Broadening the Palette for Bobbin Lace: A Combinatorial Approach

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Abstract

In Bobbin Lace, a pattern is divided into regions by shape, each region filled with a texture. Since Bobbin Lace is generally made with a single colour of thread (black, white or ecru), the textures take on the role of colour to provide interest and shading. Using a combinatorial approach, I will look at one aspect of Bobbin Lace and examine how many unique textures are possible for a specified number of grid points. I will compare the results to textures currently known and used in Bobbin Lace. In the process, I hope to rediscover some of the more complicated textures that have been lost over time as well as identify some new textures which may be of use to modern Bobbin Lace artists. Bobbin Lace provides a very high level of control over the position of threads in a material. It is hoped that by increasing the palette of possible textures, the results may also prove useful to electronic textile manufacturing or fabric designed for a specific purpose such as medical prostheses.

1 Introduction

Bobbin Lace has a long history in Europe and England. It is believed to have developed from passementerie, the art of ornamental braiding, and evolved into its current form during the last half of the 15th and first half of the 16th centuries. The lace gets its name from small wooden sticks, called bobbins, used to control the threads during production. Hundreds of bobbins may be required to make a single piece of lace. Throughout its history, Bobbin Lace has been used to create elaborate and complex designs achieving its height of diversity in the 18th century when lace was a key element in fashion. In the 19th century, due to competition from machine made copies, there was an emphasis on simplifying designs to increase the speed and volume of production. During this period, some of the knowledge and skill associated with earlier designs was lost. After World War I, Bobbin Lace production as an industry ceased completely and Bobbin Lace was relegated to the status of hobby craft. Interest in reviving the craft started in the 1970's. Initially, the focus was on basic techniques used in the 19th century but towards the end of the 20th century, interest turned to reviving some of the more advanced techniques employed in 17th and 18th century laces as well as inventing new ones. This brief summary of lace history is based on the writings of Levey [3].

Despite its interesting topology and logical construction, very little has been written about Bobbin Lace from an algorithmic or mathematical point of view. The relationship between Bobbin Lace and Braid Theory receives a brief mention in a discussion of the topology of textiles by Grishanov [6] and a passing reference by Kauffman [2] in his work on the applications of Knot Theory. In this paper, I shall frame some of the basic concepts of Bobbin Lace in a mathematical manner in the hope that this will spark others to take a closer look at the wealth of interesting mathematical properties found here.

The primary objective of this paper is to capture the basic constraints of Bobbin Lace production. For a subset of these constraints, I will use a combinatorial approach to enumerate and generate all possible solutions for a specified number of grid points. The result will be compared to the known set of stitches, as outlined by Cook and Stott [1] in their cornerstone collection of Bobbin Lace stitches. It is also expected that a wealth of new stitches will be identified. [1]

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2 Background

The term Bobbin Lace encompasses a diverse family of lace styles that share a common base technique. I will focus on the subgroup known as Straight Lace which contains well known styles such as Chantilly, Bucks Point and Torchon. The distinguishing characteristic of Straight Lace is that it is made from start to finish with the same set of threads - that is, threads are rarely added or removed and a single thread can be traced from one end of the lace to the other. This is in contrast to Part Lace in which small pieces of lace are worked as individual units that are later joined together. An interesting challenge of Straight Lace is determining how to move threads from one design element to the next so that the correct number of threads are available as required.

