SURFER in Math Art, Education and Science Communication

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Abstract

SURFER is a program designed to make everybody feel like a mathematician. The program is a bridge between art and math. Everybody can participate in the dialogue between algebraic equations and pictures of algebraic surfaces in an interactive and aesthetic way. In this paper we will introduce the program and its potential in math art, education and communication. The program was originally developed for the IMAGINARY exhibition, a project by the Mathematisches Forschungsinstitut Oberwolfach. We will present many SURFER images and project ideas by artists, mathematicians, students and visitors of the exhibition.

1 Motivation

Looking at the apple of the image series *La trahison des images* by the French surrealist painter René Magritte [13] and the lemon by the Austrian mathematician Herwig Hauser that is used as a logo for the IMAGINARY activities you can ponder on the nature of images, see Figure 1. The apple is not an apple, and the lemon is given by all points in space that satisfy the given equation. It is not a lemon; is it an equation?

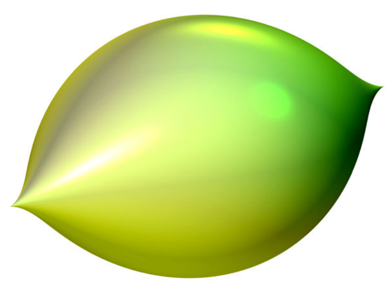

Figure 1 : *These are neither apple nor lemon, but abstractions or visualizations of the real and the mathematical world.*

2 IMAGINARY

SURFER is the core program of the traveling exhibition IMAGINARY that presents visualizations, interactive installations and their theoretical background from algebraic geometry, singularity theory and differential geometry in an attractive and understandable way. The exhibition was designed by the Mathematisches Forschungsinstitut Oberwolfach for the Year of Mathematics in Germany 2008 and has since then visited more than 80 cities in Germany, Austria, France, Ukraine, Switzerland, Argentina, Poland, Colombia, Spain, Russia, Serbia, Portugal, China, the UK and the US, see [7]. A unique feature of the exhibition is the sustainable communication of all exhibits through free software and didactical background information and the possibility for organizations or schools to copy the exhibition and organize it on their own.

Figure 2 : *Left: IMAGINARY exhibition with the picture cube at the Leibniz Universität Hannover. Right: SURFER exhibit at the IMAGINARY exhibition in Berlin.*

3 The SURFER Program

SURFER produces images of algebraic surfaces in real-time. These images form the eye-catcher of the IMAGINARY exhibition. Some artistic pictures are printed in high resolution and are presented in a walk-in cubic gallery. The program is presented on a touch screen with a printer, see Figure 2. The pictures generated and printed by the users can also be added to a pin board at the exhibition. Visitors are attracted by the possibility to contribute to the exhibition. The program shows how much fun the interplay between math and art can be. It enables users to understand and experiment the relation between formula and form in an intuitive way. The user interface of SURFER offers a simple design, see Figure 3. Behind the program there is no didactical theory or artistic approach. Its intent is to share the joy of creation in mathematics and art and to free mathematics from its image to be dry and complicated.

