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Knot Designs Based on the Hexagonal Rosette

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Abstract

The familiar hexagonal rosette pattern drawn with a compass can be used as a template to construct a variety of new rosettes with interlaced strands. Some designs are closed knots with alternating under-over strands and are chiral, with six-fold rotational symmetry. These designs can be divided into hexagonal modules of five different types. Designs of interlaced surfaces based on the same template can be drawn as layers forming Brunnian links.

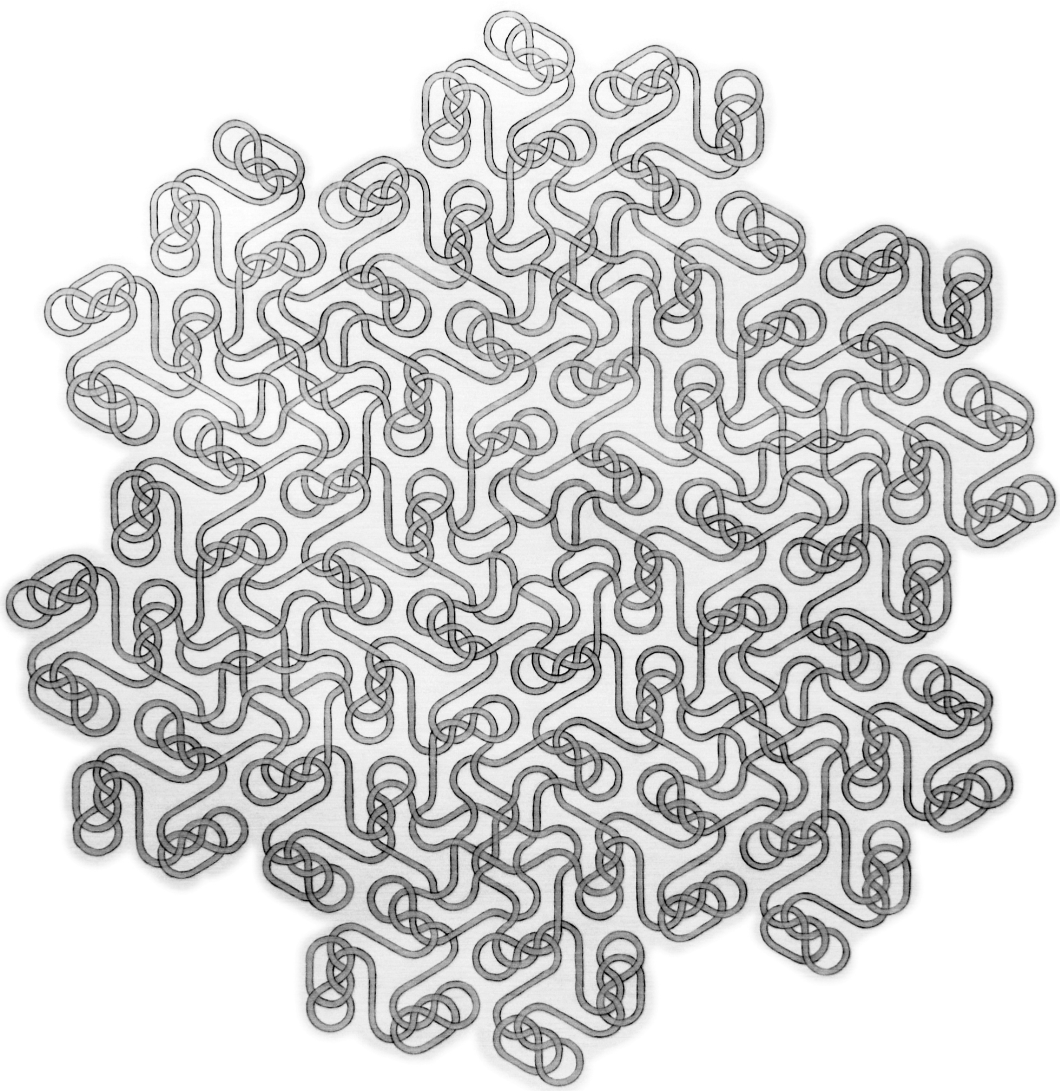

Figure 1: *Chiral knot design with six-fold rotational symmetry, alternating under-over strands, made of a single loop, crossing number 618 (pencil, marker)*

Introduction

At the School of Arts, Design and Architecture of Aalto University in Helsinki, for my master's thesis, I did artistic research as described by Hannula ed. al. in *Artistic research: theories, methods and practices* [1]. I investigated how to construct three-dimensional knot ornaments analogous to some two-dimensional ornaments I had designed earlier. This paper explains the genesis of these designs.

