Triangle Tessellation Workshop

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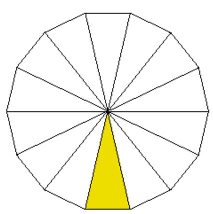
Abstract

The goal of this workshop is to construct a beautifully a-periodically tiled 14-gon by making use of brightly colored triangular shapes. By discovering and making use of the mathematical properties of the triangles, the participants discover a way to create an a-periodic tiling, making use of the principle of (perpetual) inflation.

Tessellation with Triangles

This workshop is inspired by the many wonderful mosaics that the authors have admired in mosques in Iran. These mosaics have interesting mathematical properties which can be explored at all levels. For example mosaics are often symmetric in various ways: reflection, translation and/or rotation. In this workshop we focus on an approach to construct a tessellation that does *not* have a translation symmetry This is called an *a-periodic* tessellation. [1] was among the sources of inspiration.



The a-periodic tessellation that will be constructed in the workshop is based upon a 14-gon. The participants construct a tiling of one of the fourteen slices that make up the 14-gon (see Figure 1). Through mathematical investigation of the tiles, they discover a way to make the tiling of this slice as large as they could possibly want. This can be done 14 times, for each slice, as such constructing a tiling of the entire plane.

Figure 1: *The 14-gon, of which one slice will be tiled with small triangular tiles.*

Three types of tiles are used, all of triangular shape (see Figure 2). The first step is to investigate their properties and relations. The yellow triangle plays a central role: it has the same shape as the 1/14 slice!

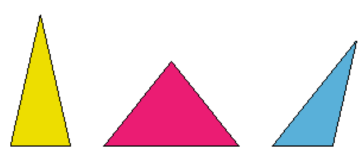


Figure 2: *The three type of tiles used to construct an infinite tiling of a 1/14th slice of a 14-gon.*

This property of the yellow (light gray) tile can lead to this insight: seven yellow triangles put next to each other with the small top angles touching each other, make a straight line (see Figure 1), so seven yellow top angles equal 180º. And: the angles in any triangle add up to 180º, thus do seven times the top angle! Question: how many ‘yellow top angles’ would fit in the bottom angles of the yellow triangle?

Next, we make a first start with the tiling: creating a large triangle, shaped just like the yellow tile, making use of two yellow, two blue (gray) and one magenta (dark gray) tile. Some effort and good thinking leads to one of several solutions, e.g. the one in Figure 3.Using this small tiling, the angles of the magenta and blue triangle can be investigated. To do so, it is helpful to know that each angle is an integer multiple of the top angle. Now, all angles can be found, either by reasoning while focusing on each triangle separately, looking carefully at its shape, or by putting several tiles next to or on top of each each other.

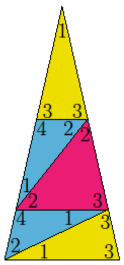


Figure 3: *A tiling shaped as the yellow tile, angles of each triangle denoted with top angle as unit.*

Now, how can the tiles be put together to make a tiling of the 1/14th -slice of the 14-gon? The trial-and-error-approach is always fun, but this will not lead to a satisfying solution so quickly. Something more sophisticated is needed.

Tessellation through Inflation

The tiling in Figure 3 is a first start of the tiling of the 1/14th -slice. To proceed and make a larger tiling of this 1/14th -slice, we can repeat this process: the inflation procedure. In Figure 4, it is illustrated that to make the next inflation step, we do not only need an enlarged version of the yellow tile, but also of the blue and magenta one. If we can make these, they can be put together to construct the next step of the tiling of the 1/14th -slice.

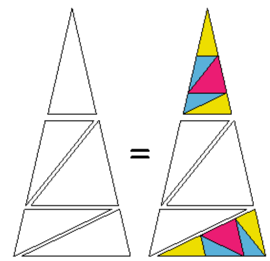


Figure 4: *The concept of inflation.*

Finding how to construct enlargements of the blue and magenta tiles requires a good thought. By comparing the shapes of the tiles and having a close look at the already found enlargement of the yellow tile, the participants are able to find the solution for the enlargements of the other two tiles, leading to the tiling in Figure 5. Now, the inflation procedure can be repeated over and over again, infinitely, to tile the full slice. To admire the full disc, the tiling can be reproduced thirteen times, or, easier and more intriguing, mirrors can be put alongside the tiling, see Figure 6.

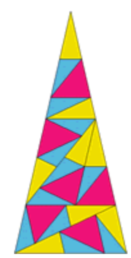


Figure 5: *The second inflation.*

Figure 6: *The disc with tiling up to second inflation (l) and the tiling-in-process as seen with mirrors (r).*

This workshop is used in the Netherlands to inspire teachers. We show them that the basic ideas of this workshop can be performed at many levels, depending on which *Math Moments* are chosen. These Math Moments vary from simple investigations on shapes, like the one presented her, to more complex reasoning about symmetry and the perpetual property of the inflation procedure.

References

[1] Non-Periodic Tilings With N-fold Symmetry, www.mathpages.com