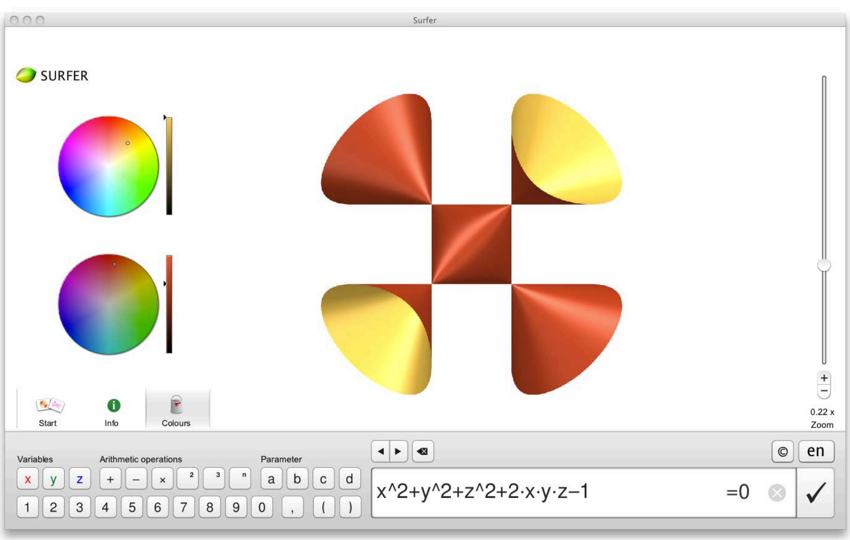

Figure 3 : *Screenshot of the SURFER program, Java version for exhibitions.*

To create an image the user enters a polynomial equation, for example $x^2 - x^3 + y^2 - z = 0$ either with the mouse, keyboard or finger on a touch screen. SURFER will then immediately calculate the points in space that satisfy this equation and display them within an invisible sphere that clips all points outside the screen. The resulting algebraic surface can be rotated. With the zoom slider the number range displayed can be changed. Additionally one can choose the colors of the two sides of the surface. To calculate the picture of the surface the program uses the ray tracing technique [6]. The surface points are computed following a ray of light that is sent from the so-called eye or camera through the image screen onto the surface. The problem to find the intersection of this ray and the surface reduces to find the zeroes of a univariate polynomial equation [16]. The colors are given through a simple light model with parameters for light intensity, position, colors and material properties.

SURFER offers several extra features. Number parameters can be added to the equation. They are controlled via sliders and allow to dynamically change the equation and thus the image. There is a large user gallery with tutorials and many surfaces given with their equations, parameters and background explications. The surfaces can be exported as pictures in high resolution. The program is available under an open source license and in several versions on the IMAGINARY website. There is a home version based on the program SURF [4] for Linux and Windows with expert settings and the option to create animations. Then there is a new Java version supporting many platforms and the web with special settings for exhibitions and museums. The galleries include explications in German, English, Spanish, Catalan, Portuguese, Serbian and Russian.

4 SURFER in Art, Science Communication and Education

There are many ways to use SURFER and the pictures of algebraic surfaces created with it. In the following sections we present a series of images by SURFER users and projects carried out in the fields of art, science communication and education. For galleries with more pictures and equations provided by our users see [9].

4.1 The equation of an object

Playing with the program the users sooner or later get to the point where they want to recreate a special imagined object. Valentina Galata, now student of bioinformatics in Germany, started to use SURFER as a high school student in 2008. She specialized in remodeling real world objects. She found equations that create images of fruits, design objects or landscapes. Generally she adds several surfaces to one image by multiplying their equations, often variations of basic geometric figures, like spheres or cones, see Figure 4. The spoon is given by the equation:

$$\begin{gathered}((3x^2+(y-1.9)^2+4z^2-1)^2+0.2z)\cdot(((((0.8z+1.2)^3+5y-6)^2+16x^2-0.5)\\ \cdot(x^2+(y+6)^2+(z-2.8)^2-0.3)\cdot(x^2+(y-1)^2+(z+3.3)^2-0.03)+290)\\ \cdot(9x^2+(y-0.1\cdot z+2.5)^2+(4z-5+y)^2-1)-400)-99=0\end{gathered}$$

4.2 Mathematics and Art

The German artist Hiltrud Heinrich used her pictures in own math art exhibitions. The pictures are of abstract and aesthetic nature. She also replicated the design of the SURFER pictures as patterns for patchwork quilts, see Figure 5. The SURFER artist Kurt Ballay explores numerical errors during visualization and their beautiful outcome by creating a series of artistic images, see Figure 6. Torolf Sauermann, a mathematical artist, explores the number of singular points for certain equations and also adds new effects to the surfaces, see Figure 7.

