Math and Dance – Windmills and Tilings and Things

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Abstract

This workshop for Bridges participants will explore integrating mathematics and dance in the classroom as well as on stage. This workshop involves several mathematical topics which will be embodied and explored by participants. These include play with circular motions, use of paper as a prop, video and rhythmic tessellations and patterns, and N-body choreographies. Participants will create, practice, and perform short dance phrases, and simultaneously explore mathematical principles and critique the work from the point of view of both the mathematics and the artistry involved.

"Math Dance"

In [4] we pointed out the following: "Dance and mathematics share many essential concerns: (1) Both deal with the manipulation and exploration of patterns. (2) Both involve defining a problem and seeking a solution. (3) Both begin with concrete problems and progress to abstract ideas – or vice versa. (4) Both involve aesthetics and are integrally connected to cultural values and biases. (5) Both can make you sweat!"

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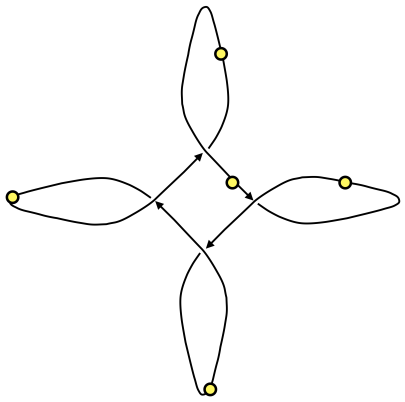
Purpose. This workshop will give participants a concrete experience of the connections between mathematics and choreography. It is in a certain sense a follow-up to a workshop given by Karl Schaffer and Erik Stern at the 2010 Bridges conference, in that it involves further explorations with symmetry, combinatorics, and rhythm; that workshop was itself based on many years of such workshops, as outlined in [3]. The introduction and overall philosophy of this workshop are similar to the 2010 workshop. Prior experience in either mathematics or dance is not necessary. The workshop alternates between creative problem solving and reflection/discussion. As in [3], the workshop participants will (1) solve problems physically in small groups, (2) discuss problems in smaller groups and as a class, (3) will both explore creatively the subject matter and examine the formalization of the concepts, (4) Discuss the use of these activities in K-12 and college classes.

Background. Mathematicians often view their work as involving creativity and imagination. Dancers/choreographers utilize analytical thinking and attend to the forms and stylistic rules inherent in their art. The two disciplines work well together, and we will see that dance compositions may be developed as "embodied mathematics," or mathematical principles may be found embedded in dance and rhythm. Several of the activities in this workshop are related to dancers created by the author in recent dance concerts during 2009-2012.

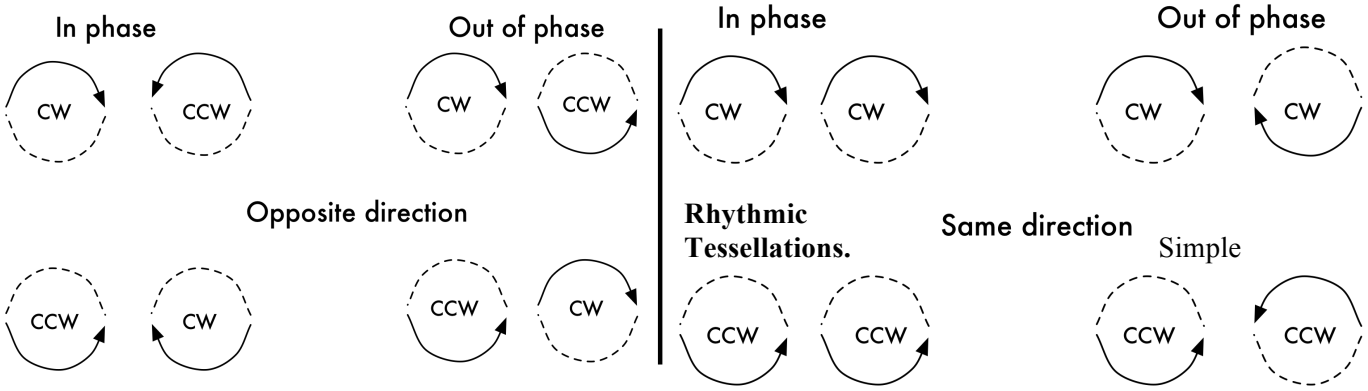
The activities do not require special training in dance or mathematics (although training or interest in these subjects can be helpful). They begin by addressing the most basic universal elements of dance and mathematics: recognizing and playing with patterns, moving or locomoting simply, counting, making shapes. The activities are designed to be flexible and can be extended to suit the level of the participants.