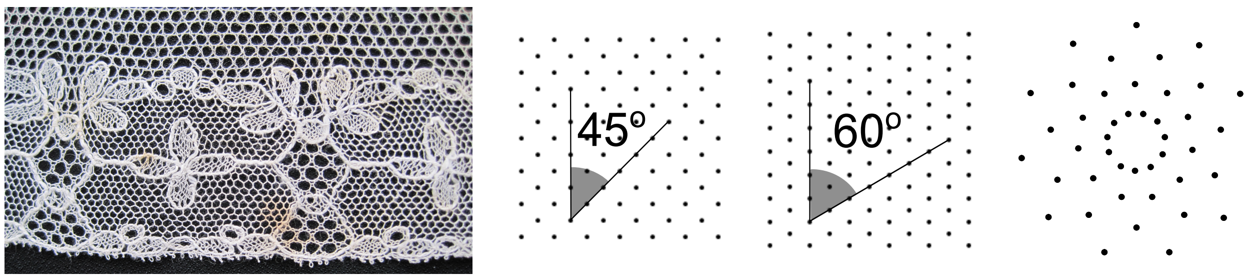

Figure 1 : *Left: Regions of varying texture in a Bucks Point edging. Right: Various grids.*

At first glance, a piece of Bobbin Lace may appear to be an impossibly complicated interweaving of threads. On closer inspection, however, we see that the lace consists of a number of small regions, each filled with a single texture. In some laces, the boundary between regions is emphasised by a thicker thread known as "gimp". Figure 1 shows an example of Bucks Point lace with a number of distinct regions [2]. Notice the different textures used in each region. Each texture corresponds to what in lace terminology is called a "stitch". Stitches range from a very dense texture called "cloth" (named after plain weave cloth which has the same topology) to very open textures like netting. Some stitches are simple and unobtrusive, like a basic net which is mainly used as a background, while others, like the Catherine Wheel, are very ornate and draw attention to an area. Since Bobbin Lace is traditionally made with thread of a single colour (black, white or ecru), the stitches take on the role of colour to provide contrast, interest and shading. A texture or stitch can be used to fill a region of any size or shape and must therefore have the property that it can tile the plane.

Bobbin Lace is worked on a grid. The angle of the grid ranges from 45° to 60° depending on the style of lace. In some cases polar grids or grids of variable spacing may be used. The grid is oriented on the diagonal as shown on the right side of Figure 1. In this paper, I am primarily concerned with the topology of the lace and not the specific geometry. For this reason, I shall use a 45° grid in the illustrations.

The procedure for making lace can be broken into three main components: Actions, Pair Traversal and Pinning. In this paper I will focus on Pair Traversal but I will describe all three components for context.

Actions: Like many other textiles, Bobbin Lace is formed from two basic actions. The actions are always performed on a set of four threads, which, for reasons that will become clear very soon, are thought of as two pairs: the left pair and the right pair. The first action, known as the "cross", is performed by taking the rightmost thread from the left pair and crossing it over the leftmost thread of the right pair (see Figure 2). In Braid Theory, this corresponds to the braid word σ_{2i} where i is any positive integer. The second action, known as the "twist", is performed by crossing the rightmost thread of the left pair over the leftmost thread of the left pair and similarly crossing the rightmost thread of the right pair over the leftmost thread of the

[2] Reproduced with permission from http://www.laceforstudy.org.uk

right pair. In Braid Theory, this corresponds to the braid word $\sigma_{2i-1}^{-1}\sigma_{2i+1}^{-1}$. In Braid Theory, when even and odd generators have opposite signs, the result is an alternating braid - that is, a braid where strand crossings alternate between over and under as in plain weave cloth [4]. Looking at the braid words for the cross and twist actions, we see that they possess this property and will therefore always produce alternating braids no matter how they are combined. Bobbin Lace is generally very open and airy and therefore relies on this alternating architecture for structural integrity.

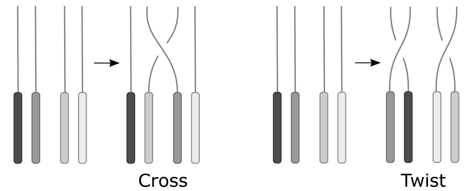

Figure 2 : *Cross and Twist: the two base actions used in Bobbin Lace.*

Pair Traversal: Once a number of actions have been performed with the current set of four threads, the lacemaker moves on to the next set. This can be done in a number of ways which is the topic I will explore in Section 3. By way of example, the lacemaker could set aside the left pair of threads from the current grouping, and start to work with the former right pair and a new pair brought in from above and to the right. This progression is shown in Figure 3. The way in which the threads travel across the design is a major contributor to the complexity of a texture.