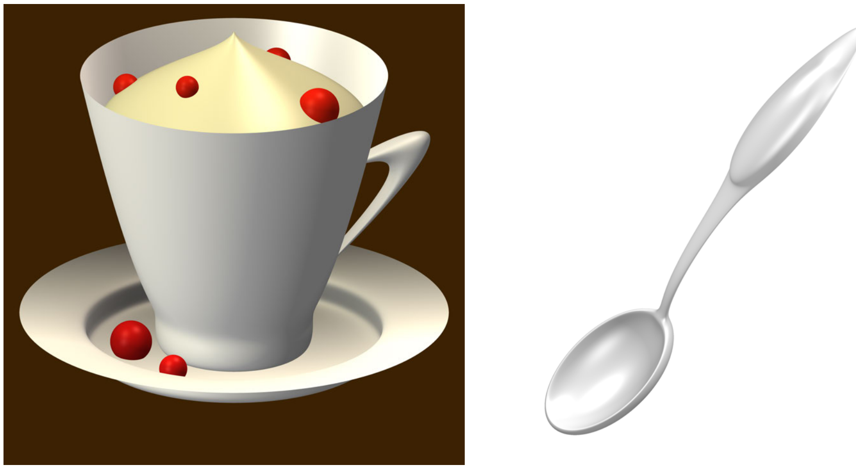

Figure 4 : *An algebraic cappuccino cup and an algebraic spoon, pictures by Valentina Galata.*

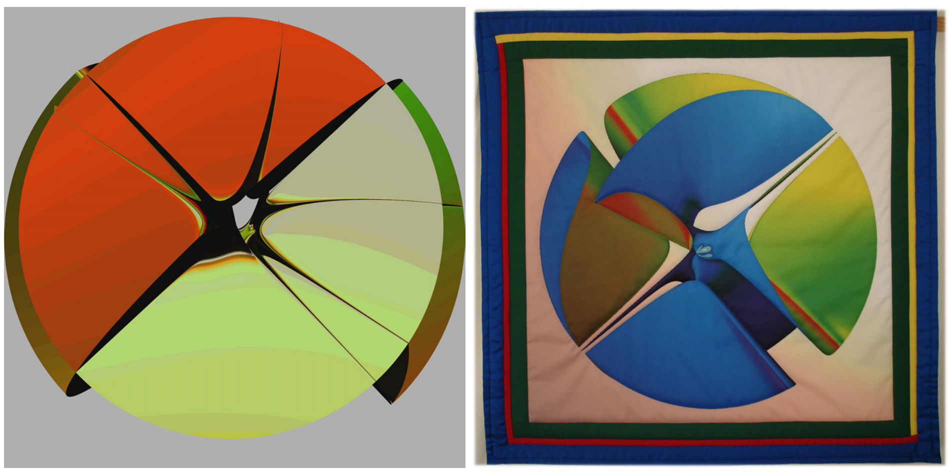

Figure 5 : *Left: winner image of the SURFER picture competition with DIE ZEIT in 2008. Right: patchwork quilt of an algebraic surface. Both pictures by Hiltrud Heinrich.*