Everyday Movement. Everyday movements, like clapping or swinging the arms in circles, are a great way to involve non-dancers in doing and creating movement sequences. In [3] and [4] we noted that, "Although many types of dance, such as classical ballet, take years of practice to master, others are built on everyday movements that anyone can do. For instance the musical performance Stomp creates dance out of everyday actions like sweeping the floor. Many hip-hop moves began as everyday gestures. All over the world folk dances are made out of the everyday movements of work and play."

N-Body Choreographies. *In this activity participants perform periodic floor patterns based on what are now known as N-Body Choreographies,* "periodic solutions to the N-body equations in which N equal masses chase each other around a fixed closed curve, equally spaced in phase along the curve."[1] Participants then convert the patterns to movement sequences. The activity helps develop spatial sense and connects to contemporary approaches to the N-body problem. Participants will also see various folk and contemporary dances that utilize these spatial patterns. The participants also look at basic choreographic principles of sequencing and transitions by creating and manipulating dance phrases along these floor patterns.
(Diagram to right based on 5-body pattern in [1].)



Windmills. *In this activity, participants explore several ways of moving limbs and bodies in circular patterns, including swinging both arms in and out of phase with each other, then create sequences of group shapes that exhibit the symmetries.* Participants will also experiment with dancing with single sheets of 8-1/2 by 11 paper, which can be swung in circular motions that keep the hand pressed against the paper and thus keep it from falling to the floor. They will then construct and perform simple dance phrases using the circles and paper. The diagram below shows ways of windmilling the arms in and out of phase.

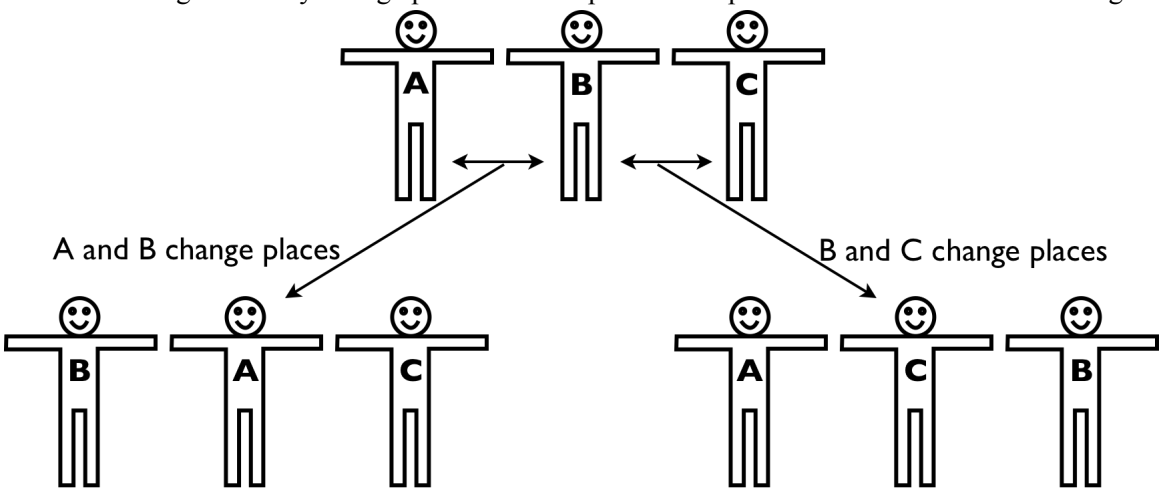


Video Tessellations. Participants will use recently created software to explore and move using live projection of video tessellations of their movements. This allows real-time creation of imagery with various symmetries. We will create and take snapshots of various poses that solve problems posed and create movement sequences utilizing the software.

