in nature.* [7]

The Institut's collection inspired Ray to produce numerous photographs and a subsequent series of 20 paintings. While these works accurately delineate the models they bear no specific mathematical meaning. They do, however, attest to the aesthetic power of mathematical form and to the intuition that these forms underlie the beauty of nature.

Geometry had re-routed into higher n-dimensions. These were abstract spaces where beings stuck in a mere three dimensions could only describe them analytically, with numbers and equations. This was to eventually change with the visual explorations enabled by computer graphics, engendering in the past few decades another cycle of mathematical representations.

Influence on Early 20th Century Art Education

The Bauhaus. Even with mathematical training all but dismissed in current art training, that training still bears the impact of the visual mathematical training of late 19th and early 20th century education. Many, if not most, first year programs at universities and art schools are largely modeled on the Vorkurs of the Bauhaus. Its founder and first director, the architect Walter Gropius, melded the Bauhaus from two prior schools: one of fine art, the Weimar Academy of Fine Arts, and one of craft, Grand-Ducal School of Arts and Crafts. By larding the Bauhaus faculty with notable avant-garde artists from Europe and Russia, Gropius succeeded in institutionalizing Constructivism, a notably geometric style of abstraction.

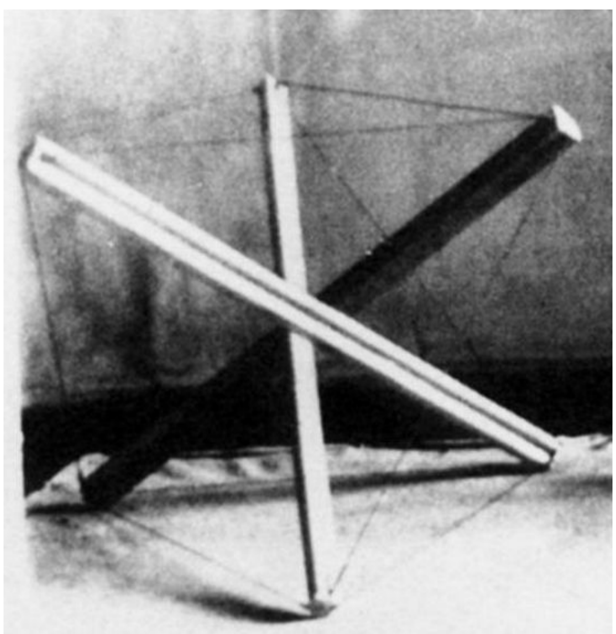
Johannes Itten, the most experienced teacher invited to the Bauhaus, designed the cornerstone first-year course of study [2]. Despite his distinctly expressionist values, Itten premised the Vorkurs on using geometry to seek out and learn visual relationships. Prior to becoming an artist, he trained as a teacher, especially in the use of Froebel blocks. At the Ecole des Beaux-Arts in Geneva Itten worked under the influence of Eugene Gilliard.

Gilliard's teaching methods fit especially well with Itten's experience with Froebel blocks. Gilliard's technique had students build the painted image by first laying down an armature of basic geometric shapes and then elaborating these into representational forms.

The geometry of the Vorkurs was, like the procedural instruction of perspective, not taught mathematically, but as a set of visual tools for propagating aesthetic research. This geometry resembled mathematics to the degree that it was formalized rather systematically, but this formalization was predicated on organizing perceptual principles, sometimes referred to as visual logic or as a visual grammar. Later advances in perceptual psychology supported such analytical approaches to art. Gestalt psychology, especially, demonstrated a predilection for the mind to organize visual data into geometric configurations.

