# A 7-Fold System for Creating Islamic Geometric Patterns Part 1: Historical Antecedents

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## Abstract

A comprehensive set of 7-fold polygonal sub-grid elements produces very beautiful Islamic geometric patterns characterized by distinctive 14 and 7-pointed stars. The tessellating characteristic of these polygonal elements represents a systematic approach to creating geometric designs with 7-fold symmetry. While a number of historical examples of 7-fold Islamic geometric patterns exist, there is insufficient evidence from the historical record to establish the degree to which designers of the past were aware of this as a *system* per se. The employment of a limited set of repetitive polygonal elements onto which pattern lines are applied aligns this *7/14 System* with other systematic approaches to creating Islamic geometric patterns. In particular, this 7/14 System has direct parallels with the historically ubiquitous *5/10 System*. As with other polygonal systems, the 7/14 System offers tremendous design potential for the creation of new geometric patterns. As demonstrated in Part 2 of this paper, the contemporary expression of this design system includes the creation of quasi-crystalline and self-similar designs with recursive symmetry.

The preeminent historical method of creating Islamic geometric patterns employs an underlying polygonal tessellation (sub-grid) upon which pattern lines are strategically located. Once the pattern has been generated, the sub-grid is discarded, leaving no trace of the initial governing structure from which the design was derived. This is referred to as the *polygonal technique* [1], and the inherent flexibility of this design methodology gave rise to the remarkable diversity found within this ornamental tradition. In particular, the polygonal technique lends itself to a category of patterns that employ a systematic approach wherein the underlying sub-grid elements are made up of a limited set of repetitive polygons that work as

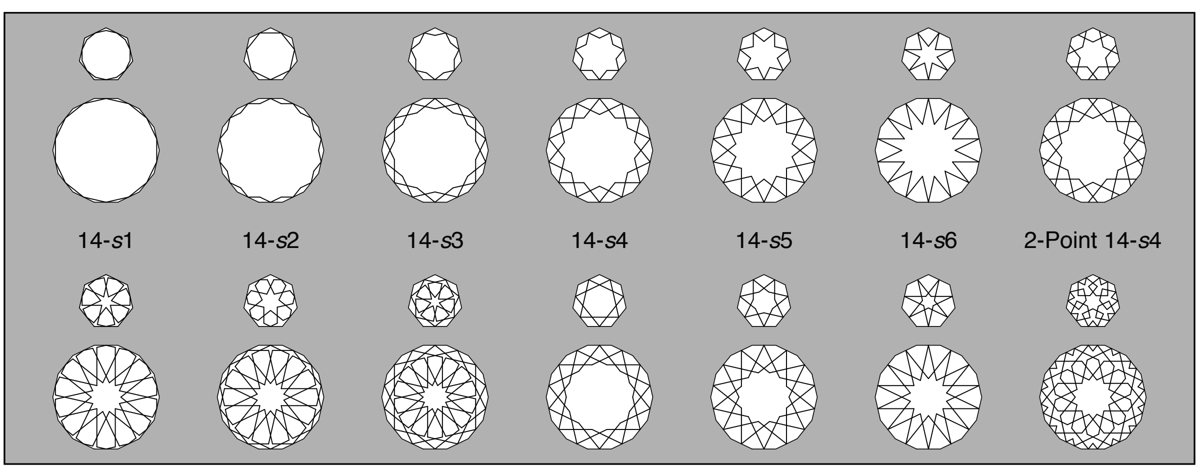


**Figure 1**: *Determining the angles of the crossing pattern lines from the 14-sided tetradecagon, with typical additive infill variations below.*

a group to tessellate in innumerable ways [2]. The earliest of these systematic approaches, dating from the tenth century, is the *6/12 System*, and is made up of regular polygons. The early eleventh century saw the development of the *4/8 System A* [1], and a century later, the *4/8 System B*: each characterized by 8-pointed stars, but using very different sub-grid elements. The invention of the *5/10 System* [1] also took place during the early twelfth century. Each of these systems produces a wide diversity of geometric patterns, and each was of ongoing significance in the history of this ornamental tradition. A number of 7-fold patterns from the historical record are similarly created from a limited set of repetitive underlying polygonal sub-grid elements. However, the paucity of historical examples belies any certainty of the extent to which these were designed consciously as a systematic process, or, in the quest for making 7-fold designs, artists merely happened upon some of the same decorated sub-grid modules without realizing their repetitive potential. Ambiguity of past methodology aside, this *7/14 System* provides for the creation of the marvelous albeit few historical 7-fold geometric patterns that have survived to the present, as well as the potential for an unlimited number of original patterns for contemporary designers.

