Proceedings of Bridges 2013: Mathematics, Music, Art, Architecture, Culture

John Cage Adores a Vacuum

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Abstract

The mathematical description of the vacuum, via quantum field theory, has revealed the vacuum to be a busy, active place, not a blank void, with this activity happening the presence of a Higgs field that permeates space, and with this activity even leading to a measurable force between conductors due to the Casimir-Polder effect. John Cage, however, recognized these features in our sonic environment, and in his work *4' 33"* has provided a concrete, experiential manifestation and representation of the vacuum.

1 Introduction

Nature, Aristotle is reputed to have said, abhors a vacuum [1]. Or, as my good friend's history of science professor would recite this phrase, "Nature abhors a va-cu-um," turning what I always thought of as a two-syllable word into a word with three syllables. Whatever the professor's motivation for this pronunciation, in saying the word this way, he was actually reflecting something key to our modern understanding of the vacuum. There, between the two *u*'s, where most of us think there is nothing, he inserted a pause, putting an extra ingredient, extra structure. And, in fact, our modern mathematical understanding of the vacuum is that it is not the featureless emptiness people had long imagined; it has substance where we thought it was a void, it has activity where we thought there was stillness, it has something where we thought there was nothing. But we do not need simply to turn to mathematical physics to understand this. Most notably, in what his perhaps his most famous work, *4' 33"*, John Cage has effectively produced a representation of the modern understanding of the vacuum, and has done so in a way that not only focuses our attention on these very features, but gives us a context in which we can directly experience what makes the vacuum what it is.

It is always a question as to how to start these cross-disciplinary analyses. I have chosen to set up the mathematical description of the physical vacuum first, and then turn to Cage's compositions, but this order is simply a matter of convention. I do not mean to suggest any kind of pecking order or prioritization of one side over the other by so doing. It is simply that something has to come first. In addition, let me stress that this is not intended as a work of intellectual history; the reason for these parallels is an interesting question to research (and the timing is rather striking), but that is not my goal here. Rather, I am interested in examining how the mathematization of science on the one hand, and the arts on the other, can explore and make manifest a common set of rich, abstract ideas.

Before delving into the analysis, I would like to offer a brief story from 1988. That year, I met John Cage, at a reception hosted by the American embassy during Cage's residency at the Royal Conservatory in the Hague. At the reception, John Cage was standing by himself, everyone else, I suppose, not sure what to say to him, so I went over to talk with him. One of the pieces on the program at the concert earlier that evening was one Cage had not heard performed for something like three decades, if I recall correctly, and so I asked him what it was like to hear this piece of his again after so many years. Cage, in his oracular way, demurred, asking instead if I had heard the piece before and what was it like for me to hear it again? As it happened, I had not heard the piece before, but his initial response has stuck with me for all these years. His immediate impulse was to turn the question around and to encourage me to focus on the sounds I had experienced. This was a valuable lesson at the time, reflecting a core aspect of how Cage approached music, and it is a lesson to remember when we turn to our analysis of *4' 33"*.

2 The Physical Vacuum

The naive notion of the vacuum is that it is simply emptiness, a region of space with nothing in it. The mathematical structure of field theory has taught us that this is incorrect, in two ways. The actual vacuum has two distinguishing features: a background and fluctuations. We will discuss each of these in turn.

2.1 Swimming Pools and Vacuum Expectation Values

The matter in the universe arises from things called fields. A field can have a different value, in principle, at each point in space and time. However, if you ask what the vacuum is, you are asking for the field values that will have the least energy, and you will find, generically, that the energy is minimized when a field has the same value throughout space and throughout time. This is because if the field changes value at some location or at some time, there will generally be energy associated with that transition.

The surprising part is that it can sometimes require less energy to have a non-zero field throughout space than to have nothing. This has profound effects.

When I was little, although I could not swim, I loved going into the swimming pool. One of my fondest memories from when I was around 5 or 6 years old was being in the shallow end of the pool with my father, who would then carry me all the way across the pool, through the "deep" section (all of 5 feet deep), over to the shallow section on the opposite side of the pool.

But another memory I have from that same time was something similar, but in reverse. While my father and I were in the shallow end, I could actually lift him up and move him around in the water, even though on land, I could do no such thing, with him weighing in excess of 150 pounds.