The Inkhuk. The state art schools of Russia, the *VKhUTEMAS*, were founded contemporaneously with the Bauhaus and shared the same principles. As Commissar of Education for the fledgling Russian republic, the painter Wassily Kandinsky kept lines of communication open with the Bauhaus [11]. Within the *VKhUTEMAS* system, Moscow's *Inkhuk* (Institut Khudozhestvennoy Kultury or Institute of Artistic Culture) expounded an even more formalist and analytic approach than did the Bauhaus. Kandinsky initiated the Inkhuk in May 1920 as a school of theory to focus on formal analysis, but it had by December of that same year taken an even more analytic bent. Inkhuk's director Alexei Babichev, a mathematician turned sculptor, introduced a reorganization of the school to that end. In one position paper Babichev declared:

> *... the form of the work and its elements are the material for analysis, and not the psychology of creation…* [17]



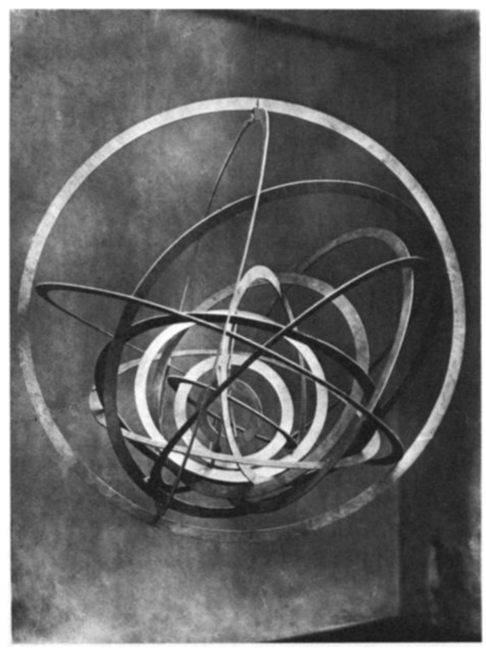


Figure 6. *Karel Ioganson, Linear Construction, 1921, wood and wire cable, PD-Russia. [8].* **Figure 7.** *Alexandr Rodchenko, Hanging Construction, 1921, plywood, PD-Russia.*

Another sculptor Alexandr Rodchenko was more direct in a similar position paper when he averred: *Art is a branch of mathematics, like all sciences*. [15]

More than any other instructors at the Inkhuk, Rodchenko and sculptor Karel Ioganson practiced these ideas in their work. The emphasis on the artist as a design scientist and researcher led to the label Laboratory Constructivism, and these sculptors were up to the task. Both worked in a style dubbed Linearism by Rodchenko, which employed only compass, straight edge and colored pencil to created diagrammatic images. Furthermore both created sculptures whose only referents were their own geometric structure (Figures 6 and 7.) In doing so, Ioganson exhibited the first known example of a tensegrity structure, almost 40 years before its re-discovery by Kenneth Snelson [8].

There was a key social dimension to the Inkhuk's conflation of art and mathematics. It was believed that this would yield an art of a universal logic, like that of mathematical formalism. In part due to decree by Lenin, this was to be an art of the collective, impersonal and objective, and not of individual expression; it was to be an art created in the factory and not at the easel. The abstract artists' purpose was that of pure theory, validated by its eventual application to product and graphic design. Regarded, then, as more research than art, this movement came to be labeled Laboratory Constructivism.

Some of the artists who believed in personal expression, Kandinsky and Marc Chagall among them, then emigrated from Russia. (Kandinsky would replace Itten at the Bauhaus.) Ten years later Lenin banned abstract art of any sort, as did Hitler with his disbanding of the Bauhaus. The resulting diaspora of avant-garde artists was to spread the new ideas in art and art education to North America.

Conclusion

Like other educated people of their time, artists benefiting from late 19th century mathematics education, possessed an appreciation for geometry that considerably exceeded that expected today. Consequently geometry provided the source materials for the development of modern abstraction.

Books on the topic by David Hilbert, Henri Poincare and H.P. Manning became best sellers, as did more occult books by P. D. Ouspensky, which argued for a very real fourth dimension from whence occult phenomena emanated. Popular lecturers on topics of higher space geometry could and did expect a working knowledge of geometry from their audiences. One outcome was the relatively quick popular reception of Einstein's ideas about time and space as functions of one another. Another was the quick adoption of alternative geometries by avant-garde artists [12].

The new ideas about Non-Euclidean geometries and the geometry of time found receptive eyes and ears in the more theoretical of these artists. In return geometries new and old gifted them with vistas into spaces previously unimagined.

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