

Bead Crochet Bracelets: What Would Escher Do?

Ellie Baker
Lexington, MA 02421, USA
E-mail: ellie.baker@post.harvard.edu

Susan Goldstine*
Department of Mathematics and Computer Science
St. Mary's College of Maryland
18952 E. Fisher Rd
St. Mary's City, MD 20686, USA
E-mail: sgoldstine@smcm.edu

Abstract

The popular handcraft of bead crochet produces elegant toroidal bracelets that are visually appealing and easy to wear. Designing bead crochet patterns is surprisingly challenging because of the arrangement of beads into a continuous spiral around the torus. In this paper, we present bracelets that we have designed in the style of M.C. Escher's tessellations. Each bracelet pattern consists of interlocking copies of a single shape, and we include patterns designed using a powerful new technique for extracting bracelet patterns from planar tilings, along with strategies for creating more patterns.

Our Design Challenge

Bead crochet is an increasingly popular craft, and bead crochet rope bracelets (shown below in Figures 2, 3, 7, and 11) are particularly elegant in their seamless construction. However, creating symmetric color patterns on bead crochet bracelets is uniquely challenging among bead crafts because of the particular geometry of the beaded surface, as described in the following section. As a result, many existing patterns are based on a few simple known motifs derived from one or two well-understood flat pattern layouts.

The bracelets presented here stem from a design challenge we set ourselves inspired by the tessellation artwork of M.C. Escher, who adapted his own designs from planes to spheres, polyhedra, and cylinders. Mathematically, could we construct bracelet designs consisting of a single shape in multiple colors tessellating the torus? And aesthetically, could we make those shapes take on recognizable forms?

Diagonal Charts

To make a bead crochet bracelet, we string the beads onto the thread in the appropriate order for the desired pattern and crochet the beads into a long spiral tube. When the tube is long enough, we sew the ends together to make a seamless torus of beads. In order to convey a pattern to the crafter, the designer must provide two pieces of information: which beads are strung in what order, and how many beads are crocheted before the spiral is closed, which is called the *circumference* of the bracelet (as in [4]). We can represent both of these data in a bead crochet chart, as shown in Figures 1 and 2.

Charts are read from bottom to top and left to right. The traditional format for a chart (as seen in [1] and [2]) is a left-leaning diagonal swath of beads, and the number of beads in each row is the

circumference of the bracelet. In such a bead chart, all of the beads that touch in the bracelet touch in the chart except for those at the top and bottom, which are sewn together after the bead tube is crocheted, and those at the edges, which are connected as shown in Figure 1.

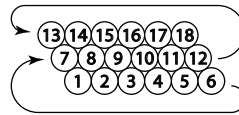


Figure 1: *The stringing order of beads in a bead crochet chart of circumference 6. Beads at either end of an arrow are consecutive in the bracelet.*

As a result of the upward shift in connecting rows, it is difficult to track how designs will flow over the edges of the chart. Most bracelets designed on this style of chart reflect that obstacle by generating a spiraling diagonal stripe along the seam, as in the bracelet in Figure 2. Since this restricts the active design region to a narrow strip within the diagonal chart, it is much more difficult to create recognizable figures, and bracelets designed in this traditional framework tend to be either more abstract, like the bracelet in Figure 2, or less symmetric.

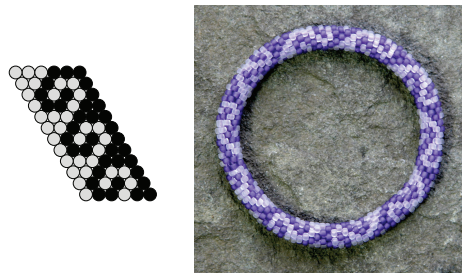


Figure 2: *A traditional diagonal bracelet chart and the corresponding bracelet.*

Vertical Charts

To overcome the limitations of the traditional chart, some designers have transitioned to a vertical chart (see [3]), in which the beads are diagrammed as though the bead tube were slit lengthwise and laid flat. While it is still difficult to align the edges of the chart properly, vertical charts give us access to patterns that would be much more difficult to design diagonally. The interlocking vine bracelet in Figure 3 was designed on the vertical chart shown on the left. Since the motifs are aligned vertically, when the bracelet is charted in the traditional diagonal layout on the right, the pattern does not appear to be regular at all.



Figure 3: *A bracelet of circumference 7 designed on a vertical chart, and its diagonal chart.*

Designs in the Bead Plane

We gain access to a broader array of bracelets in the style of Escher by switching to a design scheme that does away with the seams of regular bead charts altogether. In this technique, which we describe in more detail in [4], a paper presenting another bead crochet design challenge, we draw a periodic pattern in an infinite plane of hexagonally packed beads. In order to fit on a bracelet, the pattern must be preserved by a translation (denoted a *hockeystick translation* because of its shape, shown in Figure 4) that corresponds to the wrapping of the bead spiral. The translation captures the effect of the adjacency arrows in Figure 1.

