Approaching an Approximation of Freeform Surfaces by Developable Strips using Apparent Contours.

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Abstract

The construction of free-form surfaces is limited to what is possible by graphical and constructive control. Over a few years we have seen an important development in the control of form through graphic digital technology: software and hardware that has allowed truly spectacular constructions.

An important line of research, not only in architecture but also in engineering, even sculpture, has been the adaptation of free forms by developable surfaces using different systems, many of them based on differential geometry.

Re-interpreting some methods of projective geometry that allow the use of certain CAD software, and jumping from physical to digital drawing systems, a method has been developed that allows the adaptation of free forms by developable surfaces using the apparent contours that we can draw over these free form surfaces. By using them we can construct two types of developable surfaces (cones and cylinders) that touch the surface tangentially to the contours.

Introduction

In recent years there has been an extensive use of double curvature surfaces in architecture. At the same time there has been an important development of such surfaces in building systems supported by digital design resources, as we can see in the architecture of Frank Gehry described by J. Glymph and D. Shelden [1,2].

There has been a proliferation in systems to control form, not only CAD technology software but also in very interesting theoretical developments, such as those published by H. Pottman [3], resulting in some amazing architecture.

The use of double-curved surfaces is not limited exclusively to architecture but has been part of the history of the use of geometry in general, such as in automotive or naval construction. Artistic creators such as Ilhan Koman [4] have also used developable surfaces in a very interesting way.

Metal sheets, plywood or glass have been widely employed in engineering and building construction. These materials are industrially produced flat, and they are more or less rigid or can be bent along only one direction. So there is no doubt about the advantages offered, mainly from the economic point of view, by processes that use flat or developable surfaces in the resolution of doubly curved ones.

In order to be able to address construction using materials with the possibility of bending in only one direction, or rigid material with no possibility of being bent at all by simple procedures, an algorithm has been created that allows the determination of single-curvature or flat strips by using the classical geometric concept of apparent contours, and by the systematization of a process inspired by traditional projective geometry, using the latest outstanding software.

Background and Geometric Basis

When we draw a surface, using any system of representation, either a freehand drawing on paper or through a software-rendered model, the process needs to define a line that marks the edge of the object represented, separating the part of the object that the observer is able to see from that which is hidden. This is basically the definition of the surface apparent contour.

A developable surface is produced and it is tangent to the object along the points that define their apparent contour. It could be a cone or a cylinder depending on the relative position of the point of view.

We distinguish two common types of projection: cylindrical projection and conical projection. Projecting lines parallel to one another tangentially to any surface we obtain an apparent contour through cylindrical projection. The set of parallel lines forms a cylindrical surface that circumscribes the surface. The common set of points of the two surfaces, base surface and cylinder, constitute the apparent contour of cylindrical or parallel projection. In a similar way a conical projection can be obtained drawing several lines radiating from a point of view that touch the surface tangentially. (Fig. 1)

The concept is very simple in principle, however obtaining these apparent contours is difficult, especially for models that are far from simple primitive forms. Even for commonly used primitive simple shapes manually obtaining shadow boundary lines or apparent contours is too laborious. CAD software provides powerful tools to simplify and improve this task.

It would be impossible to draw the apparent contours over free form surfaces without using CAD software. We used the modeling tool Rhinoceros [6]. It might be possible to use differential methods, but the way we work is intended to systematize the methods used in projective geometry. This approximation is carried out by using an algorithm that automates the process that theoretically could be addressed manually by translating the operating structure and proceeding to build it in Grasshopper[7]. Grasshopper is a powerful generative design plug-in that uses Rhinoceros as graphical support.