Although my work arose from my fascination with geometric construction rather than an investigation of others' work, my designs can be examined in the context of ornamental art. For example, my work shown here can be seen to mix elements from Islamic interlace patterns [2] and Celtic knot ornaments [3]. I also have learned that my knot designs bear a strong resemblance to the work of Rinus Roelofs [4], [5]. The construction of interlaced ornaments from hexagonal modules has been previously studied by M.C. Escher [6] and David A. Reimann [7], [8]. Some board games and puzzles also utilize the same principle (e.g. *Tantrix* [9] and *Bombay Bazaar* [10]).

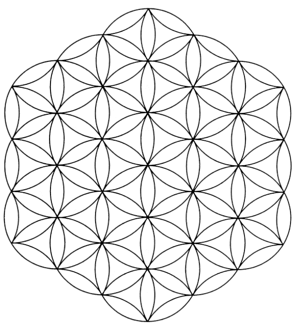

Figure 2: *Rosette pattern*

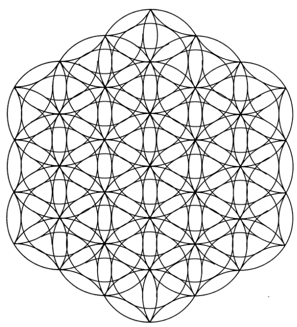

Figure 3: *Smaller circles*

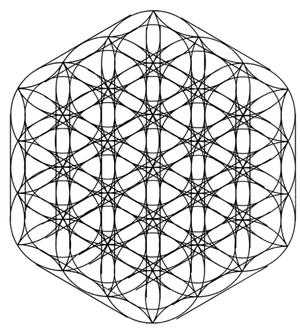

Figure 4: *Line segments*

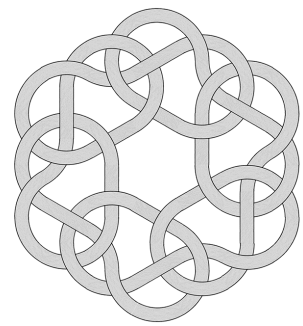

Figure 5: *Knot ornament*

The compass-constructed hexagonal rosette in Figure 2 is a template for my work; its occurrence is both ancient and worldwide. It can be found in decorated distaffs in Finland [11], ossuaries of the Greco-Roman Jews [12] and the hex signs of Pennsylvanian Dutch [13]. The pattern also appears in the sketches of Leonardo da Vinci [14] and the 'sacred geometry' of the New Age movement [15]. The functions of the pattern seem to range from mathematical [16] to apotropaic [17], to purely decorative [18].

The Template

One day I was amusing myself with the ancient challenge of squaring the circle, which requires constructing (with compass and straightedge only) a square with the same area as a given circle. Although I knew it had been proven impossible to achieve, I was still fascinated by the method of constructing geometric forms from other forms. Playing with a compass, I came up with the rosette pattern in Figure 2, which I think can occur to anyone left alone with a compass and a piece of paper.

The hexagonal rosette in Figure 2 has an ambiguous and visually fascinating quality— each of the lens-shaped pieces can be interpreted as either a spoke of a wheel or part of the rim of one of the wheels that compose the pattern. Staring at the pattern, I imagined how these 'lenses' could form ornamental strands twisting and turning over each other. I then modified the rosette by adding smaller circles (Figure 3) and line segments (Figure 4) tangent to the circles. This collection of lines and curves now included all I needed to serve as a template for my ornaments.

Restrictions

Alternation. I first noticed the possibility of entangling the strands in an alternating manner. This means that as a strand encounters other strands, it travels alternately over and under these in turn. I required this property for all my designs as its regularity pleased me and made the entanglements acquire their most knotted configuration—the topological knot described by the ornament could not have fewer crossings.

Rotational symmetry. All the ornaments were drawn in the form of rosettes having six-fold rotational symmetry. This grew out of the radial construction and the six-fold symmetry of the template.

Closing the loop. To limit the number of possible entanglements, and to increase the challenge, I also required that my ornaments should consist of a single closed loop.

Chirality. The three requirements above sought to maximize the symmetry and regularity of the designs, but I had a preference of chirality over reflection symmetry. Chiral designs gave an impression of a twisting movement, whereas ornaments having reflection symmetries appeared to be more stationary.