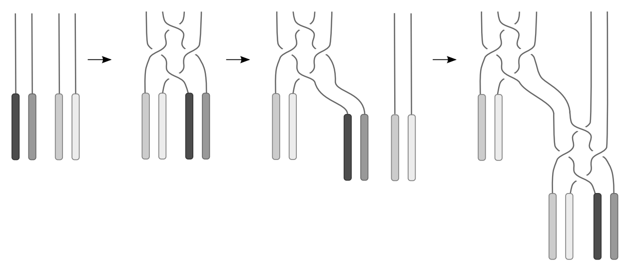

Figure 3 : *Progression of pairs of threads from one set to another.*

Pinning: Pins, placed mainly at the grid points, are used to maintain the shape of the lace during production. The lacemaker performs a series of actions with two pairs of threads then places a pin between the pairs. She may then perform some more actions before moving on to the next set of pairs (called 'closing' the pin because the pin is enclosed by crossed threads) or move directly on to the next set of pairs (resulting in an 'open' pin with crossed threads above the pin but not below it). To create a smooth result, the lacemaker regularly applies tension to the threads which the pins resist. The presence of pins has no effect on the topology of the lace but does have a significant effect on the geometry and therefore the final appearance.

3 A Detailed Look at Pair Traversal

The flow of pairs of threads through a texture can be represented as a Directed Graph. In Figure 4, the image on the left shows a braid diagram of a typical texture in which each strand represents a thread. An oval is drawn around a sequence of actions performed on a single set of four threads (in this example the action sequence is cross, twist, cross). In the middle drawing, the distance between the action sequences is exaggerated to emphasise the movement of pairs of threads from one action sequence to the next. On the right, each action sequence has been reduced to a dot - a vertex in the Directed Graph. A pair of threads moving between two action sequences is represented by a single line with an arrow indicating the direction of movement - an arc in the Directed Graph.

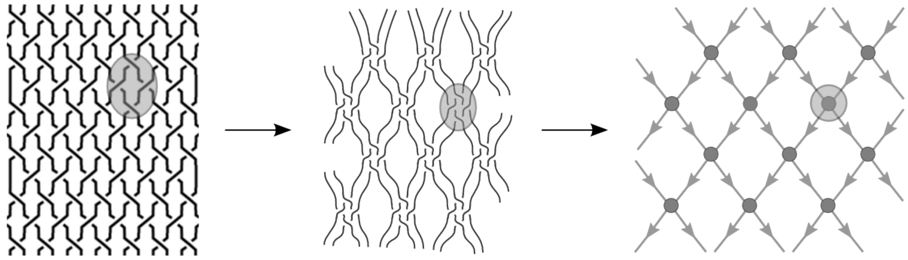

Figure 4 : *Pair traversal in a texture represented as a directed graph.*

In Bobbin Lace, physical and practical limitations on the way that pairs traverse the grid can be translated into constraints on the Directed Graph.

1. **Weakly Monotonic:** As shown in Figure 6A, Bobbin Lace is produced on a pillow with wooden bobbins. Straight lace starts at the point on the pillow farthest away from the lacemaker and is worked toward the lacemaker's body which in this paper I will represent as top to bottom. Pairs may move downward or horizontally but never upward. This results in a weakly monotonic directed graph. For long pieces of lace, the pillow may have a built in roller which is turned as the lace is produced or, on simple pillows, the lacemaker must pick up the lace once it gets close to the edge of the pillow and move it to the far end of the pillow in order to continue.
2. **Tile the Plane:** A texture can be used to fill any shape and must therefore extend in all directions. We can consider the traversal pattern as a planar directed graph with an infinite number of vertices, a small representative portion of which is shown on the left side of Figure 5. Within this infinite graph, there is a rectangular subset of vertices, a prototile, and the traversal pattern of this rectangle is repeated in all directions using p1 symmetry. Since an infinite number of vertices is difficult to model and the entire pattern can be represented by a finite subset, it is useful to consider a single prototile in which outgoing arcs from vertices along one side of the prototile wrap around to become incoming arcs on the opposite side of the prototile. The resulting directed graph is non-planar as shown on the right side of Figure 5.