The reason my ability to carry my father was so remarkable was, of course, that I spent time (indeed, most of my time) outside the water as well as in the water, and I knew that on land, carrying my father was an impossibility. But if, instead, I only spent time in the water (indeed, suppose there was not even a choice about this and there was no dry land to go to), I would no doubt have found nothing remarkable in this ability of mine, and indeed would have viewed that as the natural order of things.

The vacuum of our universe has something in common with this story. If we look at the fields in our mathematical description of physics, all but one of those fields will want to relax to be zero everywhere. The exception is the Higgs field; it turns out to require less energy for the Higgs field to be non-zero than for the Higgs field to vanish, and it acquires the particular non-zero value that minimizes the energy throughout space [2]. This value is called the *vacuum expectation value* of the Higgs field – a terminology that will resonate in interesting ways later in this paper.

The physical importance of this vacuum expectation value is that it means that as anything moves through space, it of necessity is moving through Higgs stuff. It is as if we are all in a swimming pool with my father, but with two differences. First, there is no way to get out of the pool; this Higgs stuff that is everywhere *is* the vacuum. And, secondly, the Higgs stuff works opposite to the buoyant effect of the water: a host of particles (including the electron and the various quarks) wind up with a greater mass because of this Higgs expectation value (in fact, without this expectation value, they would have zero mass).

2.2 Fluctuations and the Casimir-Polder Force

What I have just described is the base of the vacuum, so to speak. But the mathematics of quantum field theory tells us that there is more to the vacuum than this. Quantum physics tells us that things in the vacuum are not static, but that particles of various kinds appear and disappear all the time. The more massive or energetic ones disappear more quickly, but the important point is that these fluctuations – called, unsurprisingly, vacuum fluctuations – are an intrinsic part of the vacuum. That is, these are not fluctuations that take you away from the vacuum, but rather, the vacuum is constantly filled with these fluctuations – this *is* what the vacuum is. This happens because quantum mechanics says both that if there is a non-zero probability that something can occur, then it will occur, and that extra energy can appear for very short periods of time. Here, however, I am not interested in the mechanism, but rather in the phenomenon.

Now this phenomenon may seem peculiar – how can something be happening in the vacuum? – but this effect is not only calculable mathematically, it has physical consequences. In 1948, Casimir and Polder noticed an interesting effect [6] [7]. Imagine, for example, that you have two metal plates, parallel to each other, and separated by a small distance. You can imagine these in space, free from all external effects. What will happen to them? You might think nothing – gravity between two such plates is utterly negligible, and they are uncharged, so there seems to be nothing pushing them apart or drawing them together. But Casimir and Polder found, in fact, that the activity in the vacuum has a demonstrable effect: it creates an attractive force between the metal plates.

How can these random fluctuations draw the plates together? The answer is that the fluctuations are random but constrained, because they cannot pass through the metal plates. Thus the random fluctuations (and their temporary extra energy) outside the plates can occur anywhere out to infinity, but those between the plates must arise and end within those two boundaries. Remarkably, this has the effect of causing the two plates to move towards each other – to create an attractive force between them. (There are other shapes you could use that would cause the pieces of metal to push each other away [8].) And, lest one think this is just a Baroque mathematical construct, not only has this Casimir-Polder force been measured [9] [10] [3], but the vacuum fluctuations have also been directly observed [12]. In short, this mathematical picture of the vacuum as something other than a still pool, but a busy, active, buzzing place has been verified experimentally.

2.3 Vacuum Lessons

There are two lessons, then, to draw from all this.

(1) The foundation of the vacuum is the background Higgs field value, through which everything always has to move; the technical term for this value is the vacuum expectation value of the field.

(2) In addition to this vacuum expectation value, the vacuum also predictably includes a constantly churning random sea of particles, appearing and disappearing, a kind of microscopic hustle and bustle that is as much a part of the vacuum as the Higgs background.

3 John Cage's Manifestation of the Vacuum

John Cage's musical compositions pushed the boundaries of music in important ways. From his use of prepared pianos to his use of chance operations, Cage both seized and ceded control over the sonic output of his compositions. Certainly, in conceiving of pieces for prepared piano, he was deliberately seizing control over the design of the instrument to be played, rather than accepting what history had provided him.

