The Old Art of Rope Work and Fourier Decomposition

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Abstract

Mathematically there is a close connection between pendulum drawing and rope mats. This article describes a method of analysing traditional rope mats and rosettes. By sampling the rope track one finds that the rope follows a periodic function both in *x*- and *y*-directions. Having the two periodic curves, it is possible to do a Fourier decomposition in both directions. By doing this the Fourier components are found for the two curves, or, as we may call it: *The two-dimensional spectrum* for the rosettes. By knowing the spectrum for some known mats and rosettes, it is possible to categorize the mats in families based on their order (number of needed components to represent the mat), which can be different from the traditional way of categorizing them. The Fourier components for the mats may now be used to synthesize the mat with a two-dimensional curve drawing software like Matlab or Winplot. By changing the Fourier components' frequency and amplitude, it is possible to make new variants of the mats and rosettes within the same family.

Introduction

A sailing-ship cuts through the sea. There is a moderate breeze and it is time for a rest. A sailor is sitting on a barrel working with a piece of rope. The coarse fingers of the sailor makes the finest shapes with the simple rope. After a while, a work of art takes shape in his hands and a *rope rosette* is placed on the deck.

A pendulum is suspended from the ceiling. The heavy weight is carried by two cords attached to the ceiling. The pen, which is fastened to the weight, is following the movement perfectly and strokes gently across the sheet of paper which is stretched out on the floor. The pen makes arcs of the finest sort on the paper. The movements are slow and steady. A lonely spectator sitting by the side of the pendulum is spellbound by the curves becomming visible on the paper. After some time the pendulum settles down and the curve is tied up in a single point at the centre of the drawing. A beautiful *pendulum drawing* lays on the floor.

It is late in the evening, an enthusiastic scientist is sitting by his computer meditating on a mathematical problem. An irregular time function emerges on the screen. He presses a couple of keys on the keyboard and the computer starts analysing the signal. After a few seconds the time function is transformed into bars in a diagram and suddenly some of the inner secrets of the function are revealed. The computer screen shows, in sharp contrast to the dark background, a *signal spectrum*.

Are there any common denominators between these three so very different topics?

The next sections describes both my process of discovering the connection between pendulum drawing and rope works, and how my work as a researcher in communication technology inspired me to develop a method of analysing rosettes.

Pendulum drawing

By suspending the cables of a pendulum from two points in the ceiling and then leading the cables together into one point between the points of suspension and the weight, it is possible to make the pendulum oscillate with two different frequencies in two directions perpendicular to each other.