As the polygonal technique developed into a fully mature design tradition, the convention for locating the pattern lines generally called for the placement of two crossing lines at the midpoint of each underlying polygonal edge. The angle of these crossing pattern lines is a product of the inherent geometry of the specific system, and determines the character of the completed design [2]. Depending upon the contact angle, patterns are differentiated as being *acute*, *median*, or *obtuse*, and a fourth family places the pattern lines upon two points of each polygonal edge; and is hence referred to as *2-point* [1]. These are imprecise descriptive definitions that indicate the *character* of a pattern rather than the actual angle of the crossing pattern lines. Hence, *acute* patterns in the 4/8 Systems have 45° angles, in the 5/10 System they have 36° angles, in the 6/12 System they typically have 30° angles, and historical *acute* patterns made from the 7/14 System will typically have crossing pattern lines with 51.4286°, although the more acute angle of 25.7142° will also work. In each of these systems, the angle of the crossing pattern lines is determined by simply drawing lines that connect the midpoints of the edges of the primary *n*-gon. In the 7/14 System, this is the 14-sided tetradecagon. Obviously, the higher the number of sides, the more options there are for creating the crossing pattern lines. The tetradecagon allows for six natural pattern angles. In describing patterns from this system, the loose stylistic terms of *acute*, *median* and *obtuse* are made more precise by referencing the specific contact angle employed [**Figure 1**]. A convenient nomenclature identifies each of these contact angles and resulting star polygons such that an initial number (in this case 14) refers to the number of sides of the generating polygon, *s* to the connecting lines between the midpoints of the polygonal sides (*s* = sides), and the integer that follows *s* being the number of intervals between the polygonal edges that are counted before connecting the midpoints (This nomenclature is a variation of Grünbaum and Shephard's identification of star polygons [3]).

The earliest known 5-fold and 7-fold Islamic geometric designs are from the Ghaznavid minaret of Mas'ud III in Ghazna, Afghanistan (1099-1115) [4]. Remarkably, the two 7-fold examples from this building predate the next earliest extant 7-fold pattern by nearly a hundred years, and are considerably more complex. The repeat unit for one of these examples is an elongated hexagon, with four 2/7 and two 3/7 included angles, that is easily produced by arranging six regular heptagons in an edge-to-edge configuration [**Figure 2**]. These six heptagons provide the underlying sub-grid upon which the pattern is constructed. It

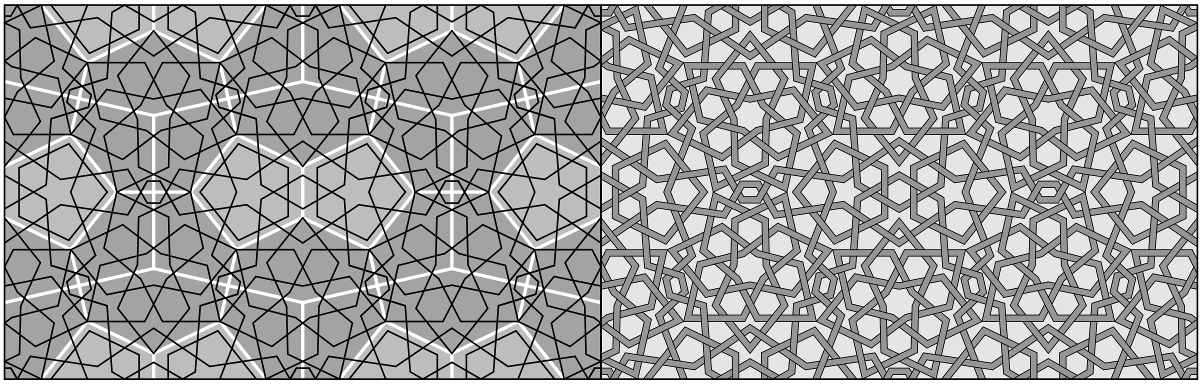

**Figure 2**: *One of two designs with 7-fold symmetry from the minaret of Mas'ud III in Ghazna, Afghanistan (1099-1115).*