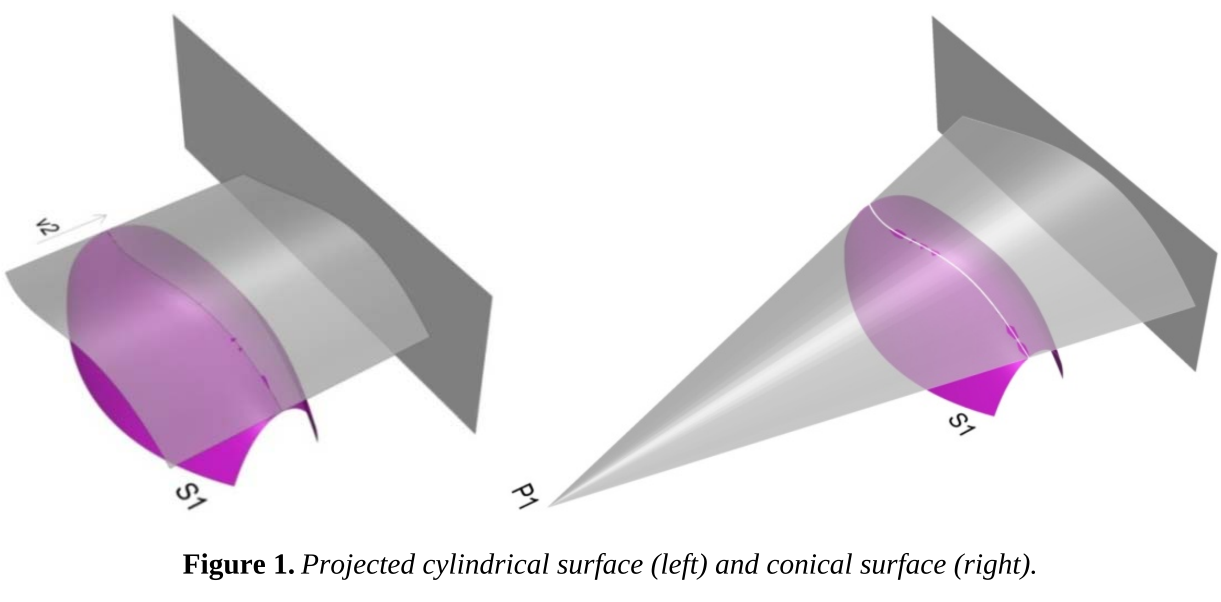


Figure 1. *Projected cylindrical surface (left) and conical surface (right).*

Approximation by Cylindrical Strips.

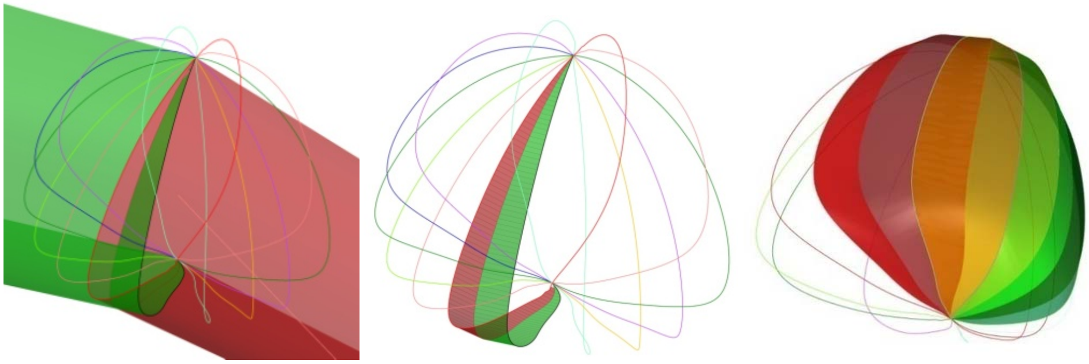


Figure 2. *Sequence of development stages to get cylindrical strips.*

Taking any positive Gaussian curvature freeform surface, we proceed to generate a family of apparent contours, by moving the point of view along a real projective line (a curve at infinity). Adjacent contours determine the intersection of the projected tangent cylinders to the base surface, and each cylinder is bounded by two intersections. Each of these parts of the original projecting cylinders contains both intersections and the tangent curve between the surface and the base cylinder. (Fig. 2)

In an analogous way we can trace a family of apparent contours using a real projective path using any kind of surface, and we can apply the method over a free-form surface with negative Gaussian curvature (Fig.3). The path used in this case, through which the point of view slides, is external to the surface so that, as can be seen, the contours are not cut. Generating apparent contour curves by moving the point of view along an improper projection produces a series of curves that match the tangent curve of the surface considered and the cylindrical surface of projection.

Each strip is obtained by transition between segments of the cylinder generatrix. We can approximate the surface as closely as we want by dividing the curve into as a large number of points as desired for plotting segments. However the surface from which the strip is taken is fully determined by the curve and the projection direction that coincides with the direction of the cylinder generatrices.