But another important, and arguably more powerful, trend in his work was the effort to remove the composer from the composition, and in the most extreme case, namely *4' 33"*, he did so in a way that leads us to focus on what is left when all extraneous entities are removed. He was, in some sense, finding the musical vacuum, and it is striking that what he found looks much like the modern conception of the vacuum I described above, rather than like the naive traditional notion of pure nothingness.

The composition on which I focus my attention — *4' 33"* — had its debut performance in 1952, but, in fact, Cage had already conceived of a composition like this in 1948 (the same year that Casimir and Polder published their results), as revealed in *A Composer's Confessions* [5]. Lecturing at Vassar (but in a lecture not published till years later), Cage said:

> I have, for instance, several new desires (two may seem absurd but I am serious about them): first, to compose a piece of uninterrupted silence and sell it to Muzak Co. It will be 3 or 4 1/2 minutes long — those being the standard lengths of canned music — and its title will be *Silent Prayer*. It will open with a single idea which I will attempt to make as seductive as the color and shape and fragrance of a flower. The ending will approach imperceptibility. And, second, to compose and have performed a composition using as instruments nothing but twelve radios. It will be my Imaginary *Landscape No. 4*. [11]

3.1 *4' 33"*

John Cage's best known piece is probably *4' 33"*, a piece that consists of four minutes and thirty-three seconds in which the performers play no notes.[1] Many people have heard of it, but fewer have actually heard it. Back when I was in college – coincidentally, in the same year in which I learned the modern mathematical formulation of the vacuum – I had the chance not simply to be at a performance of *4' 33"*, but to perform in it. Experiencing the piece live is a revelatory experience, so I would like to describe that experience; this is key to understanding how the piece works and thus what Cage has achieved in this work.

As part of an experimental music festival, I played the flute in an orchestral performance of *4' 33"*. The conductor had performed the piece before, but nearly everyone else in the room had only heard of the piece, never actually heard it. The experience of the performance was striking, lifting the piece, for me at least, from an interesting intellectual exercise that would appeal to a mathematician (in the way a mathematician notes that the empty set is a set) to an experience that made me appreciate sound and music in new ways.

As the conductor raised his baton, we raised our instruments, and sat there silently. The audience was in their rows of chairs, also sitting silently, facing forward, watching us. We in the orchestra sat up front, our seats in curved rows so we could see each other and look out at the audience; the conductor had his back to the audience. In these positions, we sat, and did nothing.

After a few seconds of calm, little bits of laughter broke out here and there. I tried to keep a straight face (fortunately, I wasn't trying to get my flute to play any notes, so lip control was not an issue), but was certainly a bit bemused. But somewhere about thirty seconds in, when everyone realized that, yes, we really were going to do this, something changed. We began to experience something new.

There were two sides to the experience. First, there were the conventions, providing the foundation on which our experience took place. We all had certain expectations, because this was, after all, a concert, a performance of a piece of music. Our behavior was constrained: audience members sat in their seats, orchestra members sat in theirs, orchestra members played the notes on their score (none), audience members sat quietly waiting for the end of the piece of music. The conductor marked the beginning and end of each

[1] Strictly speaking, the title of the composition is not *4' 33"*. As Cage says in the score for this work, the title of the piece is the number of minutes and seconds that the particular performance lasts. Following the first performance by David Tudor, however, it has become conventional to use the timing that Tudor used [4].

movement with his baton. Everything that occurred in those four minutes and thirty-three seconds was forced to occur in the context of those expectations.

The second side is what we experienced besides those expectations, besides what convention demanded of us. As we all sat (allegedly) silently, we found that we and the room were not really silent, and all the minute sounds that came and went in the course of any performance became the focus of our attention. We could hear a chair creak as someone moved, and then a cough, a sound from outside, and a piece of paper falling. This supposedly silent world was anything but silent; that thing we thought of as silence was actually a busy place of micro-noises, appearing and disappearing with no discernible pattern, but inevitably present.

3.2 The *4' 33"* Vacuum

Comparing the description of what the experience of *4' 33"* was like to the mathematical description of the vacuum, you will see that the parallels are powerful; indeed, we can naturally map one onto the other.