is important to note that this design, along with the other patterns from this building, originated during the early formative period of this geometric design tradition, and the technique for applying the pattern into the underlying sub-grid had not yet developed into its more formal systematic methodology. Apart from the 7-pointed stars that locate their points upon the vertices of the heptagons, this pattern is the product of a series of arbitrary design decisions. While beautiful, this is in marked distinction to the more rigid systematic approach that soon followed. This more systematic methodology is seen in three 7-fold examples from the Seljuk Sultanate of Rum in Anatolia; each of which employs the same hexagonal arrangement of underlying heptagons that was used in the earlier Ghaznavid example. Each of these three Seljuk examples was designed in a different pattern family [**Figure 3**]: a 14-*s*5 *acute* design from the Great Mosque of Dunaysir near Kiziltepe, Turkey (1200-04); a 14-*s*4 *2-point* pattern at the Great Mosque of Malatya in Turkey (1237-38); and a 14-*s*2 *obtuse* pattern at the Egridir Han in Turkey (1229-36). While the 14-*s*4 *median* pattern from this sub-grid may not have been used historically, it nonetheless makes a very successful pattern.

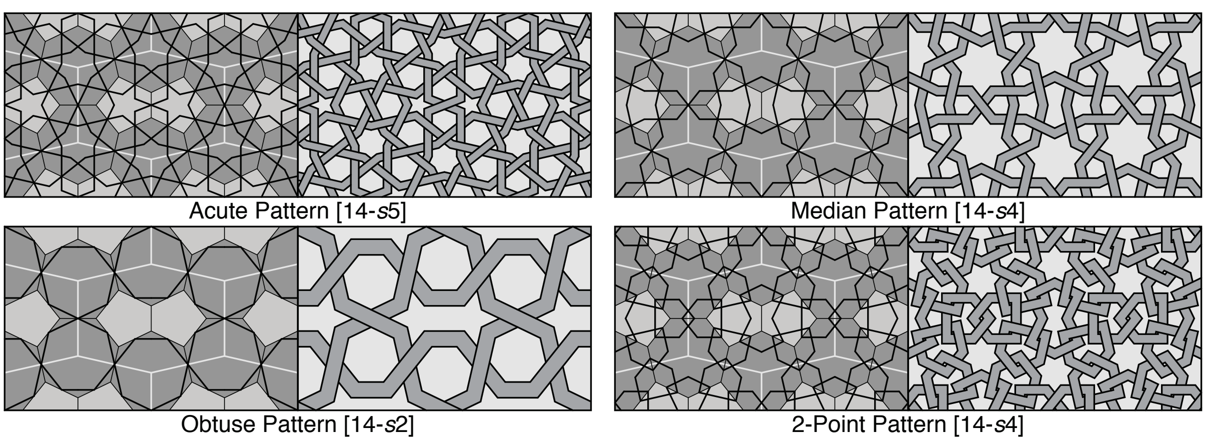


**Figure 3**: *The four pattern families as applied to the sub-grid of six edge-to-edge heptagons that form an elongated hexagonal repeat unit.*

The only sub-grid elements used in these early examples are the regular heptagon and mirrored irregular pentagons that are the interstice of this particular heptagonal arrangement. The many additional polygonal components of the 7/14 System are created by either (1) *interstice*, or "left-over" spaces produced when arranging the various sub-grid elements into