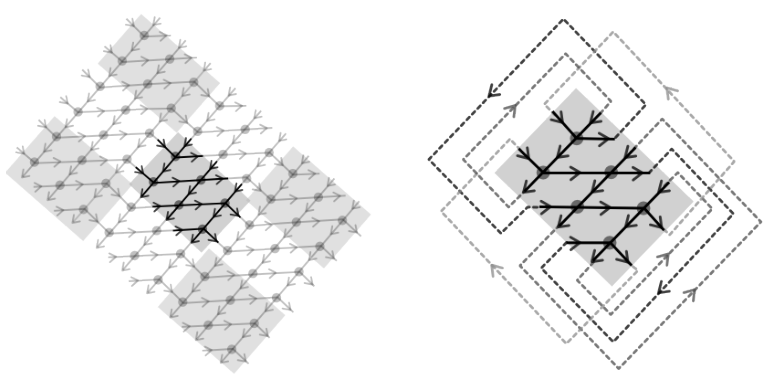

Figure 5 : *Tiling the plane. Left: a representative subset of an infinite graph. Right: A wrapped prototile.*

3. **2 In, 2 Out:** At each node, two pairs of threads arrive and two pairs of threads leave. The associated Directed Graph therefore has in-degree 2 and out-degree 2, also known as a 2-regular digraph [5]. The only exception to this is an empty node in which no pairs arrive and no pairs leave – in the directed graph, this corresponds to an isolated vertex.

4. **Conservation of Pairs:** In Straight Lace, threads travel the entire length of the lace. Unless the width of the lace changes significantly, no pairs are added or removed during the working of the pattern. In order to achieve this, the number of pairs used in a single tile must be conserved. Figure 6C shows a traversal pattern in which pairs are conserved by weaving back and forth. Figure 6D shows a very similar pattern but in this case pairs are not conserved: at each row a pair is added on the left and removed on the right. A balance is required between the number of left and right oriented arcs.

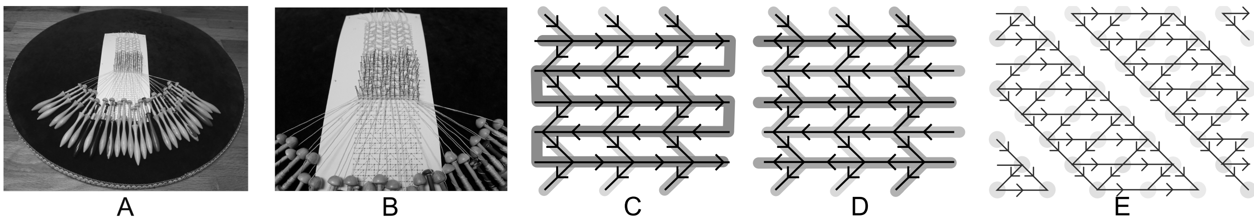

Figure 6 : *A) Bobbin Lace being worked on a pillow. B) Close up of pins and threads. C) Threads conserved, 4 pairs used. D) Threads not conserved, 8 pairs used. E) Pattern not connected.*

5. **Connected:** Like a piece of cloth, lace must hold together on its own; it must not be a collection of disconnected pieces. Figure 6E shows an invalid, disconnected traversal pattern. A traversal pattern is connected if a path exists between the top left vertex and every non-isolated vertex in the wrapped, directed graph of the prototile. In addition, there must be at least one outgoing arc from a vertex along either the left or right border of the prototile that wraps around to the opposite border.
6. **Simple:** In the traversal pattern there are no arcs that start and end at the same vertex and no two arcs share the same endpoints (independent of direction). The related directed graph is therefore described as simple. Note: Bobbin Lace is commonly decorated with "picots" which are small loops formed by two threads. As these loops play no role in the traversal pattern, the directed graph is still simple.