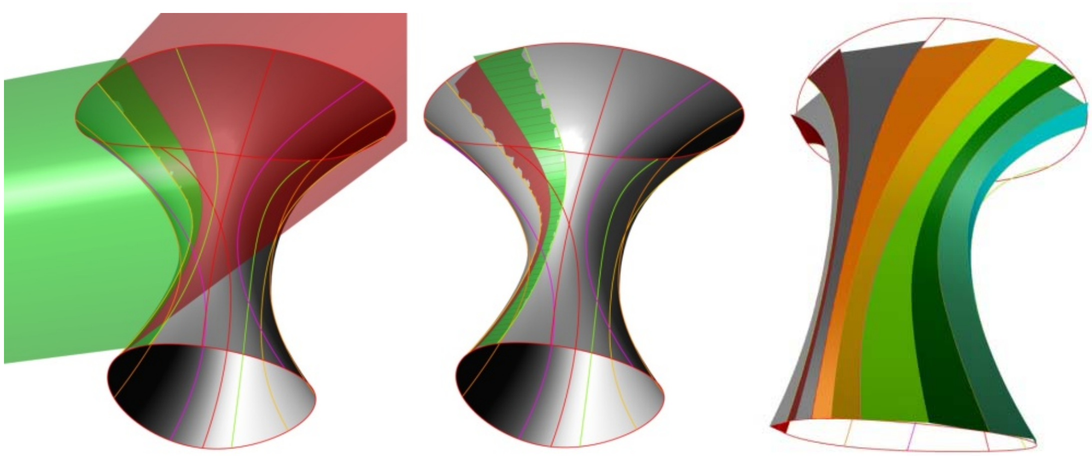


Figure 3. *Projection of cylinders supported in selected contours.*

Approximation by Conical Strips.

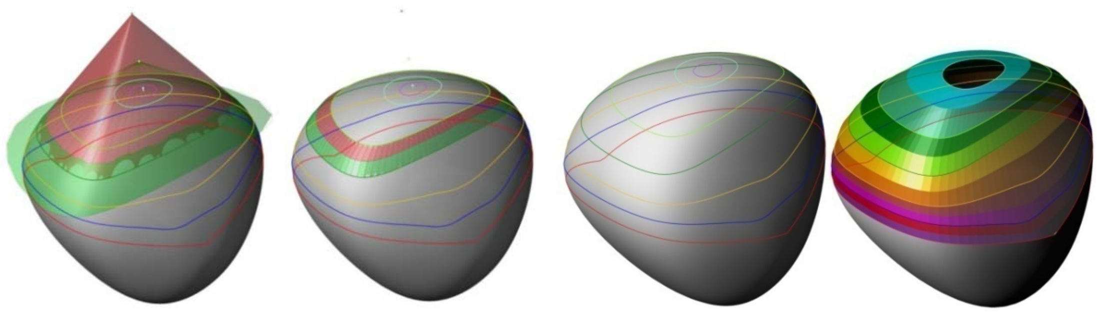


Figure 4. *Projection of cones supported in selected contours. Approximation by conical strips results.*

A similar approach can be applied over an elliptical, positive curvature surface, here using a family of conical contours drawn on its surface as support. In this case a design decision has been made to generate the contours by moving the viewpoint along a straight path so it cuts across the surface like a secant axis. (Fig. 4)

In the same way that the view points along the trajectories are arbitrarily chosen by a subdivision of the path, conversely we can obtain the paths that pass through certain points on the surface, locating for the intersection of the surface tangent plane at that point and the line used as the path. Thus we can select various specific contours passing through certain points that offer singularities on the surface or that may be interesting due to any design motif. We could likewise use the surface section through the plane containing the path, and divide the section produced on this surface. By plotting the tangents at several division points of the intersection curve we obtain the trajectory of the vertex of the cone that generates the contour passing through these points.

The process to obtain strips is similar to the cylindrical contour method explained above. The cones that rest on adjacent contours intersect between them. The intersection of two adjacent cones, external to the surface considered, together with the two contours which belong to the surface and each of the respective cones, determine a conical strip.

As we can see the choice of the kind of projection path and its relative position to the base surface generates strips that differ substantially, not only in the morphology of the strips themselves, but the appearance and the interpretation of the object also varies substantially. The versatility of this method in the treatment of the surface offers the possibility of addressing the modeling of it with a freedom that can have a positive effect on the interpretation of the original surface, expanding the design possibilities.