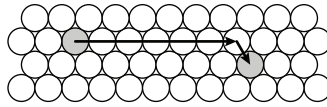


Figure 4: A hockeystick translation corresponding to a bracelet of circumference 6. The two shaded beads in the bead plane both represent the same bead in the bracelet.

In Figure 5, we show a bracelet pattern that was designed in the bead plane, along with the vertical and diagonal charts for the same pattern. The asterisks indicate accent beads in the middle of each motif. In this design, we chose the flower-shaped tile first, then tiled the plane with it, and finally used the plane tiling to deduce that the appropriate circumference was 9. Not only is it impossible to design a bracelet on a standard chart without choosing the circumference first, but also, as Figure 5 shows, it is quite difficult to see that the flower motifs align properly at the edges.

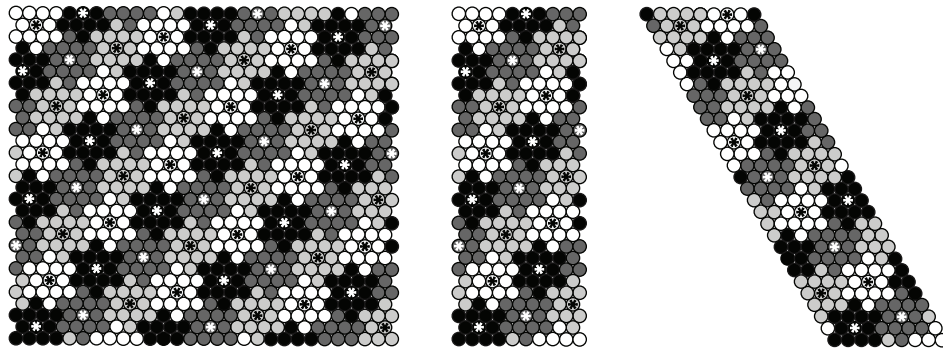


Figure 5: A bracelet design in the bead plane, with the corresponding vertical and diagonal layouts.

Figure 6 shows two more designs created in the bead plane, and Figure 7 shows bracelets produced from the patterns in Figures 5 and 6. The pattern on the right of Figure 6 has the unusual property that the congruent motifs are rotated by 120° , a phenomenon that we have not seen in the work of other bead crochet designers.

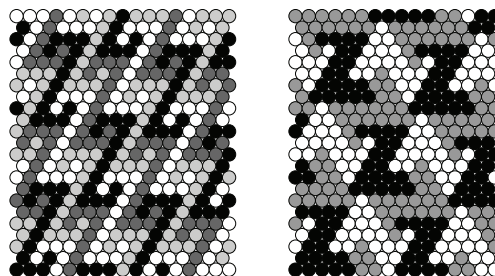


Figure 6: Two more bead plane bracelet designs; the circumferences are 8 and 7.



Figure 7: Bracelets made from the bead plane designs in Figures 5 and 6.

Making Tessellated Bracelets

The best way to learn to make bead crochet ropes is to work with an expert in person. Many bead stores offer lessons in bead crochet, and we encourage you to check your local stores for classes. There are also bead crochet pattern books (such as [1]) that include step-by-step instructions, as well as many online videos and tutorials.

Part of the challenge of making bead crochet bracelets is that the beads, thread, and crochet hook are very small. One way to master bead crochet techniques without undue strain is to start by using plastic pony beads, which are sold for children’s crafts. The resulting bead ropes, like that shown in Figure 8, while too large and inflexible to form bracelets are quite useful as a learning tool, and we especially recommend them if you are attempting to master bead crochet without a class or workshop.

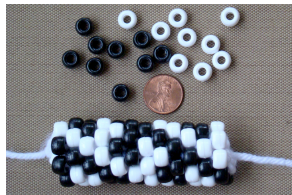


Figure 8: A stub of bead crochet rope made with plastic pony beads.

Once you are ready to work in scale, you need seed beads, thread, and a tiny crochet hook (size 10 or 11 works well). As a general rule, the larger the bracelet circumference, the smaller and more uniformly sized the beads should be. Size 8/0 seed beads work well for circumferences 4 and 5, whereas circumferences 6 and 7 are better suited to smaller size 11/0 round seed beads. For circumferences 8 and larger, we have had great success with Miyuka Delica size 11/0 seed beads and Toho Aiko Precision Cylinder beads, which are close enough in size and shape to be used together. The thread must be durable enough to sustain flexing in the finished bracelet; we use Gütermann Top Stitch heavy-duty thread.

The best way to learn to *design* bead crochet ropes is to experiment with bead charts. While the bead plane may seem more daunting than diagonal or vertical charts at first glance, it is so much easier to decipher how patterns fit together in the bead plane that we strongly encourage trying it out. The simplest way to begin is to choose your circumference n in advance and start coloring in a grid of hexagonally packed beads. Each time you color a new bead, you must color the beads that are a hockeystick translation of length n away from it (Figure 4) the same color.