3.1 Variations

While there are always exceptions, the constraints described above apply to most Straight Bobbin Lace stitches. By adding some additional restrictions we can identify common stitch subgroups. I will describe two main subgroups but there are several others.

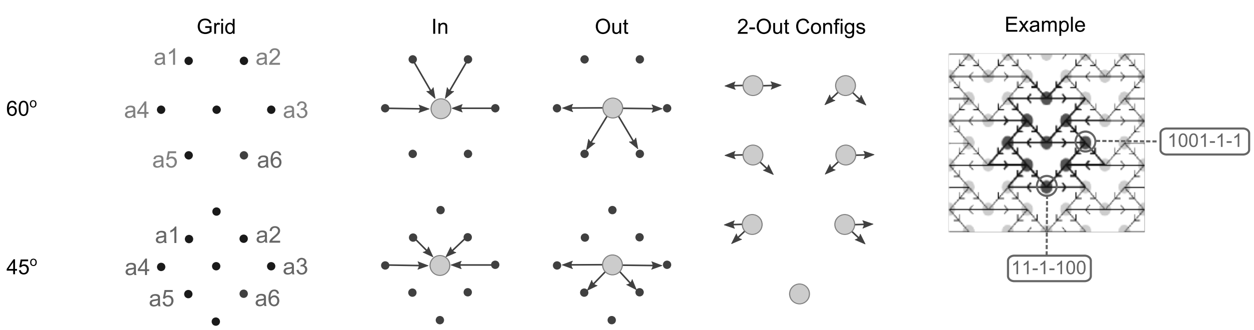

Figure 7 : *Basic traversal patterns. Each node has a label of the form $a_1a_2a_3a_4a_5a_6$ where $a_i = 1$ is an incoming arc, $a_i = -1$ is an outgoing arc and no arc is $a_i = 0$. The label 11-1-100 indicates that the node is connected by incoming arcs to a_1 and a_2 and by outgoing arcs to a_3 and a_4.*

The first group I will call **Basic** because it is the most simple. In the Basic group, pairs can move to nodes immediately left or right and diagonally below as shown in Figure 7. In a 60° grid, this corresponds to arcs between the six nearest neighbours of the node. Note that in a 45° grid the node immediately below is not included even though it is at the same distance as the nodes immediately left and right.

The second subgroup I will call **Interleaved** because it uses two interleaved grids as shown on the left side of Figure 8. Pairs can travel to nodes immediately left or right, immediately below and diagonally below resulting in arcs between eight adjacent nodes. In this configuration, we also add the constraint that pairs must only cross at nodes. As a result, two horizontally adjacent nodes may not have incoming diagonal arcs that cross.