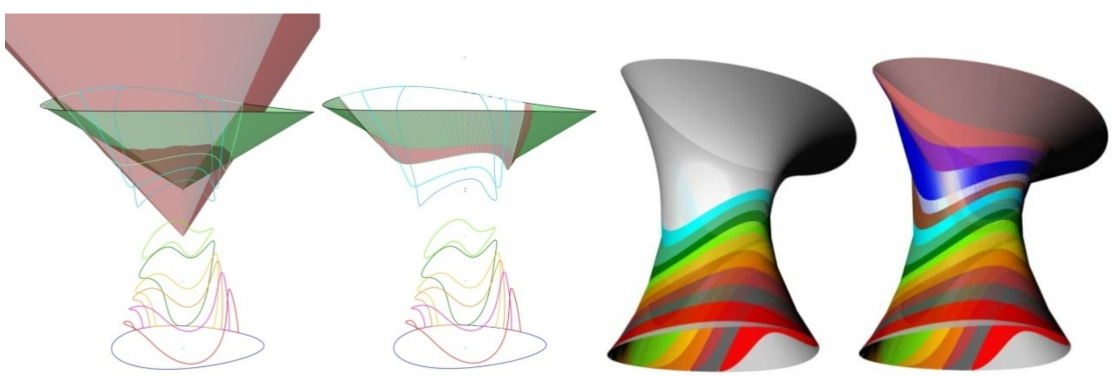
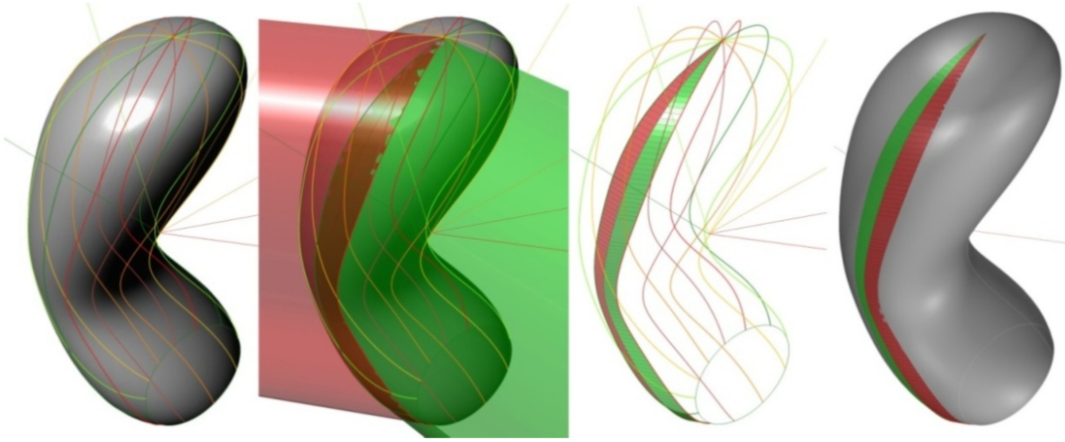


Figure 5. *Projection of cones along proper axis. Conical developable strips from hyperbolic surface.*



Thus we take advantage of one of the fundamental qualities of parametric design. The same process with different start values can yield different results using the same logical construction or algorithm. This allows the designer to choose the option that is most appropriate to the particular project, based on his requirements. As noted, these apparent contour curves have great expressive power, so that the choice of apparent contour groups is closely linked to the design process, which is another advantage that this method offers. Unlike divisions that rely on other mechanisms, a correct decision can contribute to the enrichment of the shape, producing significant and singular results with a visual interest.

The basic objective of the research is to find results that can be extrapolated to the physical and constructive environment so we have proceeded to physically check the function of the system, the approximation of the shape and the behavior of the material. In this way several cases have been studied graphically and physically, and we have been built prototypes using different materials. In all of them the faithfulness in the physical reproduction of the final prototype compared with the graphic model can be seen. (Fig.6-7)

The construction was carried out using a laser cutting machine EPILOG 36EXT Legend - Model 9000, which allows fast cutting directly from the design software. It thus anticipates the difficulties that may arise due to material nature, thickness, cutting tolerance, so on. The cutting system we employ limits the types of material which we can use, reducing them to cellulosic sheets (paper and cardboard) and plastics that are not chlorine compounds, mainly polyethylene and polypropylene.