The ornament in Figure 5 fulfils all of these criteria: it is alternating and chiral with six-fold rotational symmetry, and consists of only one strand. You can trace the path of the strand following along parts of the circles and straight segments of the template in Figure 4.

Fulfilling all the criteria proved to be quite challenging in designing larger ornaments. After making many ornaments of growing complexity, I succeeded in producing for my graduation exhibition (MoA'12 in Helsinki) the intricate design in Figure 1. This, unlike my earlier designs, contains repetitions inside it.

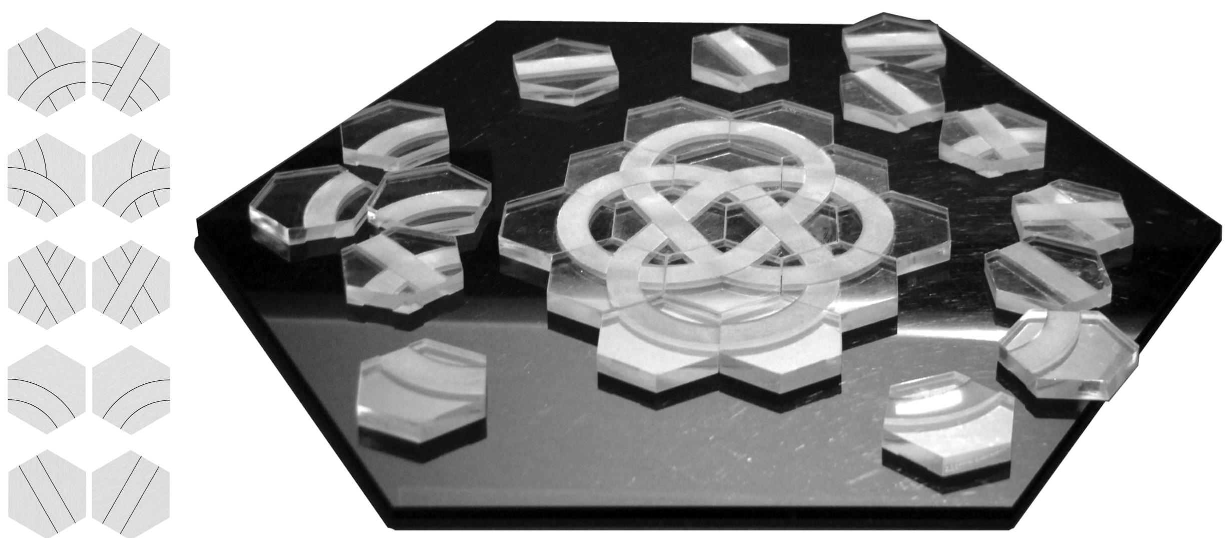

Figure 6: *Layout for five different hexagonal tiles (left) and the puzzle (laser cut and etched acrylic)*

Hexagonal Tiles

To eliminate the need for lines that always have some arbitrary weight, I tried to construct from the underlying geometry a colouring for the ornaments. I divided the plane into hexagons, each of them corresponding to a crossing point, or a potential crossing point in the ornament, and in each hexagon, I painted the ‘over’ strand, the ‘under’ strand and the background with different elementary colours. From this exercise, I noticed how the designs could be partitioned into hexagonal tiles of just five essentially different types, with the use of a blank tile excluded (Figure 6, left). For the MoA’12 exhibition, I produced a puzzle that offered visitors a chance to construct ornaments of their own (Figure 6, right).

One potential application for the tiles could be a game in which players compete against each other or solve a puzzle together. Or it could be a zero-player game, a cellular automaton evolving into ornaments of growing complexity according to some random input and appropriate logic.