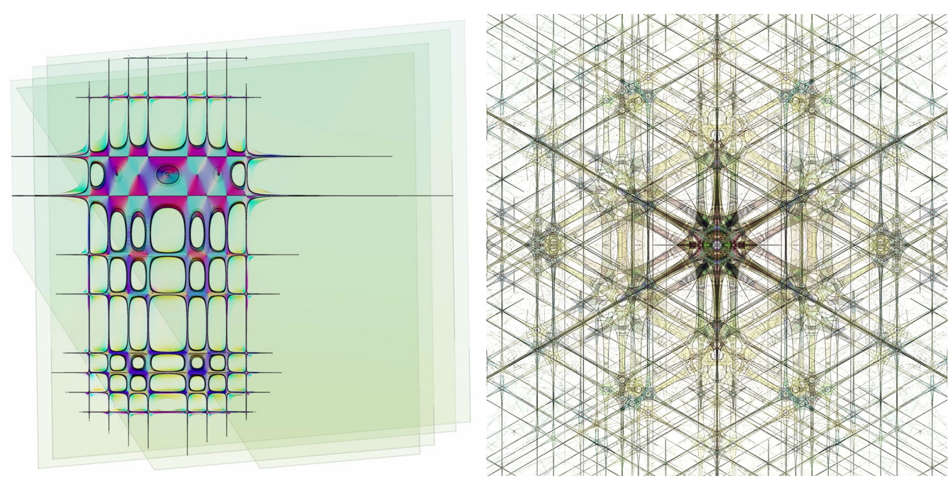

Figure 6 : *Images of numerical errors in the visualization with SURFER by Kurt Ballay.*

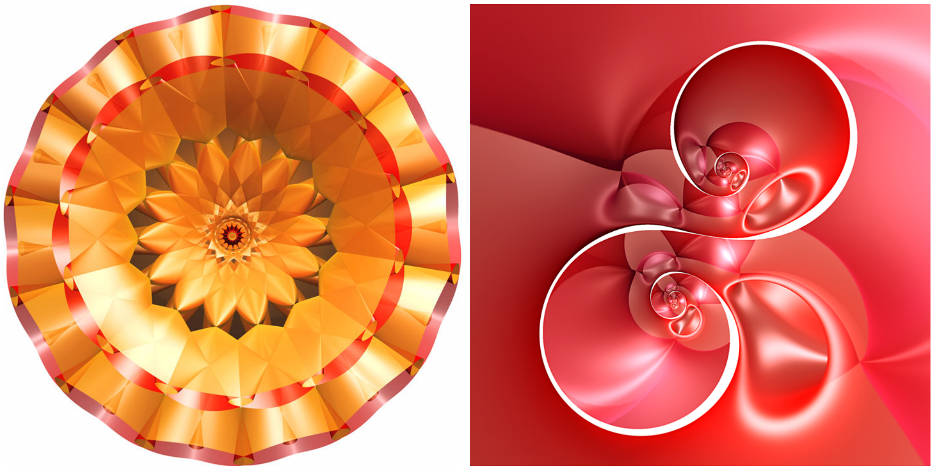

Figure 7 : *Sunflower, a septic surface with 99 singularities. Right: an algebraic surface combined with the Droste effect.*

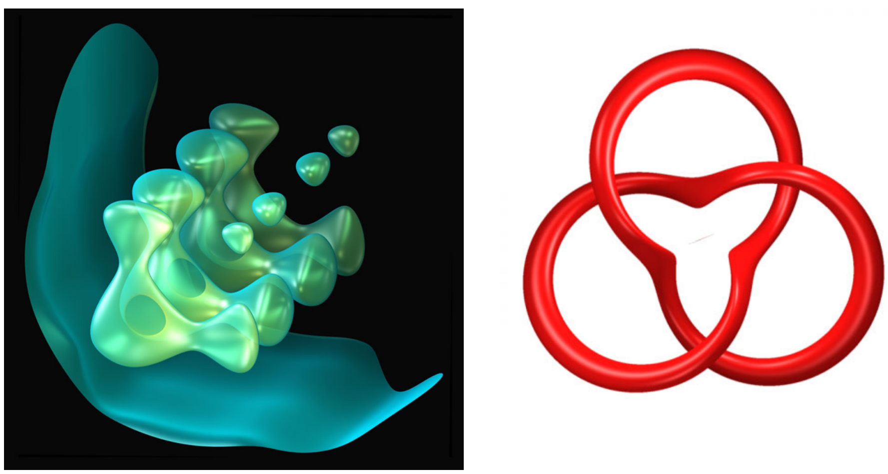

Figure 8 : *Left: Buddha by Mehrdad Garousi. Right: Trefoil knot as algebraic surface by Stephan Klaus.*

Bianca Violet, mathematician and film editor, created animations with SURFER, that were used at the film LPDJLQH D VHFUHW [11]. The film gives visual insights into elliptic curves and cryptography that are essential for the proof of the Fermat Problem. It is a production by the Centro Internacional de Matemtica (CIM) in collaboration with the Casa da Animação in Portugal and the Mathematisches Forschungsinstitut Oberwolfach. Mehrdad Garousi an Iranian artist and scientist is dealing with mathematical and digital forms, especially fractal pictures and sculptures. In the program SURFER he found new inspiration for his pictures, see Figure 8. He published a paper on SURFER and its use in math art [5].