The first item we found in trying to understand the vacuum is that it is filled with a uniform expectation value of the Higgs field. (Interestingly, this idea, in the form of spontaneous symmetry breaking and the Higgs mechanism, did not emerge in physics until the early 1960s [2].) That is to say, we cannot speak of the behavior of particles as they move through the vacuum independently of understanding what the default field values in the vacuum are. In physics, this is mathematically explicable by recognizing that all the action takes place in the presence of these default values. But this notion is already captured by Cage's *4' 33"*. It is perhaps a terminological coincidence, but the expectations we bring into the concert hall play the role of the expectation values that fill the vacuum. It is absolutely critical to the effectiveness of *4' 33"* that it take place in a context in which there are certain rules governing our behavior. A musician simply not playing any notes in some random setting would not have generated the experience I described above. It is essential that the default framework requires audience and performer to stay in one place, making little to no deliberate noise, for the duration of the piece. As a consequence, Cage's piece draws our attention to the critical role expectations make in specifying the background and thus shaping what actually happens. The analogous lesson had to be learned in physics as well, but once learned, has come to play a central role.

The second feature of the vacuum that Cage's work captures is the presence of ongoing minuscule fluctuations, a sea of microscopic activity and turbulence, in which there is more than a uniform, unchanging default background. The effect of *4' 33"* is to draw our attention to the same kind of fluctuations that are always and endlessly present in our sonic environment. It is the structure of the piece that forces our attention to this activity. As we sit "silently" but listening, we become attuned to every sound, however minute, in our environment. We find that "silence" is not silent, but is rather a realm in which small sounds routinely appear and disappear. (There is a chance that there will be a large sound, but this is much less likely.)

You will notice that this array of sound fluctuations mirrors the particle fluctuations that fill the vacuum. Whereas the naive vacuum is purely empty space, we learn mathematically that the real vacuum is filled with fluctuations, and is neither empty nor still. Cage's *4' 33"* shows us the same features of our sonic environment. But what is most striking here is not simply that this parallel exists, but the Cage has created, through his composition, an environment in which we viscerally experience what the vacuum is like. It is rare to be able to construct an experience to mirror quantum reality, but I think there is no mistaking that Cage, whether he intended to do so or not, has succeeded in doing so.

One can even make a case that, metaphorically, *4' 33"* creates a kind of Casimir-Polder force among those in the concert hall. The concert hall is a closed environment. There are sound fluctuations inside the hall as well as outside, though obviously the ones inside have a greater impact on the audience and the performers. Remember that the vacuum fluctuations can create a force between two metal plates, attractive in the typical case. In a performance setting, the shared experience of those in the auditorium creates a bond among those who were present (or, in the case of an offending performance, perhaps a repulsion for some, but either way, it is not neutral). In the case of *4' 33"*, that shared experience is generated by the sonic fluctuations, since there is nothing else. Note if there were really a void of utter silence, just as if there were really an empty vacuum, there would be no experience to have shared, just as there would be no Casimir-Polder force. What Cage has pointed out is that even when you remove overt performance, there is still an experience to be shared and a bond to be formed, just as Casimir and Polder showed a force can arise physically in the not-so-empty vacuum.

4 Conclusion: Physical Silence

The connections drawn here between the mathematical description of the vacuum in physics on the one hand and what Cage's *4' 33"* teaches us about our experience of the sonic enviornment on the other hand are striking because they reveal such a natural fit. Certainly, neither one nor the other of these parallel constructs can replace the other. Rather, something much more powerful is going on. In very real ways, on both sides of this fence, common ideas are being explored and common insights are being revealed. Often, scientists seek to explain ideas with demonstrations, systems designed to reveal one or another property. What is fascinating here is that we can meaningfully turn to Cage's *4' 33"* to obtain an analogue representation of something that cannot otherwise be directly experienced on a human scale. Cage's art makes our understanding of the science not only richer, but better. And, in turn, when I listen to *4' 33"* again, I will find myself automatically heeding John Cage's question to me at that reception a couple of decades ago, and reflect on how my experience of *4' 33"* is different, now that I have understood these connections.

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