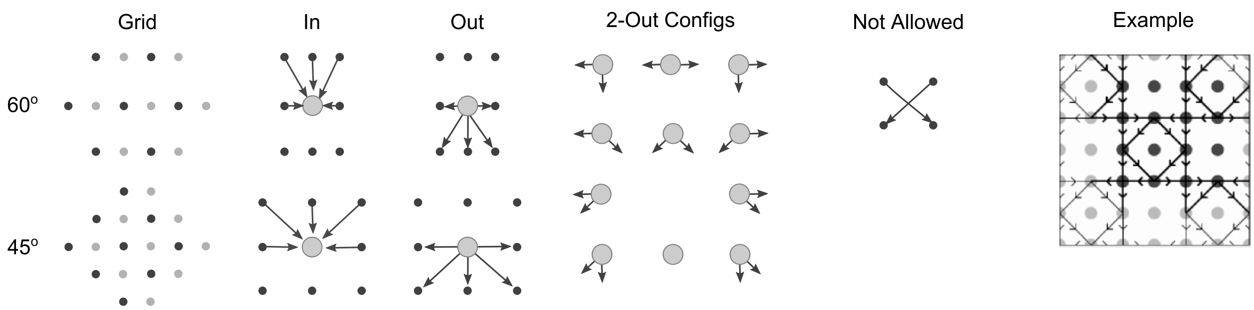

Figure 8 : *Two interleaved diagonal grids and allowed traversals.*

4 Algorithm for Generation and Enumeration

Like the enumeration of 2-regular directed graphs conducted by Ramanath and Walsh [5], I will use a backtracking approach to enumerate and generate the pair traversals for Basic and Interleaved stitches. My algorithm identifies isomorphically unique traversal patterns by iterating over outgoing arc configurations for each vertex. Incoming arc configurations automatically follow from the assignment of outgoing arcs. There are 7 possible outgoing choices at each vertex for Basic stitches and 11 for Interleaved stitches.

Several early termination conditions are used to reduce the overall order of the algorithm:

- As each arc is added, test the vertex at the incoming end of the arc. Terminate branch if a vertex has more than two incoming arcs.
- When all incoming neighbours have been processed, test a vertex for 2-in 2-out or isolated vertex.
- When a row of vertices has been processed, check for the conservation of pairs.
- Each internal vertex must be lexicographically greater than the top left vertex. This condition will eliminate textures that are equivalent by translation. The lexicographical order is determined by assigning an index to adjacent nodes and constructing a label based on the nodes to which the vertex is connected as illustrated in Figure 7. Whenever a vertex has a full set of incoming and outgoing arcs, it is compared to the top left vertex. Since the top left vertex may not yet have its incoming arcs fully determined, the comparison is based on the lexicographically lowest possible configuration of the top left vertex given its currently defined arcs.
- For interleaved stitches, if an internode crossing is encountered, the branch is terminated.

Additional validation is performed when a branch of the backtracking algorithm successfully terminates. A check for isometric uniqueness under rotation requires that all nodes are valid under rotation which can only be determined on completion. While a partial test for isometric uniqueness under translation or reflection is done at each step, final validation requires the top left node to be fully determined which may only happen after the last node is processed.

5 Results

In their comprehensive reference book, Cook and Stott list 262 stitches [1]. Several stitches share a common traversal pattern but use different combinations of pinning and action sequences. For example, 34 stitches use a Basic 1x1 traversal pattern like the one shown on the right side of Figure 4 and 15 stitches use the 4x4 Interleaved pattern shown on the right side of Figure 8.

As expected, my algorithm demonstrates an exponential growth in the number of traversal patterns as the number of nodes increases. The results in Table 1 for “Basic” traversal patterns and Table 2 for “Interleaved” traversal patterns, show that with as few as 16 nodes we have exceeded the number of patterns reported by Cook and Stott. A few samples of new traversal patterns discovered using this algorithm have been worked using a simple “cross, twist, pin, cross, twist” action sequence and are shown in Figures 9 and 10. Following the work of Cook and Stott, samples have been worked in a coarse thread (Coats No. 30 Cotton) to show the detail.

$m\backslash n$	1	2	3	4	5
1	1	1	1	1	1
2	1	3	1	3	1
3	1	1	24	1	1
4	1	3	1	199	1
5	1	1	1	1	>13000