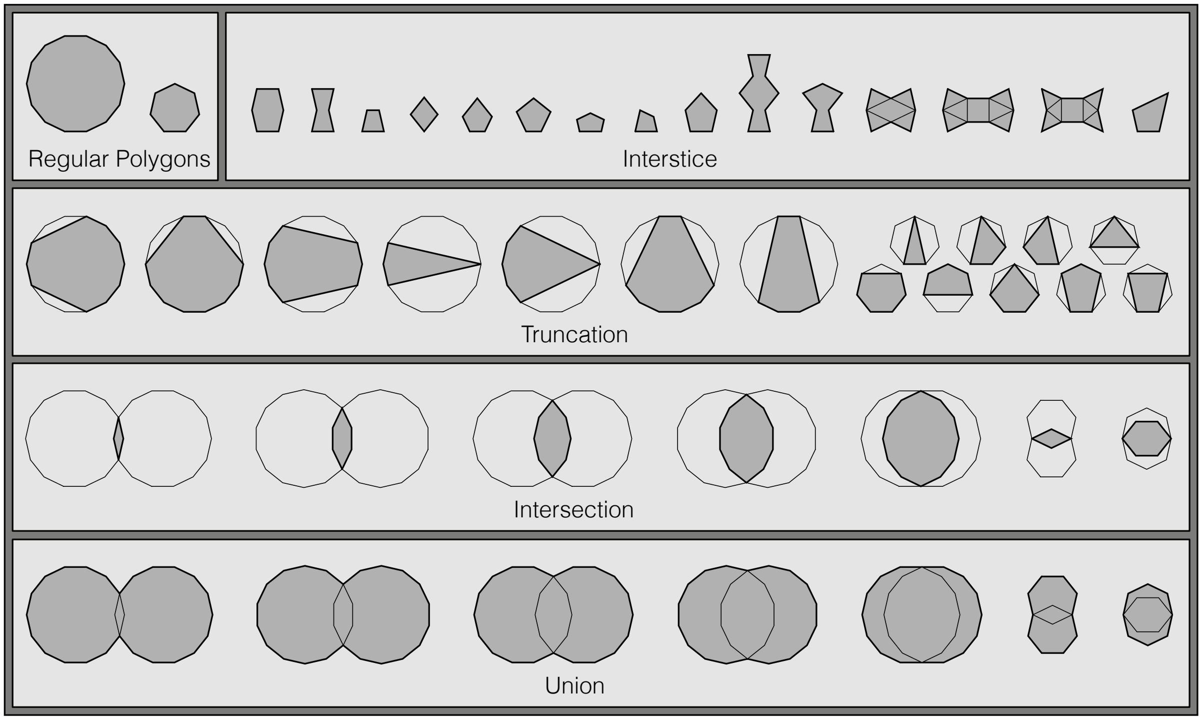


**Figure 4**: *The underlying polygonal sub-grid elements of the* 7-Fold System.
*(Note: this is not an exhaustive set of the interstice and truncation elements.)*

a tessellation; (2) the *truncation* of the heptagon and tetradecagon; (3) the *intersection* between superimposed heptagons and superimposed tetradecagons; and (4) the *union* of heptagons and tetradecagons (as per Kepler's "monsters") [**Figure 4**]. This is directly analogous to the origin of the sub-grid elements within the *5/10 System*, with the added factor that the greater number of 7 and 14 polygonal sides results in more truncated, conflated, and intersected elements within the 7/14 System. The greater number of exterior angles of the heptagon and tetradecagon also gives rise to significantly more interstice elements (the examples in Figure 4 are only representative, and by no means exhaustive).

One of the earliest patterns to utilize sub-grid elements from the more expansive 7/14 System is from the Seljuk section of the Friday Mosque of Isfahan ($11^{th}$ or $12^{th}$ century)

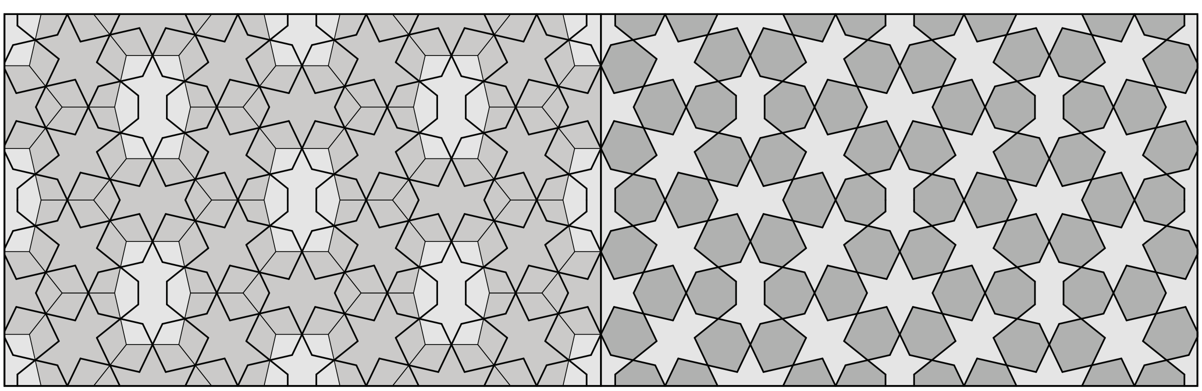

**Figure 5**: *A* 14-*s*5 acute *design from the Friday Mosque of Isfahan that is also illustrated in the anonymous treatise* On Interlocking Similar or Congruent Figures.

[**Figure 5**]. This is a 14-*s*5 *acute* pattern that employs two types of irregular hexagon as the only two sub-grid elements. The absence of tetradecagons or heptagons creates a *field pattern* without 7 or 14-pointed stars. Interestingly, this exact same design is illustrated, along with its generative underlying sub-grid, in the anonymous 11th to 13th century Persian treatise *On Interlocking Similar or Congruent Figures* [4]. Despite their closeness in time and place of origin, no causal relationship has been established.