When you have a design that you like, you can extract a beading pattern by taking a region in the shape of either a diagonal chart (Figure 2) or a vertical chart (Figure 3) for a bracelet of circumference n and reading it in the usual order, left to right and bottom to top. Figure 9 shows an example of this process for a bracelet of circumference 7. In a pattern that is periodic in a direction other than that of the hockeystick translation, the bead color pattern will eventually repeat, and writing out the numbers of beads of each color in sequence can help determine how long it takes for the stringing sequence to repeat itself. To make the bracelet charted in Figure 9, which is the pattern for the pony bead stub in Figure 8, you would string the 12-bead sequence listed on the right over and over until you had enough beads for a bracelet (around 70 repeats in size 11/0 round seed beads).

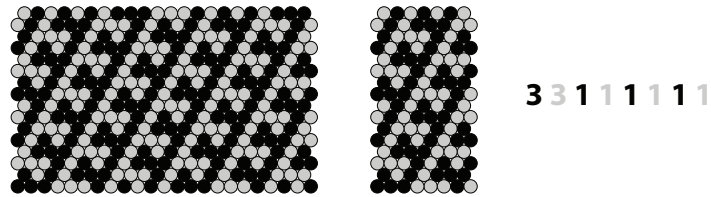


Figure 9: A design in the bead plane along with its vertical chart and bead color sequence.

For a bead plane novice, constructing a pattern that is also a tessellation of the plane with a single shape takes practice and cleverness. One strategy is to work out a tiling by polygons and then deform the tiles into more interesting shapes, in the manner of Escher's symmetry studies [6]. In Figure 10, we offer a tiling of the bead plane by parallelograms as a springboard for new designs. The pattern is for a bracelet of circumference 7, and any alteration in shape that is performed identically on every parallelogram will necessarily preserve the hockeystick translation to produce another viable bracelet pattern. The region on the right contains the color sequence that is repeated to string beads for the entire bracelet, and extracting a region of the same shape from a modified pattern will give the color sequence for the new pattern.

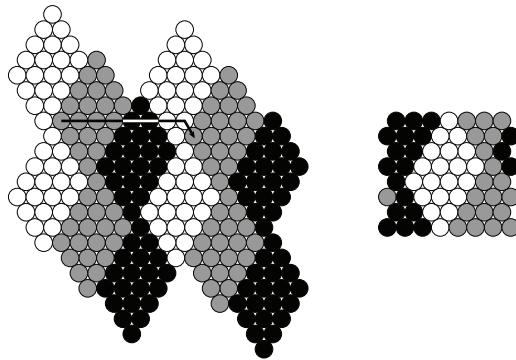


Figure 10: A polygonal tiling of a circumference-seven bracelet that can be deformed into more intricate designs. The chart on the right gives the bead sequence for the bracelet.

As you grow more comfortable with bead plane design and bead crochet, you may be inspired to move to bracelets of larger circumference and their increased design space. Although bracelets of circumference 9 or higher take *much* longer to make, they accommodate enough extra detail to make them worth the time and effort, as shown in Figure 11. The pattern for the bracelet on the right is in [4].

In our exploration of bead crochet rope patterns, the translation between bracelet charts and planar tilings has pushed our designs in exciting and unexpected directions. We hope that the techniques outlined here inspire you to extend the art of bead crochet in your own way.



Figure 11: *Lizard and fish bracelets of high circumference.*

Acknowledgements

We are grateful to Katie Sheena for her invaluable assistance with bead crocheting and for inspiring the pattern in Figure 5, to Kira Isaacs for her bead stringing, to Robert Fathauer for his suggestions in the early phases of our work, and to Randall Munroe for inspiring the title of this paper with his classic *xkcd* cartoon, reprinted below.

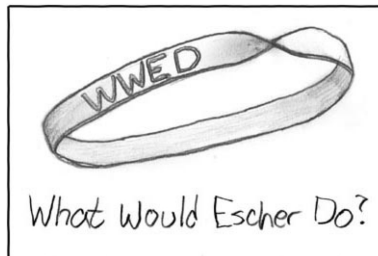


Figure 12: *Thanks again, xkcd!*

References

- [1] J. Bertoglio-Giffin, *Bead Crochet Ropes*, Glass Cat Books LLC, Manchester, NH, 2002.
- [2] J. Bertoglio-Giffin, *Patterns & Graphing for Bead Crochet Ropes*, Glass Cat Books LLC, Manchester, NH, 2004
- [3] J. Bertoglio-Giffin, *Triangular Bead Crochet Ropes*, CreateSpace, 2011.
- [4] S. Goldstine and E. Baker, *Building a better bracelet: wallpaper patterns in bead crochet*, *J. Math Arts* 3 (2012), pp. 5—17.
- [5] R. Munroe, “Escher Bracelet,” <<http://xkcd.com/88>>.
- [6] D. Schattschneider, *M.C. Escher: Visions of Symmetry (New Edition)*, Harry N. Abrams, New York, 2004.