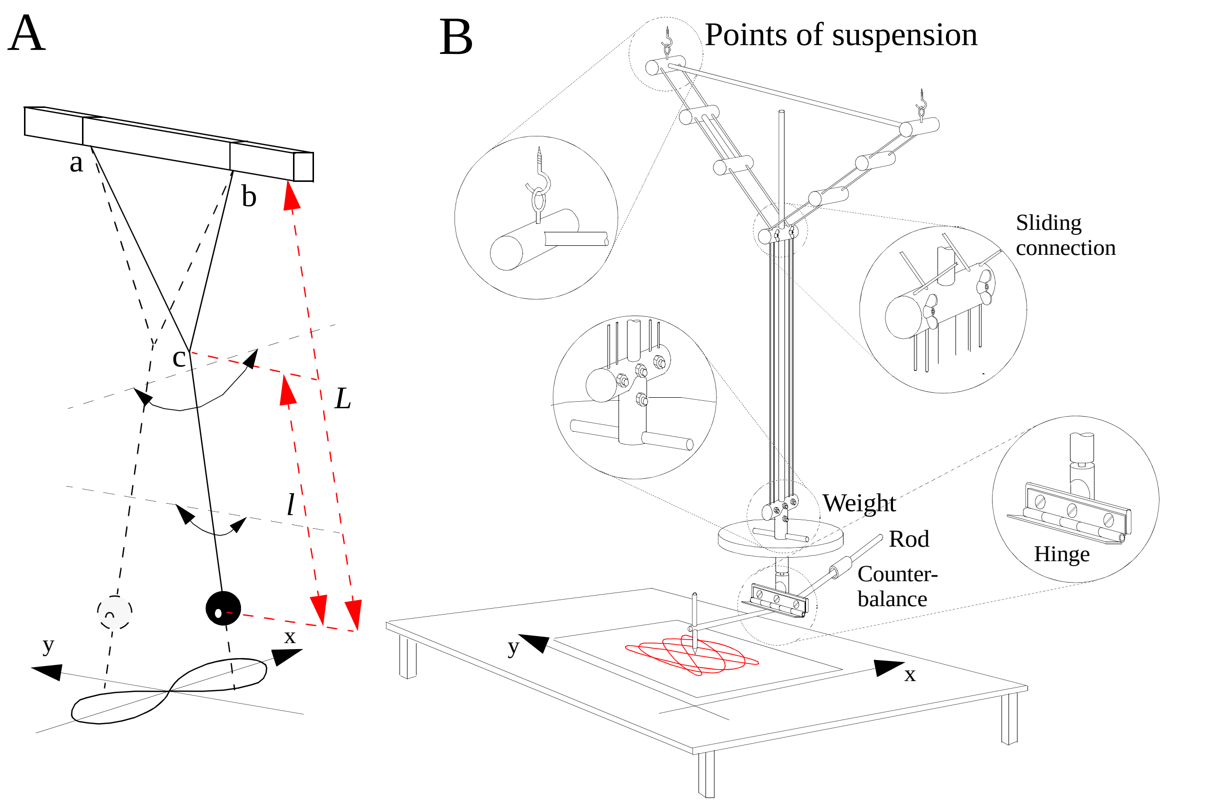


Figure 1 *A. Simple two-point pendulum. B. Two-point pendulum drawer.*

Figure 1 A shows how the two-point pendulum works in principle. When the square root of the ratio between the long part, *L*, and the short part, *l*, of the pendulum, is 2, the pendulum weight describes a figure of eight, if the phase between the two oscillations is suitable. Such a pendulum was described by Dr. ***Nathanael Bowditch*** (1773–1838) [4] and Prof. ***James Dean*** (1776–1849) in the beginning of the 19th century. Prof. Dean used the two-point pendulum to illustrate the apparent motion of the earth, observed from the moon [3]. Figure 1 B shows a two-point pendulum *drawer* first described by ***Hubert Airy*** (1838–1903) in *Nature* 1971 [6]. By using a sliding connection between the two wires, it was possible to change the frequency ratio between the two oscillations, and a great variety of figures could be created on the paper. The pen is attached to a rod fastened to a hinge so the pen can tip up and down and keep a constant pressure on the paper. The pressure may be adjusted by the counterbalance. These figures are also called *Lissajous (or Bowditch) curves* after the French physicist ***Jules A. Lissajous*** (1822–80) [5]

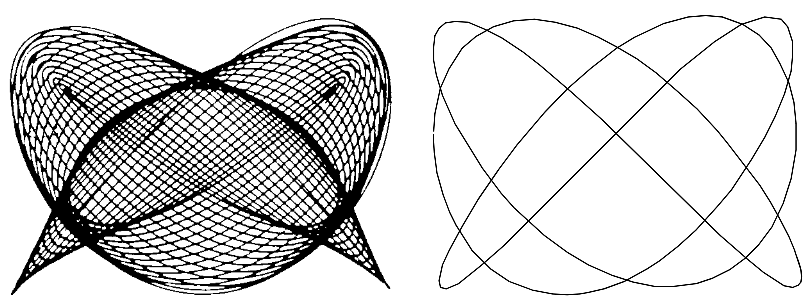


Figure 2 *Examples of Lissajous curves drawn by the pendulum.*

Mats of rope

The Danish cordage artist and writer *Kaj Lund*[1] has for a number of years collected and recreated old rope mats and knots. Some of them can be traced back to the time of the Vikings (800–1100AD) [7]. One of the most simple models is the *Rectangular mat (or Expansion mat)* shown in figure 3. Comparing figure 2 and 3 one may see that the Rectangular mat and the Lissajous curves have almost the exact same shape.