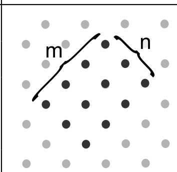

Table 1 : *Number of unique pair traversals for “Basic” stitches*

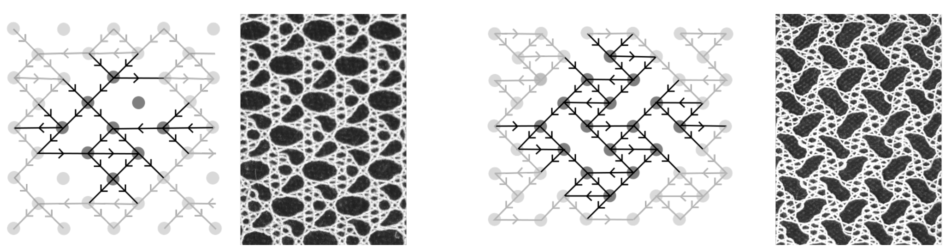

Figure 9 : *Two new “Basic” textures discovered using my algorithm.*

$m\backslash n$	1	2	3	4	5
1	1	5	11	32	92
2	2	15	44	140	507
3	3	51	265	1518	9756
4	7	243	2316	27938	
5	17	1243	23945		

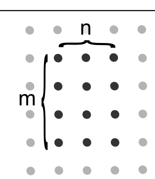

Table 2 : *Number of unique pair traversals for “Interleaved” stitches.*

In some stitches, it can be a challenge to identify the pair traversal pattern. For example, when many actions are performed at a single vertex, as is the case for a tally, the resulting node may be elongated or fat and the internodal distances very short or non-existent. Some stitches do not insert a pin at every pair crossing which allows the nodes to migrate from strict grid positions and, under tension, pull toward each other in a central position. However, by abstracting a sequence of actions performed on four threads to a vertex, it is possible to tease out the pair traversal pattern.

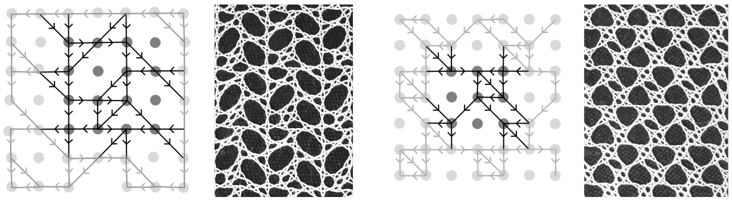

Figure 10 : *Two new "Interleaved" textures discovered using my algorithm.*

Cook and Stott document several stitches that do not follow the pair traversal patterns or actions described here. They report 5 stitches that use an action called "sewing" which involves a thread looping back on itself in violation of the monotonic constraint. They present 11 stitches in which a pair of threads is treated as a single thread in a manoeuver known as a lazy join. This results in a pair traversal that is no longer 2-regular. Several stitches in their reference book follow the common constraints outlined in Section 3 but do not belong to the subgroups described in Section 3.1. These include stitches using spider motifs, diamonds and stars. Like the Basic and Interleaved subgroups, the traversal patterns for these stitches can be described by an additional set of constraints.

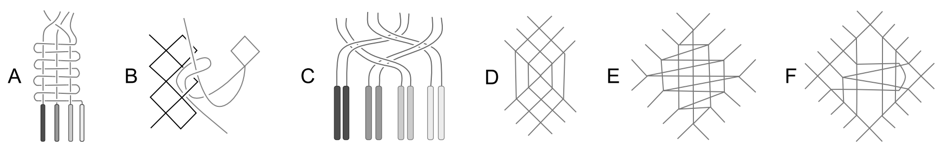

Figure 11 : *A) Tally B) Sewing C) Lazy join D) Spider E) Diamond F) Star*

6 Conclusion

Using a combinatorial approach, I have isolated one aspect of Bobbin Lace production and demonstrated that there is a much broader range of possible textures than is currently being used. The advantage of exploring these new possibilities is many-fold. For the Bobbin Lace artist, it extends the possible "colours" available, potentially enriching the designs created. The impact, however, can extend beyond the art of lace making. Each Bobbin Lace stitch has unique physical properties – some are flexible while others are rigid; some have many holes while others are solid; holes can be large or small. The lace maker has fine control of individual threads and can easily change from one texture to another. For example, a rigid texture can be applied around the border of a circle with a flexible texture at the centre – all without seams or knots. This might have useful application in electronic textiles and the development of medical prostheses.

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