Spelunking Adventure III: Close-Pack and Space-Fill Octahedral Domains

Curtis Palmer
Edmonton, Alberta, T6J 4N2 Canada
clpalmer@shaw.ca

Abstract

Twelve polyhedra with octahedral symmetry are modeled within a unit radius circumsphere. These are used as blocks to populate two matrices that share a 12-around-1 packing strategy: a close-pack array of unit spheres and a rhombic dodecahedral space-fill. Each polyhedron is modeled with a tree of virtual layers containing geometric identifiers: points, point-clouds, vectors, surfaces, and polysurfaces. With programmatic selection of layers a vast number of constructed spaces can be visualized, printed and animated. Visual examples are presented.

Introduction. Since 2005 [1], I have used a database of polyhedra constructed within a unit radius circumsphere to inform my inquiry into geometry and computer-aided design software practices. To explore the CAD data structure known as a *block* I modeled 12 polyhedra with octahedral symmetries to create the *octablock* (Fig. 1). I copied it into two 12-around-1 spatial domains: a Close-Pack of spheres (CP) that contains convex rhombic dodecahedral voids [2], and a Rhombic Dodecahedral space-fill array (RD) with no voids. Visualizations of selected *octablock* layers in both spatial domains are compared three views at a time corresponding to the axes of symmetry of the octahedron. Each frame of the triptych records a 2D projection of a 3D assemblage. The rotational symmetries intrinsic to these octahedral systems are seen through a filter of select *octablock* layers. These 2D projections resemble tesselations. Pairs of triptychs document the before and after views of the spatial translation: the omnidirectional deflation / inflation of the *octablock* across the void. A void dance, as it were.

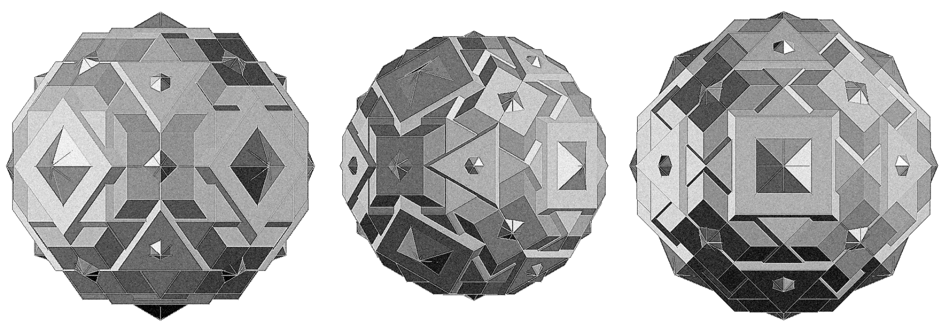

Figure 1: Three *octablock views, left to right, aligned to an octahedron's edge, face and vertex.*

Spherical Close-Pack and Rhombic Dodecahedral Space-Fill Domains. For this paper the matrices were of frequency f=3 where f refers to the number of spherical shells radiating outward from a nucleating sphere. In closest packing, the number of spheres accumulating to each shell equals ($10f^2$+2). For f=3,2,1 the total number of spheres is (92+42+12) +1 (for the nucleus) = 147 (Fig. 2).

Rhombic dodecahedra like cubes can combine to fill all space [3]. In the CP domain each *octablock* plus sections of neighboring voids can be encased by a rhombic dodecahedron of circumsphere radius cr=√2; proof that the CP and RD share the ability to aggregate 12-around-1, omni-directionally. This capacity for 12-around-1 packing is my muse for constructing these comparisons. To construct the RD from the CP without scaling (cr=1), I need to *squeeze out the void* by translating the *octablocks* until the faces of neighboring rhombic dodecahedra are conjoined.