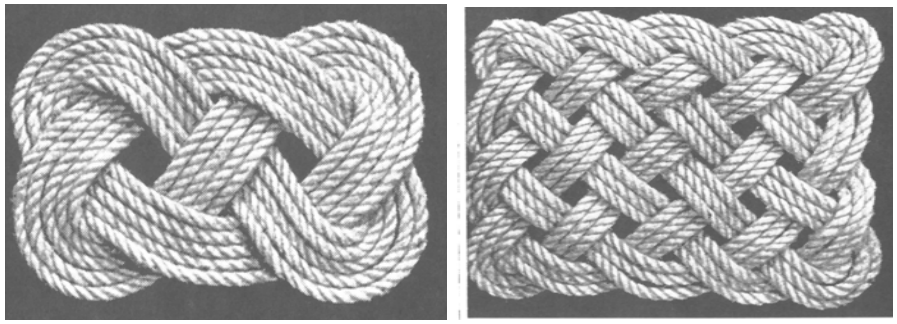


Figure 3 *Rectangular mats.*

The mathematics of Lissajous curves

Lissajous curves can easily be described mathematically by two sine-shaped functions which are oriented perpendicular to each other. This is shown in figure 4.

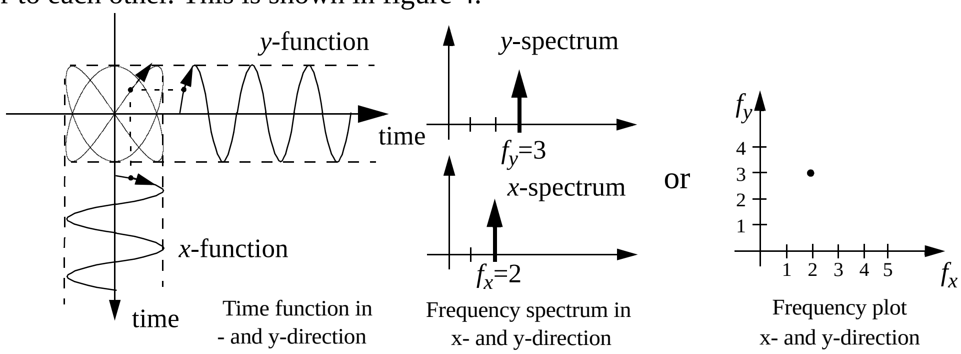


Figure 4 *Lissajous curves can easily be described mathematically by two sinusoidal functions.*

Mathematically the Lissajous figure may be expressed by two equations:

$$x = A_x \cos(2\pi f_x t) \tag{1}$$

$$y = A_y \sin(2\pi f_y t) \tag{2}$$

Different Lissajous curves can be obtained by changing the ratio between the frequencies, f_x and f_y (figure 4) of the two sine-shaped functions. The shape of the Lissajous curves may be changed by the amplitudes A_x and A_y.

It is also possible to represent Lissajous curves with two spectra, one for the *x*- and one for the *y*-function.

Rectangular mats

We have observed that the Rectangular mat and the Lissajous curves are almost exactly the same. For this reason we may represent the Rectangular mat with the same mathematical expression as we did for the Lis-

1. *Kaj Lund* published several books on rope work from1969 to 1979 (Borgen – Danish publisher).

sajous curves. Figure 5 shows some examples of simulated Rectangular mats. The shape of the mat is, as for the Lissajous curves, mainly determined by the ratio between the two frequencies, f_x and f_y.

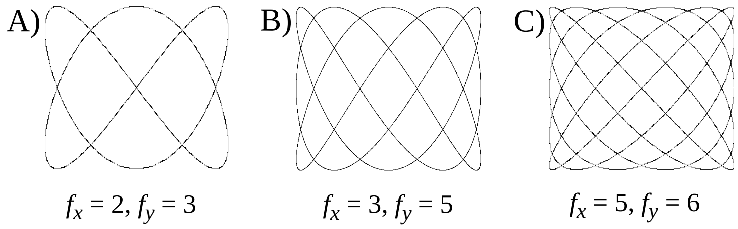


Figure 5 *Examples of simulated Rectangular mats.*

However, the Rectangular mat is one of the most simple mats. A natural question is: Is it possible to find equations describing more complex mats and rosettes. Let us take a look at the Turk's Head [9].