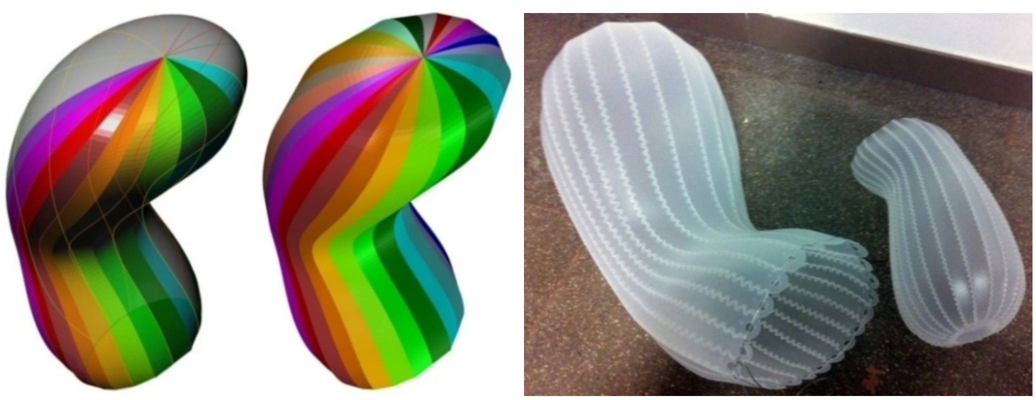


Figure 6. *Above and below. Developable generic freeform surface using flat strips.*

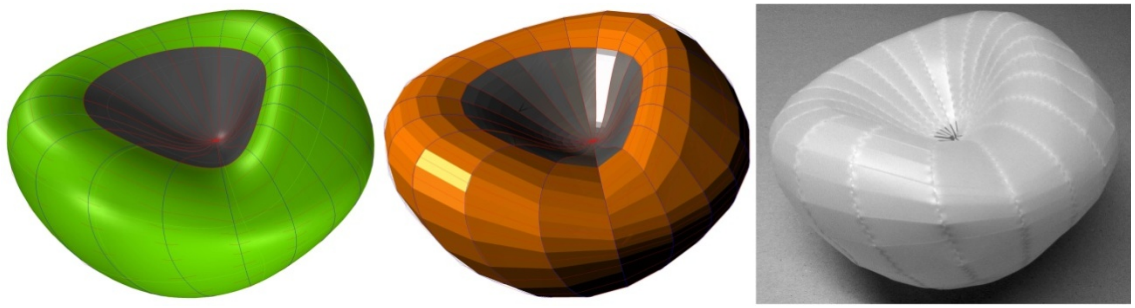


Figure 7. *Different adaptation phases from the digital model to the physical one built from plastic sheet.*

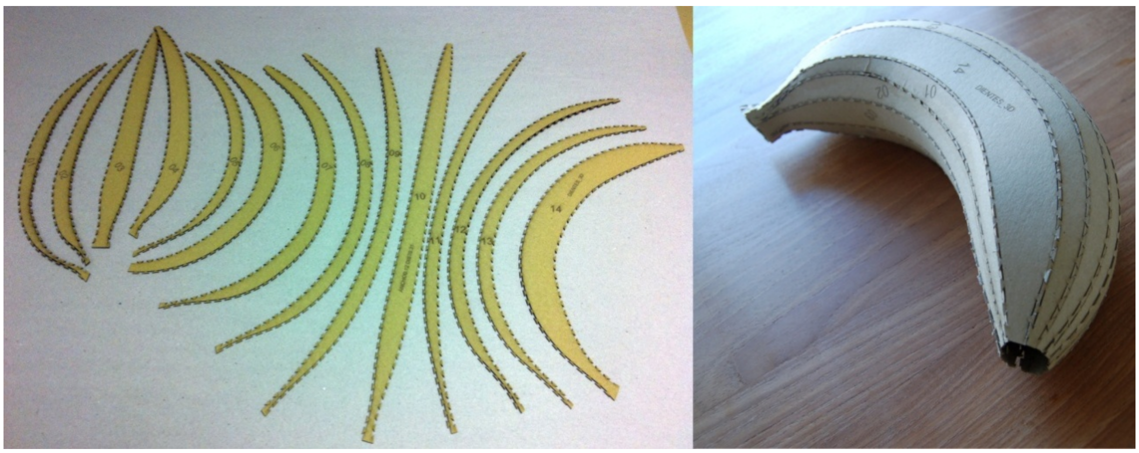


Figure 8. *Development of strips of flat cardboard that form the tridimensional form.*

Figure 9. *Spiral module prototype.*

We have tested the results that can be reached by this method in a Workshop about geometry and digital fabrication. Various models, seen in the next page, made by the School of Architecture students, have been used to check first the flexibility in design that CAD software based in NURBS surfaces such as those introduced by Rhinoceros, and on the other hand the versatility of the method applied to the solution of free form surfaces. It can be seen from the results which have been built, inspired by organic forms understood as freeform shapes, how by using apparent contour curves we can obtain a very faithful result in the reproduction of the form.