We will also explore movement and rhythmic tilings in time, using binary clapping patterns composed of claps with the hands and slaps on the thighs can to construct rhythmic tessellations. These employ patterns in which one clap by a participant occurs on each beat, and every participant plays the identical pattern, translated a certain number of beats in time from the next person. We will learn and practice several of these patterns, and also learn simple slapping, clapping games popular with children, noting

their mathematical content, and use them to construct movement sequences and movement tessellations. For example, a clapping, finger-snapping routine called "biddy-bam" is an "almost" palindrome with a nice rhythmic pattern. Other common patterns involve different ways of dividing up 8 beats.

Change-Ringing Three people standing in a row can accomplish all 6 permutations by repeatedly switching two neighbors, returning to the starting position without any repeats. At any step the first two or the last two neighbors may change places. This duplicates the process used in church bell change-ringing.



Four people in a row. ABCD, can change to any of the following, by switching two neighbors:
BACD (A and B switch), ACBD (B and C switch), or ABDC (C and D switch)
Again, it is possible to accomplish all 4! = 24 permutations and return to the starting position, without repeating. The mathematics for this involves finding a Hamiltonian cycle through the vertices of what is known as the "permutahedron." This problem, and that with larger numbers of participants, was solved by change-ringers before it was addressed by mathematicians [2]. We will attempt to move through these patterns, and will also explore performing 3 or 4 movements in a permutahedron sequence.

The larger picture. We often refer to two quotes by William Thurston as applying to the larger purpose of these activities [5]:

> "An interesting phenomenon in spatial thinking is that scale makes a big difference. We can think about little objects in our hands, or we can think of bigger human-sized structures that we scan, or we can think of spatial structures that encompass us and that we move around in. We tend to think more effectively with spatial imagery on a larger scale: it's as if our brains take larger things more seriously and can devote more resources to them."

> "One-on-one, people use wide channels of communication that go far beyond formal mathematical language. They use gestures, they draw pictures and diagrams, they make sound effects and use body language. Communication is more likely to be two-way, so that people can concentrate on what needs the most attention. In talks, people are more inhibited and more formal…in papers people are still more formal. Writers translate their ideas into symbols and logic. And readers try to translate back… Mathematics in some sense has a common language: a language of symbols, technical definitions, computations, and logic. This language efficiently conveys some, but not all, modes of mathematical thinking."

Background. The author and Erik Stern had been choreographing works together for three years when we began to discuss the similarities between the processes which underlie mathematics and dance. The performance which resulted, "Dr. Schaffer and Mr. Stern, Two Guys Dancing About Math," premiered in

1990, was performed over 500 times throughout North America, and led to the creation of numerous other performances exploring the connections between mathematics and dance. In 1993 Schaffer and Stern collaborated with Scott Kim, noted puzzle designer and mathematician, on the performance "The Secret Life of Squares." As a natural outgrowth of their work on stage, Schaffer, Stern and Kim created workshops which allowed students to experience in the classroom the connections between dance and mathematics. The activities in this workshop are recent ones created by the author, but in the spirit of the workshops created in earlier years with Erik Stern and Scott Kim.

The goal of these activities is to provide a jumping off place for teachers and artists to use to create classroom activities and/or performance works that simultaneously explore mathematics and dance. Participants or readers might consider dance forms with which they are familiar, and look for connections between mathematical concepts and the ways that dancers form shapes with their bodies, connect with other dancers, follow spatial paths, make use of rhythm, or perform sequences of movements.

References

[1] Montgomery, Richard, "N-Body Choreographies," in Scholarpedia, http://www.scholarpedia.org/article/N-body_choreographies, 2010.

[2] Dermot Roaf and Arthur White, "Ringing the changes: bells and mathematics," in *Music and Mathematics: From Pythagoras to Fractals*, ed. by John Fauvel, Ramond Flood, and Robin Wilson, Oxford Univ. Press, 2003.

[3] Karl Schaffer, Erik Stern, and Scott Kim, *Math Dance with Dr. Schaffer and Mr. Stern: Whole-Body Math Movement Activities for the K-12 Classroom,* published by MoveSpeakSpin, 2001.

[4] Karl Schaffer and Erik Stern, *Workshop on Mathematics and Dance*, Bridges, 2010.

[5] William P Thurston, "On Proof and Progress in Mathematics," in *18 Unconventional Essays on the Nature of Mathematics*, ed. By Reuben Hersh, Springer, New York. 2006.