Turk's Head rosette

The sign of *The International Guild of Knot Tyers* is a Four-Bight Turk's Head rosette (figure 6). A "bight" is a knot tier's term for a loop.

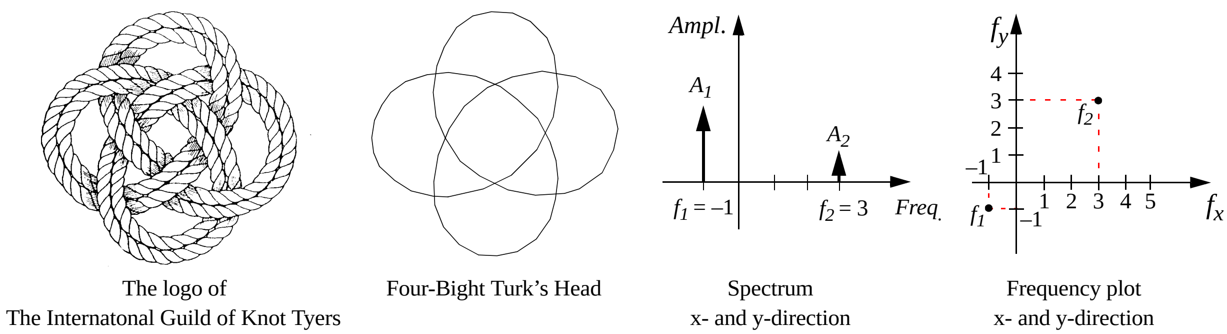


Figure 6 *Four Bight Turk's Head.*

Curves like the Four-Bight Turk's Head are known from spirograph curves, ornamental turning [10] and Guilloché patterns [2]. By studying the spirograph it is quite obvious that simple spirograph curves appear from two rotating vectors, which can be expressed mathematically by equation (3) and (4):

$$x = A_{x1} \cos(2\pi f_{x1} t) + A_{x2} \cos(2\pi f_{x2} t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t) \quad (3)$$

$$y = A_{y1} \sin(2\pi f_{y1} t) + A_{y2} \sin(2\pi f_{y2} t) = A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) \quad (4)$$

As we see, the expression contains two frequency components in x- and y-direction, f_{x1}, f_{x2} and f_{1y}, f_{2y}. Due to symmetry the components are equal in both x- and y-direction ($f_{x1} = f_{y1} = f_1$ and $f_{x2} = f_{y2} = f_2$ and $A_{x1} = A_{y1} = A_1$ and $A_{x2} = A_{y2} = A_2$). By manipulating these components, it is possible to make more complex Turk's Head's rosettes.

The distance between the two frequency components reflects the number of bights (figure 7). By drawing a line from the centre to the periphery, the value of the positive frequency (f_2) defines the number of strands crossing the line. By changing the distance between and location of the frequency components, we can get several variants of the Turk's Head rosette as shown in figure 7.

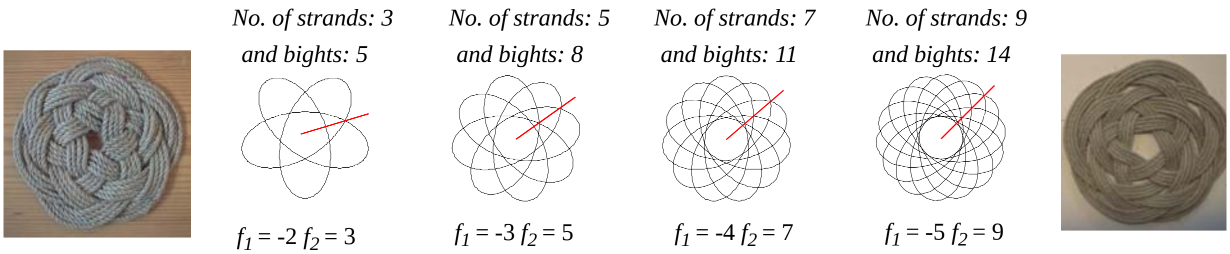


Figure 7 *Examples of simulated Turk's Head rosette.*

Simular patterns can be achieved by a *twin pendulum* described by J. Goold et.al. [11].