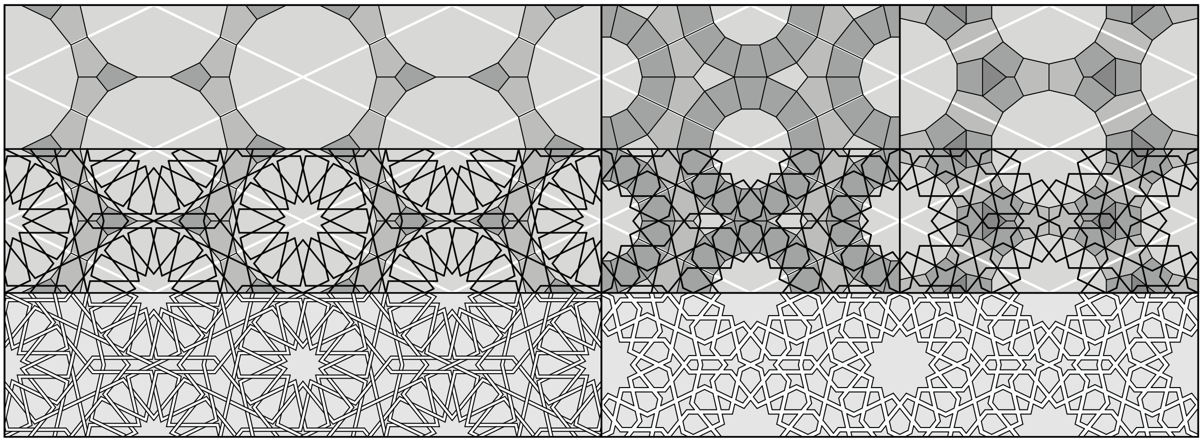

**Figure 6**: *A* 14-*s*2 median *design from the Qawtawiyya Madrasa in Tripoli, Lebanon (1316-26), and variation from the same sub-grid (and alternative dual sub-grid) from the Amir Burunduq Mausoleum at the Shah-i Zinda Complex in Samarkand (1390-1420).*

The most stunning designs produced from this system are characterized by 14-pointed stars within their overall pattern matrix. This variety of design originated during the 14th century under the auspices of the Mamluk dynasty of Egypt, and spread from the Mamluks to the Timurids and Ottomans. Most of these utilize either an acute rhombic repeat unit with 2/14 and 5/14 included angles, or an obtuse rhombus with 3/14 and 4/14 included angles. Examples with the acute rhombus include two 14-*s*2 *median* designs produced from the same sub-grid, but differ in the treatment of their central rosettes [**Figure 6**]. These are from the Mamluk Qawtawiyya Madrasa in Tripoli, Lebanon (1316-26), and the Timurid mausoleum of Amir Burunduq at the Shah-i Zinda Complex in Samarkand, Uzbekistan (1390-1420). The

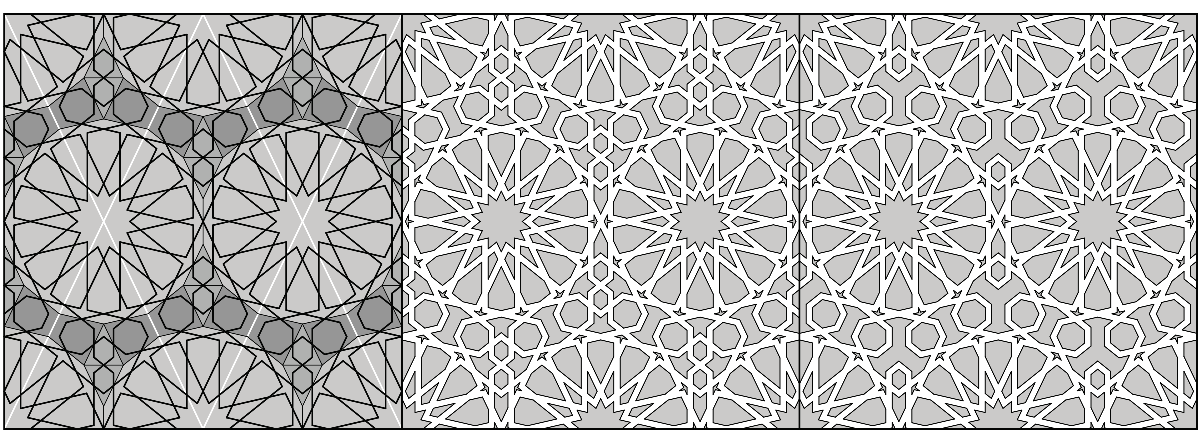

**Figure 7**: *A* 14-*s*2 obtuse *design with subtractive variation (right) from a wooden door of the Bayezid Pasa Mosque in Amasya, Turkey (1414-19).*