Figure 10. *Different models built by UPV-EHU School of Architecture students*.

Conclusion and Future Works

Following the initial objectives, economy of construction and fidelity in the reproduction of the form, we can conclude that the method forms an adequate basis of a geometrical method to adapt double curved surfaces through developable surfaces, which could be extended to the resolution of free-form surfaces. Satisfactory results at a prototype level have been shown, which achieve the stated objectives: graphical shape adaptation and construction.

The use of developable strips that have been extracted from cylinders or cones is applicable when using a material that allows bending in a single direction, so we are presenting a geometric system that underlies the construction of double curvature surfaces with metal sheets and products with similar behavior.

The use of apparent contours is not limited only to the generation of the developable surface strips but also faceted strips that overlay the form. Curves have been found, conjugate curves to contour curves, that are closely tied to these apparent contours since geometric points of view and pathways have been developed that rely on these properties, and approximations found to allow the resolution of quadrilateral flat facets or flat panels with utility in constructions that use rigid materials that cannot be curved (or curved only with difficulty), such as glass. The extent of this paper does not allow this development to be described. For the same reason (lack of space) we cannot appropriately describe the algorithm developed using Grasshopper.

It is relevant to note that it is possible by the method that we present to generate guidelines to design structural elements that could be built using developable strips, (Fig.11) so the method appears versatile enough for this purpose. In the same way that we found solutions both by faceted surfaces or flat strips or a solution combining both on a same model, we can propose various types of structural design solutions to adapt freeform surfaces without geometric contradictions.

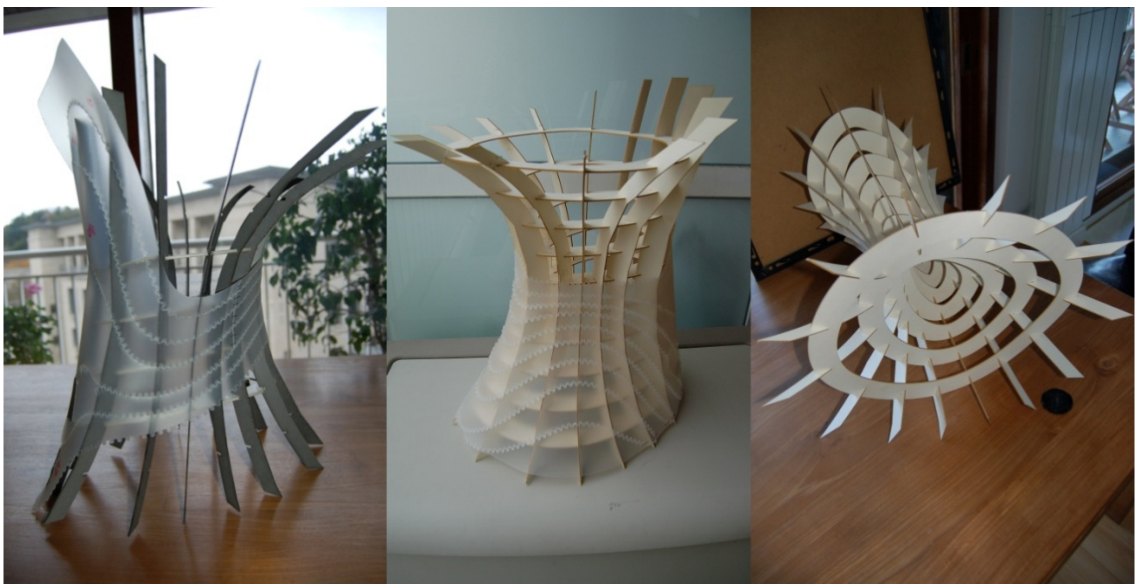


Figure 11. *Resolution of a full model, structure and skin enclosure using developable surfaces for both elements supported through the use of apparent contours.*

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