Let us now go one step further and look at even more complex mats.

The Twisted rosette

Figure 8 shows examples of the Twisted rosette.The twists (pretzels) on the left mat overlap in the centre. The midle and the right ones do not overlap. To find the mathematical expression of these rosettes we may use Fourier decomposition.

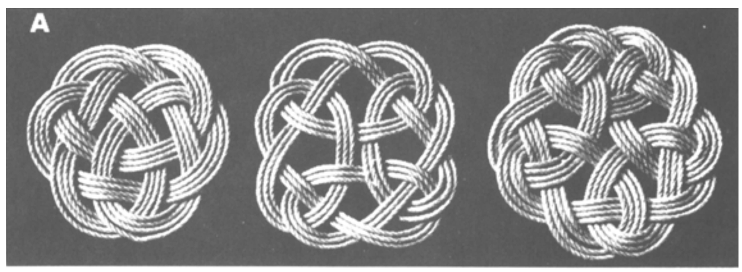
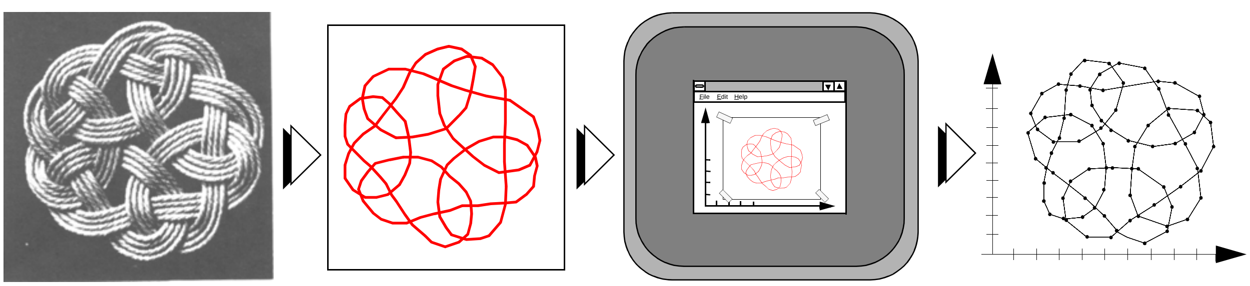


Figure 8 *Examples of Twisted rosettes [8].*

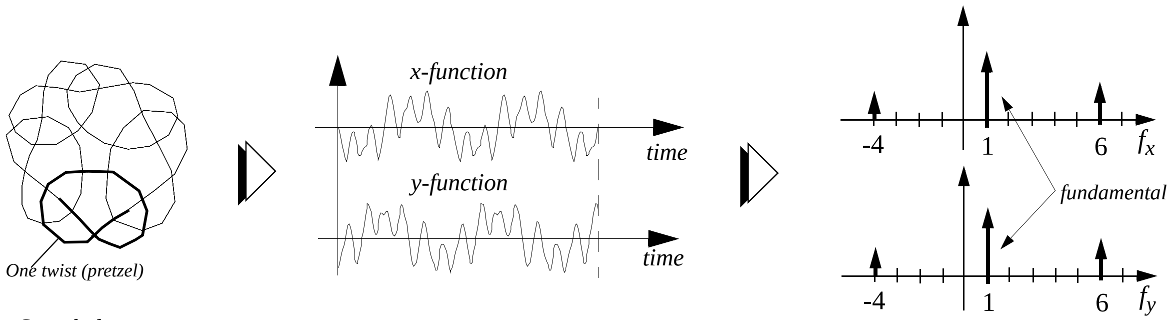
To do the analysis we transfer the rosette curve to transparent plastic which is taped onto the computer screen. The running Matlab program shows a coordinate diagram behind the plastic with the rosette drawing. The sampling is done by clicking the mouse cursor along the rosette curve, from the beginning to the end (figure 9).



Original rosette — *Rosette copied to plastic* — *Plastic placed on the computer screen and sampled by the mouse* — *Sampled rosette*

Figure 9 *Sampling of the rosette.*

Next the curvature of the rosette is transferred to a set of coordinate points, from which the computer calculates the x- and y-function. The spectra of the two functions are found by Fourier decomposition. The rosette shown in figure 10 is made by five overlapping twists. From the x- and y-spectra we can see that the twisted rosette may be described by *three* frequency components. The distance between the frequency components is five and equal to the number of twists in the rosette.



