

Image Generation from Magic Squares

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Abstract

The paper is departing from a position in aesthetic theory, in which order of some sort is considered a contributing factor to aesthetic value. It is likewise departing from a position, that generative art, drawing on rule based systems and algorithms, is a legitimate art practice. Instead of generating order by falling back on an artistic master mind or the sensibility and skill of an artist, we explore a highly structured mathematical entity, the *magic square*, as an engine to generate fine-art by designing representation schemes, which will transform the inherent mathematical order of magic squares into images. A number of such schemes are demonstrated, and an experiment is described to verify the conjecture, that images constructed as mappings of magic squares are valued as more pleasing than similar images constructed randomly.

Introduction

An art experiment on generative art was carried out in 2010 at the Media Research Center of the School of Communication and Design, Sun Yat-sen University, Guangzhou, China¹, and the results have been published as a book, see[1]. Generative art, based on premeditated rule systems, can by now be considered an established practice for the generation of fine-art work. Both, the published book and this paper are parts of the project. However, in this paper we focus on some particular content of the project, which is dealing with *magic squares*.

Our intention here is to expand the matter with some technical aspects, which are only touched in the book, and we regard this paper as a supplement to the project. As discussed in the book, a particular challenge in any art practice is the necessity to make abstractions, to simplify, to reduce, to find or even invent essentials, or to design order of some sort. Art is always and foremost an activity of abstraction, see [1]. When we practice art as *generative art*, it becomes mandatory to formulate the properties of e.g. *order* in a formal rule system. The rule system then is the generative engine for the pursued artistic intentions.

Magic Squares as Generative Engines

The idea comes from the interest in a chapter of an old Chinese book Lo Shu, shown in Fig1, see [2]. Circular dots of numbers were arranged in a 3*3 grid pattern, the sum of the numbers in each row, column and diagonal was 15. It is probably the oldest known form of a magic square. As we know from mathematics, the structure of magic squares contains peculiar properties of order and balance, and from this order multifold of possibilities can be generated, based on the inherent structure of the system. The Chinese traditional aesthetic concept He Er Bu Tong (harmony in diversity) is reflected in such transformation processes. We suppose that works based on the abstract mechanism and the inherent order of magic squares may be in accord with the abstract

¹The project group was formed by members of the following institutions: School of Communication and Design, Guangzhou; Kunsthochschule, University of Kassel, Germany; School of Information Science and Technology, Sun Yat-sen University.

notion of design, and may be better suited than those using disordered random generators. Thus more harmonious images may be created.

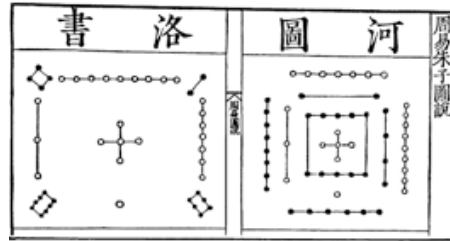


Figure 1: The old Chinese divine image showing the 3 x 3 Lo Shu square

A $n \times n$ magic square is an arrangement of $n \times n$ numbers in a square, of usually distinct positive integers. And the n numbers in all rows, all columns, and both diagonals sum to the same constant, sometimes called the magic number. In our project we only made use of 4×4 magic squares. As an algorithm to construct the magic squares, we tried to use lozenge method, symmetrical method, diagonal method, but finally we decided to use *De La Hire's Method* [3]. Because *De La Hire's Method* is an easier method for the programmers, especially generating even Magic Squares. With *De La Hire's method*, we finally coded the program in *objective C* on Apple Mac and constructed all theoretically possible 880 magic squares of size of 4×4 . By rotation and reflection, 7 more magic squares can be generated from each one of the 880 magic squares. So we can get 7040 different magic squares in total.

Verification of Aesthetic Conjecture

With the conjecture, that the mathematical order inherent in magic squares will show, we conducted an experiment to verify whether visual arrangements following the rules of magic squares would be received as aesthetically more pleasing to a viewer than those of randomly chosen arrangements of the same elements. The transformation scheme from integer to image in the experiment is based on the following rules:

- Generate a 4×4 square grid M with 16 cells.
- Algorithmically, generate a 4×4 magic square, using the integers 1 to 16, and assign the calculated integers to the cells of M .
- Inscribe into each of the 16 cells of M a 4×4 grid according to the mapping rules of Fig. 2, dividing each cell into 16 small squares.
- Starting on the top left corner of each cell of M , blacken out small squares according to the magnitude of the integer encountered in the cell.
- Repeat for a randomly filled 4×4 square.