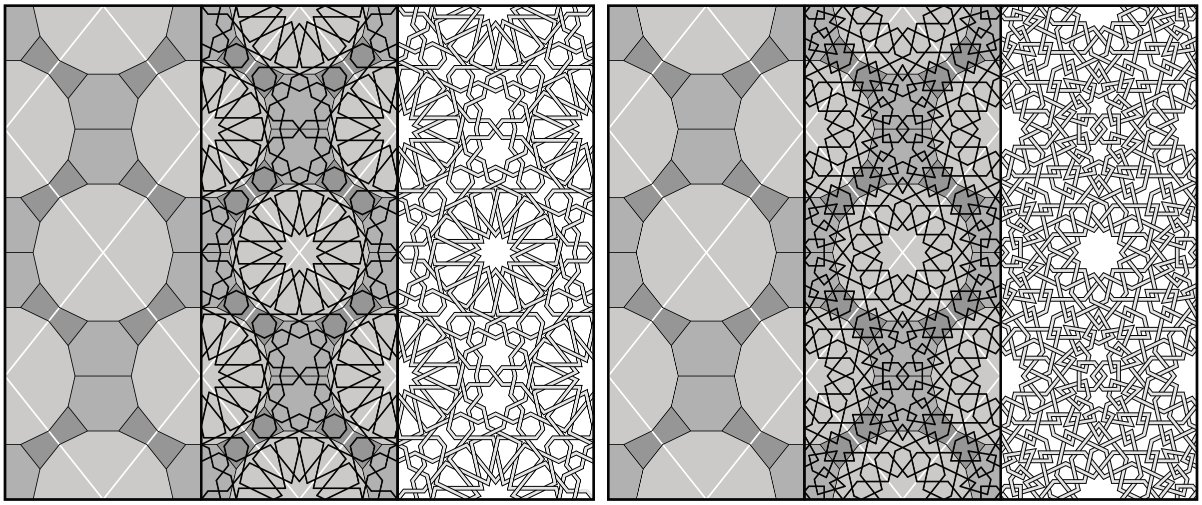

**Figure 8**: *A* 14-*s*2 obtuse *design from the ‘Abd al-Ghani al-Fakhri Mosque in Cairo, Egypt (1418), and a* 14-*s*4 2-point *design from the same sub-grid on the minbar door of the Quran stand at the Sultan Qansuh al-Ghuri Complex in Cairo, Egypt (1503-05).*

conventions for creating rosette variations in primary *n*-gons is an interesting subject for which there is not adequate space within this paper to do justice. Suffice it to say, and as demonstrated in comparing these two examples, such variations will significantly alter the visual quality of a design. The illustration of the Timurid example also demonstrates how a single design can be created from more than a single underlying sub-grid: a reciprocal feature commonly associated with 10-*s*2 *obtuse* patterns and 10-*s*3 *median* patterns in the 5/10 System. The illustrated alternate sub-grid in Figure 6 identifies this design as a 14-*s*5 *acute* pattern. Another historical design that employs the more acute rhombic repeat unit is from a door panel in the Ottoman mosque of Bayezid Pasa in Amasya, Turkey (1414) [**Figure 7**]. Bourgoine represents this design without its generative sub-grid in plate 166 [6]. This is a 14-*s*2 *obtuse* design with an additive 14-fold dart-rosette that is identical to that of the Qawtawiyya Madrasa example. As illustrated, this lovely Ottoman example was given a subtractive treatment: the elimination of the hexagon contained within one of the underlying triangles, creating the three-pronged elements within the pattern. Historical designs that repeat with the more obtuse 7-fold rhombus include two Mamluk designs produced from the same underlying sub-grid [**Figure 8**]. The earlier is a 14-*s*2 *obtuse* pattern from the minbar door of the ‘Abd al-Ghani al-Fakhri Mosque in Cairo (1418). E. H. Hankin first identified the underlying sub-grid for this pattern in his seminal work *The Drawing of Geometric Patterns in Saracenic Art* [7]. The later example is a 14-*s*4 *2-point* pattern that decorates the