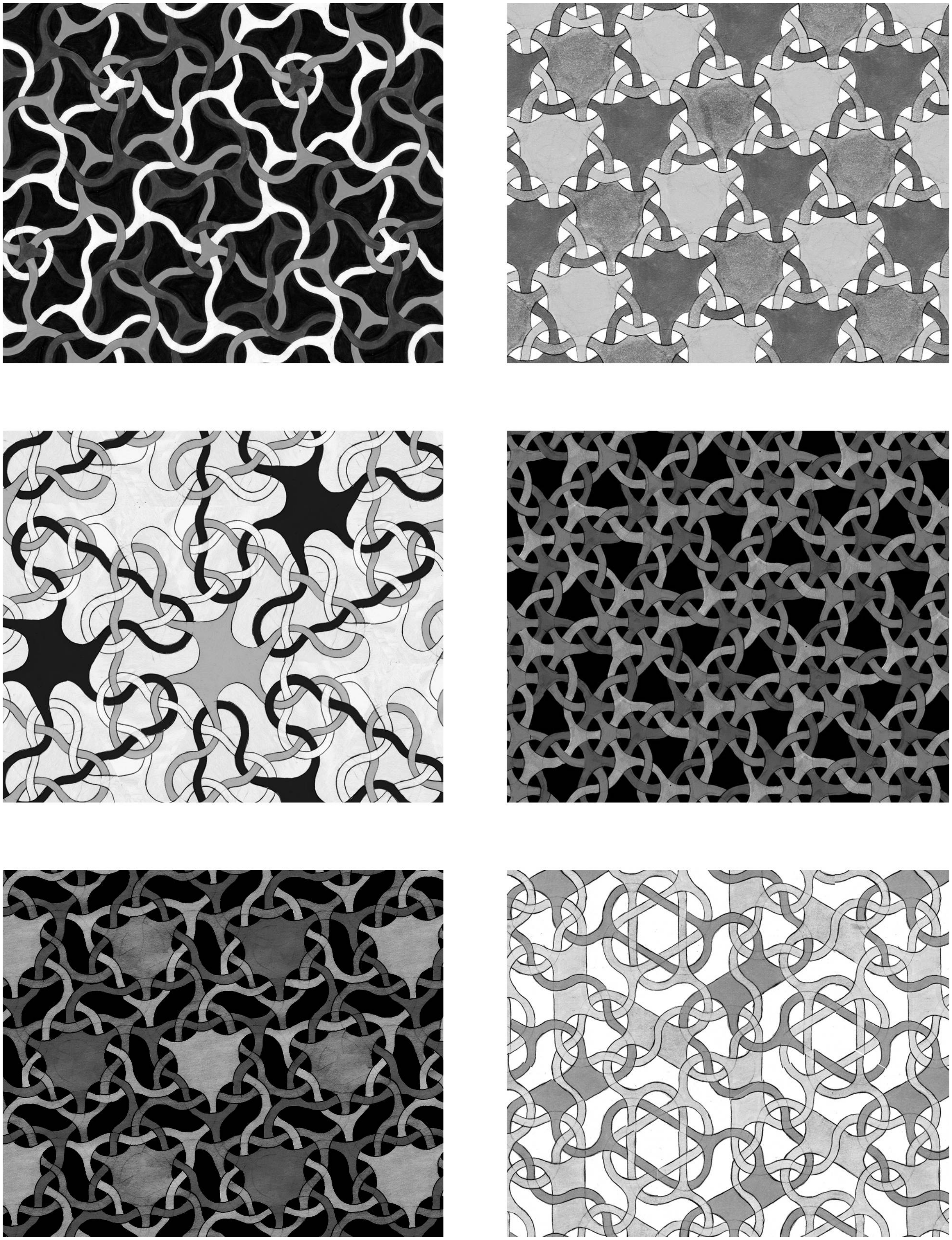

Figure 7: *Knotted surfaces filling the plane*

Knotted Surfaces

Next I decided to relax the conditions a bit, and allowed the strands to branch out. Now I was now dealing with knotted surfaces. My initial designs were knotted holes on surfaces and designs that appeared like knotted disks. What inspired me the most, however, was to construct plane-filling ornaments that are links of three identically punctured surfaces (Figure 7). All of them have the interesting property that if you could pull out one of the three surfaces, the other two would no longer be linked. In knot theory entanglements of this kind are called Brunnian links, of which the most famous is the Borromean rings. Links instead of knots allow colouring that contributes to the graphic clarity of the ornament.

Finally I designed rosettes where the repetitive interior demonstrates the possibility for a plane-filling, and the rim has six-fold rotational symmetry. One of the most complex these is in Figure 8. Here the surfaces are coloured with subtle tones and the loose ends of the fringe are secured by circular hoops, disks, and bulges.