Stephan Klaus is professor of topology and works at the Mathematisches Forschungsinstitut Oberwolfach. He used SURFER to visualize knotted surfaces, see Figure 8. He found a method to construct such polynomials by Fourier decomposition and algebraic variable elimination for every knot type [10].

4.3 Math Communication and Popularization

The combination of a strong visual presentation with possibilities to creatively interact using free software proved to be incredibly fruitful to communicate the underlying mathematics which reaches from basic geometry to open questions in current research [8]. Since the German Year of Mathematics the IMAGINARY exhibition and SURFER were presented in more than 80 cities worldwide. The exhibition was among others hosted in universities, schools, art galleries, shopping centers and railway stations. We estimate the total sum of visitors about 750.000 including more than 1700 school classes. Diverse media picked up the contents of the exhibitions. An example is the BILD, a German yellow press paper that reaches a wide audience which is not frequently addressed with scientific topics.

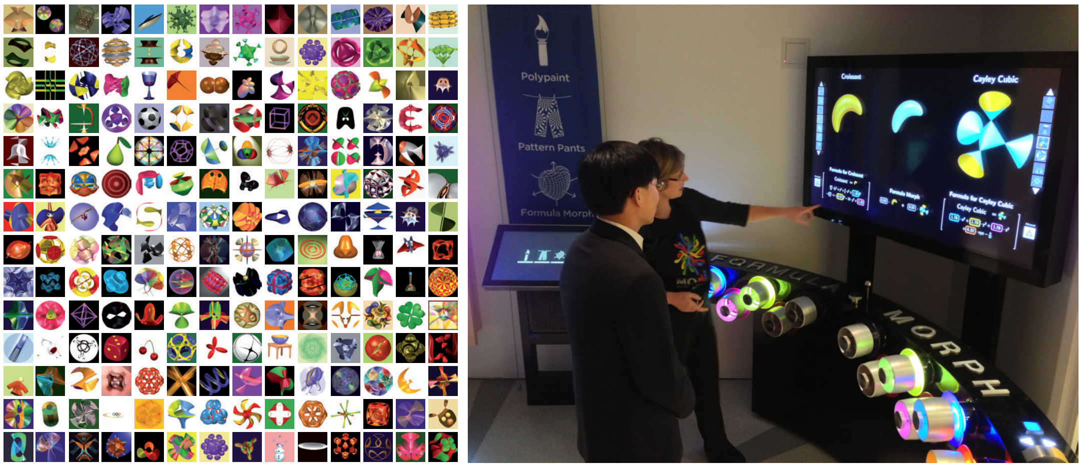

Figure 9 : *Left: mosaic of entries at the Spektrum SURFER competition. Right: SURFER as base for the exhibit "Formula Morph" at the MoMath in New York*

Besides the exhibitions, SURFER picture competitions were held in collaboration with newspapers and magazines. The audience was asked to download the program SURFER and create pictures, see the sample gallery of the competition with Spektrum der Wissenschaft [14] in Figure 9. The pictures, together with their titles and formulas, were uploaded to a web gallery and then evaluated by a jury, consisting of artists, scientists and journalists. A community of SURFER users started to interact, share images and contribute with own ideas to the project. These disseminators or independent ambassadors made the project popular in many ways. Workshops were held at universities and schools and it was a surprise to see a SURFER image to be included in a nationwide AIDS awareness campaign in Germany [1]: the algebraic surface formed part of a classical student room with skateboard and rock poster.