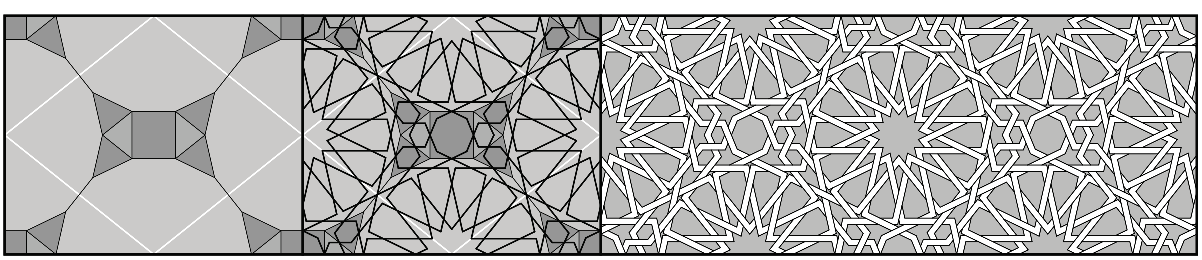

**Figure 9**: *A* 14-*s*2 obtuse *design on the minbar door of the Haram al-Ibrahimi in Hebron, Palestine (14th century).*

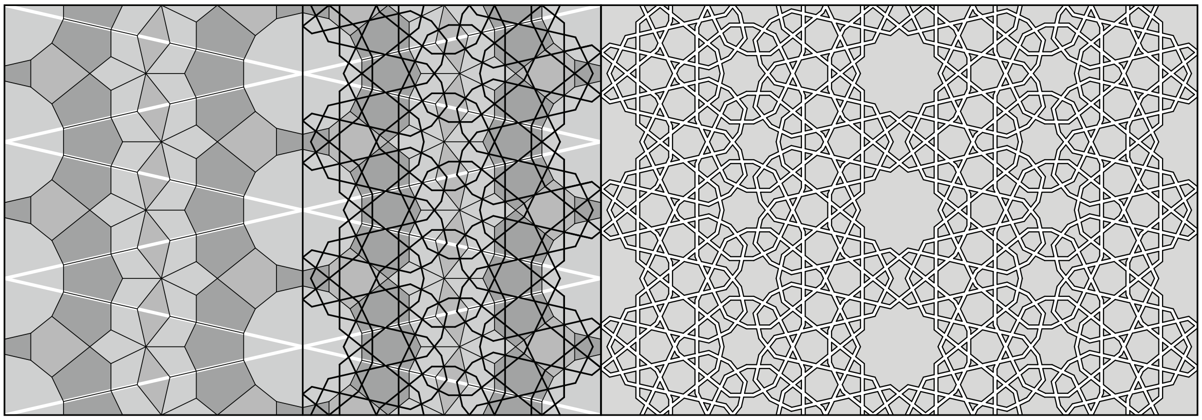

**Figure 10**: *A* 14-*s*4 median *pattern that repeats on an acute rhombus with 1/14 and 6/14 included angles* (© Jay Bonner, 2012).

congregational Quran stand at the Sultan Qansuh al-Ghuri Complex in Cairo (1503-05). Bourgoine represents this design without its generative sub-grid in plate 168 [6]. Another pattern that repeats with the obtuse rhombus is a 14-*s*2 *obtuse* pattern from one of the side panels of the minbar in the Haram al-Ibrahimi in Hebron, Palestine (14th century) [**Figure 9**]. There is a third rhombus that can be used within the 7/14 System, although there do not appear to be such patterns within the historical record. This third rhombus is the most acute, with 1/14 and 6/14 included angles [**Figure 10**]. It is worth noting that, similar to the two 5-fold Penrose rhombi, these three 7-fold rhombi can be used together to create 7-fold non-periodic Islamic geometric tilings [**Figure 11**]. In addition to patterns that repeat with elongated hexagons and rhombi, Muslim artists also occasionally utilized the rectangle as a repeat unit for 7-fold designs. An outstanding example of such a design is from the minbar of the Sultan al-Mu'ayyad Shaykh Complex in Cairo (1415-22) [**Figure 12**]. Bourgoine represents this design without its generative sub-grid in plate 169 [6]. This design places 14-pointed stars at each corner of the approximately square repeat unit, and an additional 14-pointed star at the center of the repeat unit. The skewed orientation between the generative