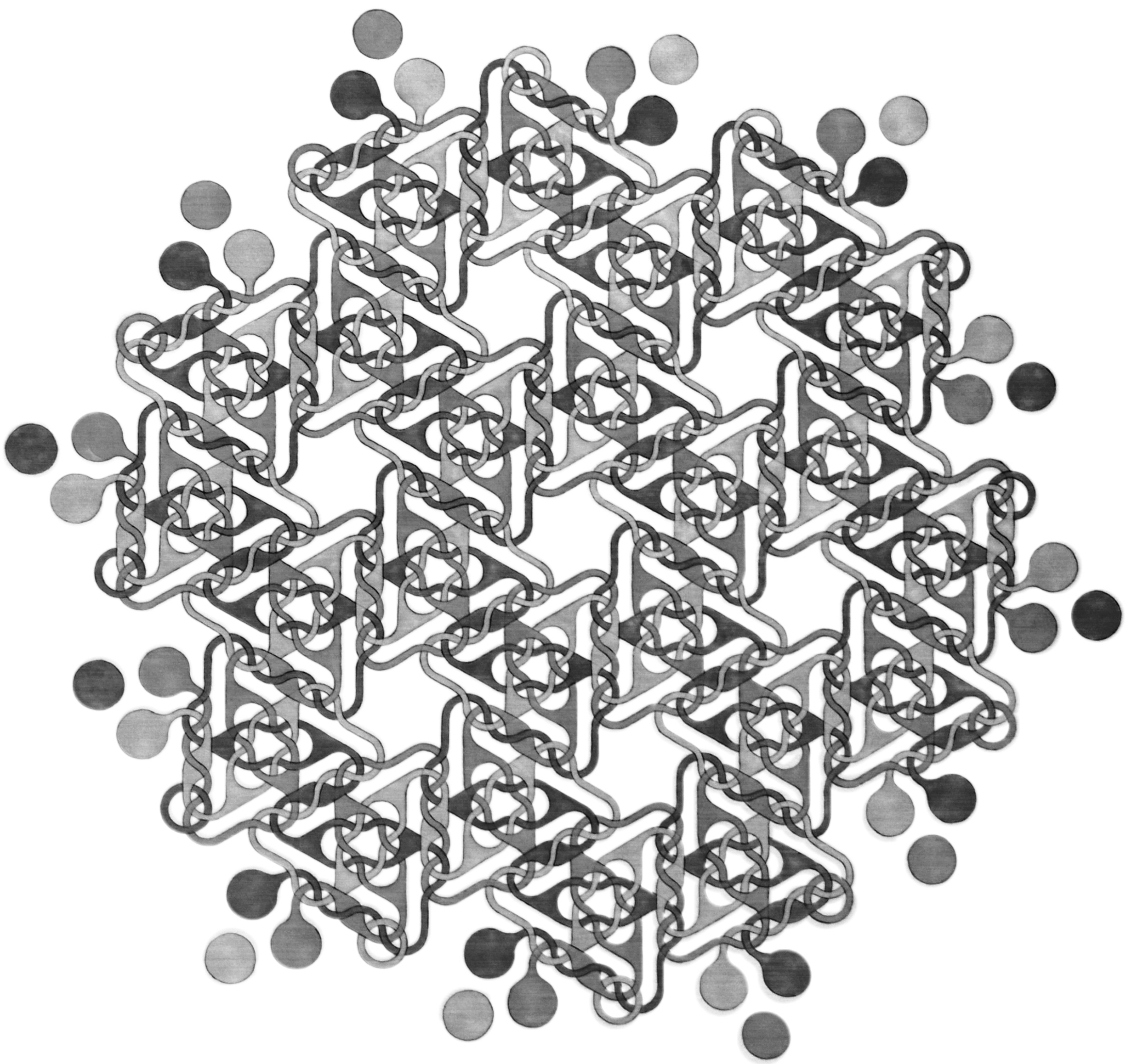

Figure 8: *Knot design of three surfaces forming a Brunnian link (pencil, marker)*

I described earlier how all of my strand ornaments can be constructed from only five different inscribed hexagonal tiles. It would be possible to design tiles that could produce the surface ornaments, but there would be roughly sixty of them. If the crossings were constructed from see-through tiles stacked on top of each other, I could reduce the total to eleven.

Conclusion

The simple hexagonal rosette, used as a template for knot ornaments, gives rise to designs of graphic clarity and harmony in proportion. These designs can have interesting spatial qualities that can be described in the language of mathematics. Although spatial concepts such as symmetry and proportion are commonly considered as inhabiting the domain of mathematics, they can also serve as a subject and inspiration for artistic investigations.

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[17] Goodenough, vol. VII, p. 191.
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Acknowledgements

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