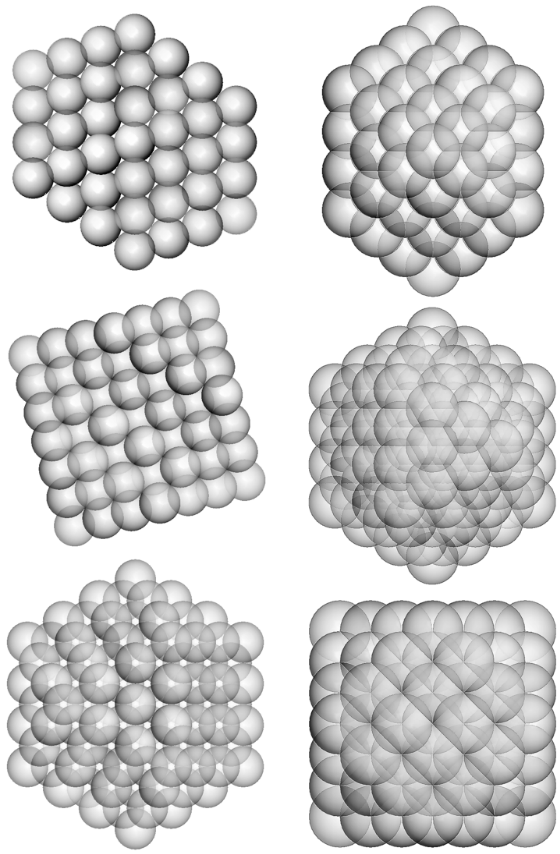

Figure 2: *CP & RD Sphere Packing.*

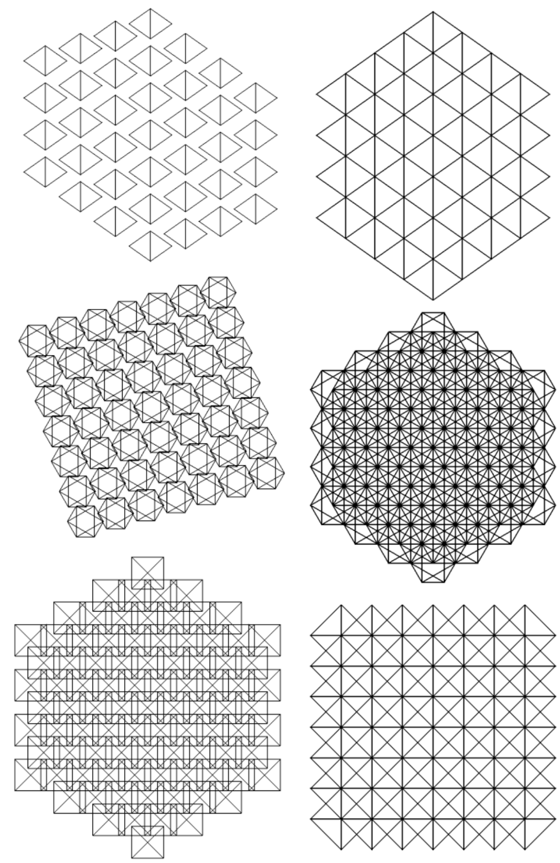

Figure 3: *CP & RD Octahedra.*

Close-Pack and Space-Fill Patterns. Unique patterns arise when plotting, shading or rendering these domains according to the axes of symmetry of the octahedron. Other views that do not exploit this symmetry are messy and hard to read. In the CP domain each *octablock* is isolated in its own circumsphere. The polyhedra as defined, overlap within the *octablock*. In the CP they appear to overlap. These overlaps create 2D effects that read like the boundaries of tiles in a mosaic (Figs. 2 & 3). By keeping the visible layer count low, interesting patterns are produced, particularly when visualizing duals.

In the RD domain, neighboring *octablocks* actually overlap. The circumspheres (Fig. 2) and some polyhedra (not the rhombic dodecahedra) encroach on each other's space, so as to add real boundary effects to the projections. The side by side triptychs allow for easy comparison of these 12-around-1 spaces with and without voids and contentious boundaries.

Conclusion. By following my muse I have gained experience with the use of blocks and produced a dazzling variety of forms. By varying the packing strategy of the *octablock* I have documented a radial inflation / deflation of octahedral polyhedra along 12 axes simultaneously. That much was easy. How to select graphic elements, pleasing to the eye and yet informative? There's the rub!

References

[1] C. L. Palmer, *Digitally Spelunking the Spline Mine,* Renaissance Banff, pp. 309-312. 2005.
[2] R. B. Fuller, *Synergetics,* Macmillan Publishing Co., p. 108. 1975.
[3] R. B. Fuller, *Synergetics,* Macmillan Publishing Co., p. 599. 1975.