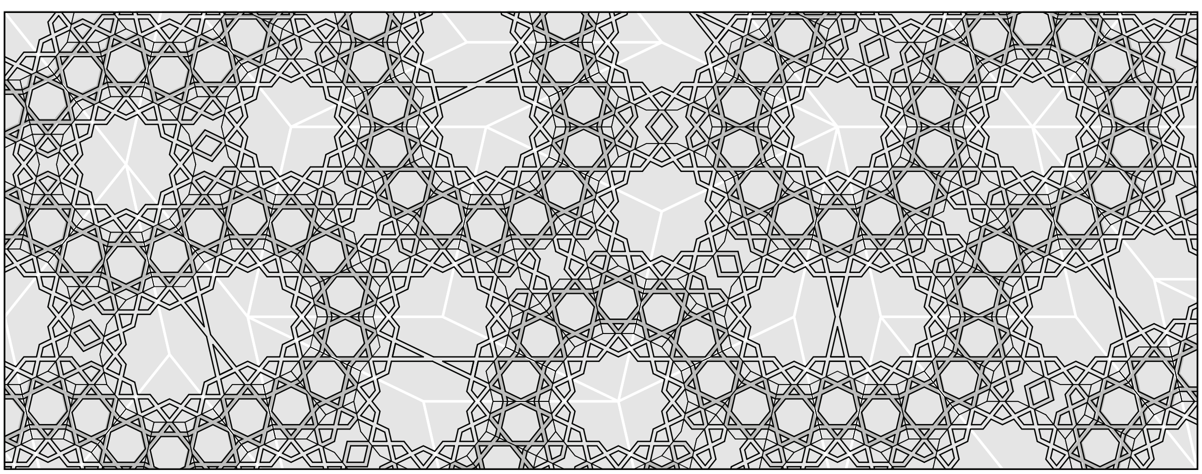

**Figure 11**: *A non-periodic* 14-*s*4 median *pattern that uses all three 7-fold rhombi as repetitive units* (© Jay Bonner).

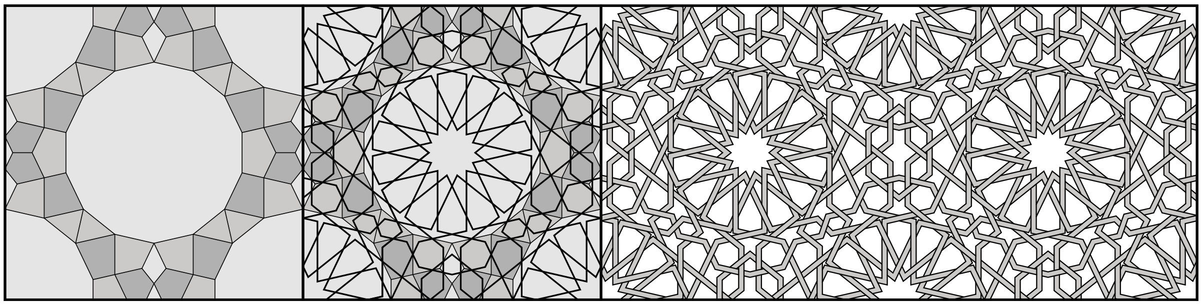

**Figure 12**: *A* 14-*s*2 obtuse *design with a rectangular repeat unit from the minbar of the Sultan al-Mu'ayyad Shakh Complex in Cairo (1412-22)*.

underlying tetradecagons in each corner and the one in the center has an unusual snub-like quality that creates the interesting dynamic of this design.

## Conclusion

The 7/14 System represents another example of the exceptional geometric resourcefulness and innovation of Muslim artists from the distant past. Durer's forays into the esoteric realm of 7-fold tiling were some 500 years after the building of the minaret of Ma'sud III in Ghazna. While remaining hidden all these years, the 7/14 System offers new opportunities for design innovation – especially as pertains to its suitability for producing 7-fold self-similar geometric designs and true quasi-crystalline patterns with recursive symmetry. Part 2 of this paper reports on the authors' investigations and discoveries while working with the 7/14 System. In particular, this second paper identifies a wide selection of polygonal elements, not necessarily found within the historical record, that are included within this system, as well as partially reports on the analogs of individual 7/14 polygonal elements with the elements from other *n*-fold systems for generating Islamic geometric patterns. Part 2 concludes with a demonstration of the recursive use of the 7/14 System to create very beautiful multi-level quasi-crystalline designs.

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