Sampled rosette *X- and y-functions of the rosette* *The spectra of the x- and y-functions*

Figure 10 *The spectrum of the twisted rosette.*

Normally spectra consist of positive frequency components only. Studying the original phase output from the Fourier analysis, we can conclude that the components in the *x*- and *y*-spectra represent vectors rotating in both directions. This is illustrated graphically by both positive and negative spectral components, and show very clearly the constant gap between the individual components. This is evident in the next examples.

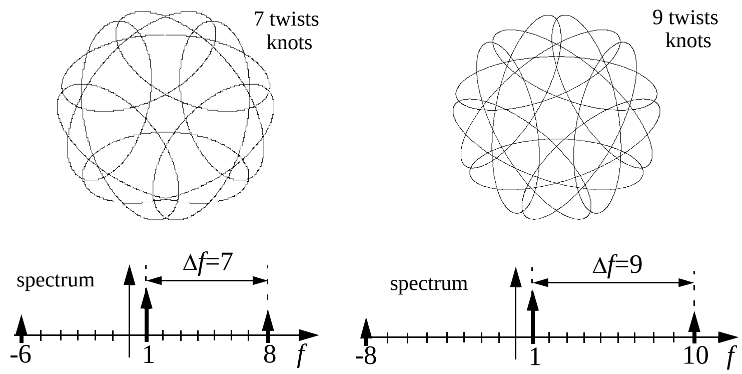


Figure 11 The number of twists is equal to the distance between the frequency components in the spectrum.

It is now easy to experiment with the parameters. By changing the amplitudes, the distance between the spectral components or by moving the x- and y-components along the frequency axis, we may design new versions of the rosette. Figure 11 and 12 show four examples of the twisted rosette.

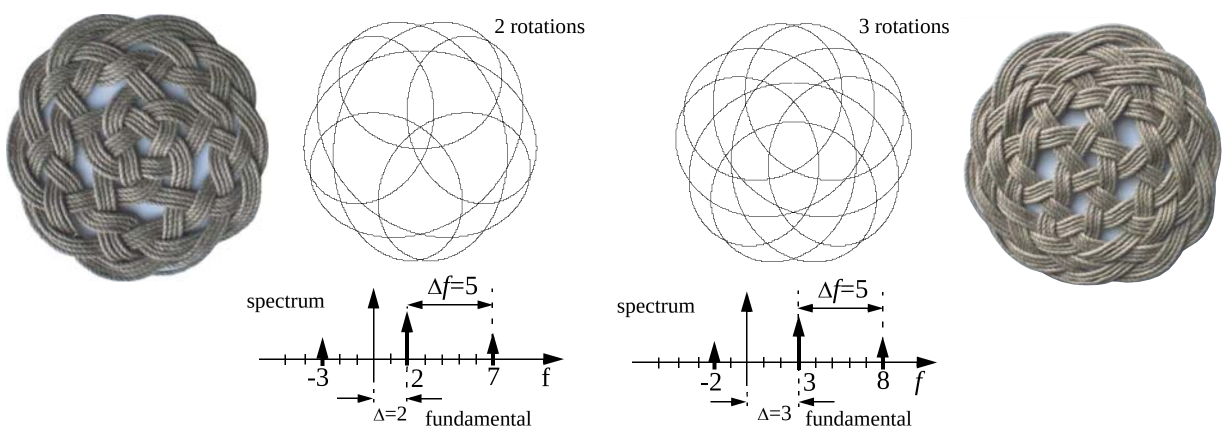
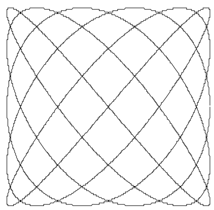


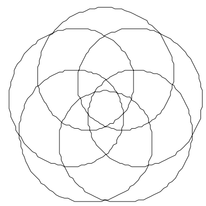
Figure 12 *The value of the fundamental frequency is equal to the number of laps the curve is running around the centre of the rosette.*

Rosettes of different order

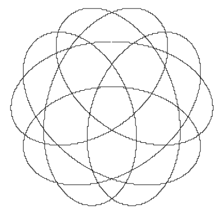
Since the Rectangular mat can be described by one frequency component in each direction *x* and *y*, this class of mats can be called *mats of first "order" (authors notion)*. In the same manner we can say that a Turk's Head rosette is a mat of *second order* and a Twist rosette a mat of *third order*. Using the Fourier technique it is quite easy to analyse mats of higher orders. The figure below presents some examples.