To make SURFER available in a permanent form museums have been approached to include it as a permanent interactive station in their maths sections. At the moment SURFER can be experienced in the Deutsches Museum in Munich, the MiMa museum in Oberwolfach or at Tecnópolis in Buenos Aires and the MoMath in New York, next to temporary installations in the Science Museum of the University of Coimbra and the National Museum of History of Nature and Science in Lisbon, see Figure 9 for an example of a museum exhibit.

4.4 School Projects

Many schools visited the IMAGINARY exhibitions and received a free guided tour. The students and teachers were fascinated by the intuitive and colorful approach to mathematics. As mentioned before several schools copied the contents and held their own exhibitions. To extend school participation and to communicate more mathematical content a series of school workshops with SURFER were conducted in Germany, Spain and Argentina, see image in Figure 10. The students were very interested in the mathematical background of SURFER and started to carry out research activities motivated by simple questions while using the program.

Figure 10 : *Left: school workshop in Berlin with surfaces as cardboard sculptures. Right: picture of the exhibition "The taste of mathematics" in Malaga. Photographs by Pedro Reyes Dueñas.*

Examples for such questions are: "How can I move the surface to another place, how can I rotate only one part of the surface, how can I create a morph between two surfaces, etc.". We are also following a general approach to add more images into the math curriculum to motivate calculations or exercises, e.g. to solve algebraic equations by checking if they contain specific points or not.

4.5 Music, Photography, Food, Literature, Architecture

The idea to connect algebraic equations with its visual aspect resulted in a general motivation to explore the connection between formulas, images and other art forms: Students at the ITBA University in Buenos Aires developed a plugin for SURFER, where names and sentences are translated into equations and thus displayed as colorful surfaces [15]. Another project called SoundSurfer adds the possibility to generate sound files with SURFER, where music samples are combined depending on the shape of the surface. In Malaga, Spain, as part of the IMAGINARY tour through 15 cities organized by the Royal Spanish Mathematical Society (RSME), the SURFER images were connected to fine cuisine: chef Jose Carlos Garca together with mathematician Mercedes Siles Molina and photographer Pedro Reyes Dueñas created unique algebraic dishes and presented their creations in a photo exhibition - some of them were also offered in a restaurant. Additionally the creative process of cooking and doing maths was explored in a second exhibition, see Figure 10. There is a strong connection between architecture [2] and sculptures of the algebraic surfaces. We printed algebraic sculptures with rapid prototyping 3D printers and also built them in form of cardboard slices, a technique introduced by Mara G. Monera from Valencia, see Figure 10.

5 Conclusion and Future

We introduced SURFER and showed its possible use for art, science communication and education. Its secret lies in encouraging the users to create their own mathematical artwork and to explore the underlying mathematics in a playful way. Once the curiosity about formulas and forms is raised to a certain level, there is no limit in diversity of the image or interest by the media. Within the project "IMAGINARY - open mathematics" which started in 2011 and is funded by the Klaus Tschira Stiftung [18], we are currently developing an open web platform to collect all SURFER images and further involve the community [12].

It is planned to offer all modules of the IMAGINARY exhibition on this platform and to allow users to upload own digital exhibits. This follows the idea of more participation in the math communication process:

not only is the audience involved in creating images using free programs but also in creating own and new exhibits and even whole exhibitions. As an example look at the competition of virtual modules for an open source exhibition for the international initiative Mathematics of Planet Earth 2013 [3] which will be hosted at the new platform. All exhibits of this new exhibition were collected through a competition with cash prizes. Several museums and exhibition organizers already agreed to produce and show the winning modules. The first exhibition is held in Paris in March 2013.

In the future we will put a focus on schools through workshops and free material co-authored by the participating students. Following the experience from a 20 hours workshop conducted with artists at the Centro Cultural de España (CCEBA) in Buenos Aires we think there is a big potential in further improving the aesthetics of the visualization. Some artists started painting algebraic surfaces and also combining them with digital art forms, as live visuals for events. The program SURFER will be further developed and we hope that we can attract new users and also software developers to support us. We invite you to use SURFER, explore the vast image world of algebraic surfaces and share your creations.

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