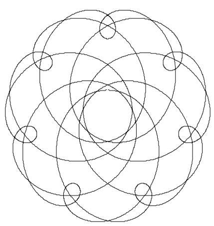
1st order Rectangular mat



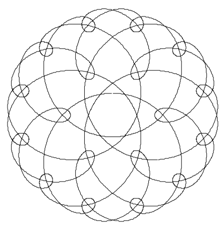
2nd order Turk's Head rosette



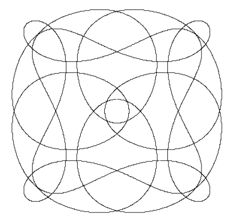
3rd order Twist rosette



5th order Simple eye rosette



8th order Jens Kusk Jensens rosette



9th order alternating eye rosette

Figure 13 *Traditional and new rosettes of orders 1, 2, 3, 5, 8, and 9 generated mathematically.*

However, to characterize rosettes by its *order*, is not an exact branch of knowledge. The order is a kind of minimum number of Fourier components that can describe the rosette or mat up to a certain level of visual quality. Some rosettes may apparently be described in more detail by adding components, and some can be described by fewer components but with reduced quality. So order may, up to certain extent, characterise a mat or rosette.

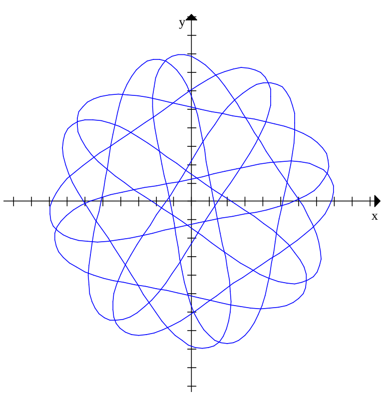


Figure 14 *Fourth order Rotting rosette.*

Based on the knowledge of the Fourier components of some traditional mats and rosettes, we can synthesize their patterns with a two-dimensional curve drawing software like Matlab or Winplot. By changing the Fourier components in frequency and amplitude, it is possible to make new variants of the mats and rosettes within each individual family (or order).

The tradition of making fancy rope works originates mainly from sailors. In their spare time they had little to do, and a lot of suitable material, and in addition few of them could read. Making mats and rosettes was therefore mainly for pleasure and decoration. In some cases mats were used to protect the deck, and to reduce slipperiness. P.P.O. Harrison writes:

> *The art of Fancy Ropework reached the peak of its excellence about the middle of last century (19th century) when a slump in trade brought about a reduction in the number of crew in a ship with the consequent reduction of the spare time that had permitted sailors to occupy themselves with their handicraft. An observer at that time considered that in the way of knots, especially for use at sea, there could be nothing more to invent. He was wrong [12].*

Harrison concludes that much more has been invented since then. Probably more than existed at that time in 19th century, but probably mostly by hobby sailors and rope work enthusiasts. Similar patterns can be found in Celtic, Tamilian (Indian) [1] and *Sona* (Angolan) tradition [13].

The main purpose of this work has been to establish a method suitable to explore and understand the inner properties of traditional mats and rosettes, and to develop a tool for expanding and redesigning traditional models into new versions of rope works.

I have approached the subject more as an engineer rather than as a mathematician. Consequently I would like to focus on developing ICT-tools rather then developing the mathematics. For me a further improvement would be to make an integrated ICT-tool for analysing and synthesising mat and rosette patterns. The analysing part of the software should be able to scan and sample the patterns, subsequently decompose them into their Fourier components. With the synthesising part, based on the results from the analyser, it should be possible to develop new patterns and prepare them for handicraft production.

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