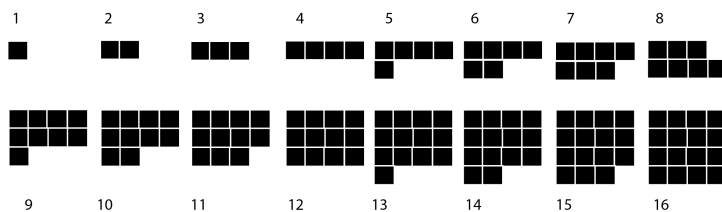


Figure 2: each image represents an integer

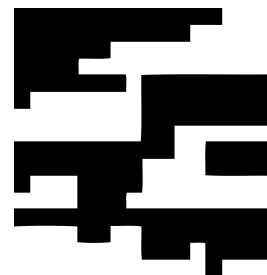


Figure 3: one image in the experiment

20 magic square and 20 random square images are used in verification experiments, both based on 4×4 squares, as one example in the Fig.3. Magic squares and random squares are presented in completely arbitrary sequence to 92 students coming from art study class. They grade aesthetic of images based on 1 to 5 scale. Fig. 4 shows the experiment statistics, horizontal axis (x) represents

the number of students, vertical axis(y) represents the mean value of students' grading on the 20 magic squares and random squares. The blue is for magic squares, the red is for random squares;

On the basis of the statistics data, we compare each student's grading between magic and random square. Then we mark the student in blue who has higher mean value of grading on magic squares than random squares and mark the others in red (see Fig.5). And we define higher mean value of grading as tendentiousness. Fig. 5 shows that the 61% students prefer images generated from magic squares. These findings verify our conjecture, that image constructions along the properties of magic squares will be valued over the randomly generated ones.

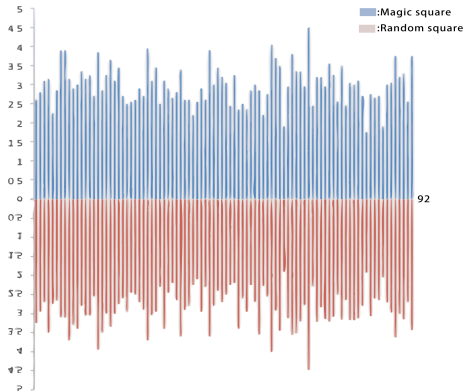


Figure 4: the diagram shows the 92 students's grading of magic squares and random squares

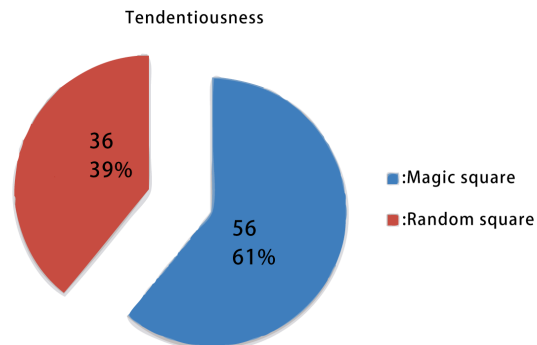


Figure 5: the diagram shows ratios of 92 students's tendency

Mapping the Inherent Order of Magic Squares to Art

On the basis of our conjecture, the artists were trying to find visual representations for transforming the numbers in the magic squares into images. We think, a large number of possibilities for such transformations do exist, and we have experimented with a few of them: abstract symbols, lines, geometrical entities like triangles and squares, number representations, circles and a few others. In addition, color, transparency and movement were considered for enhancements. The images using lines and squares are interesting because of their simplicity and symmetry. Images made up of simple elements often transmit an abstract sense of rhythm. In order to gain this sense in images, we tried a great many and quite different schemes. In the process, we did work out each scheme in an interactive fashion, where the artists were working together with the programmers. Before the programmers started their work, each scheme was carefully considered and tested by the artists. After some rough prototyping, the artists advised the programmers to modify their code again and again in order to improve the abstract sense of rhythm. For the close interaction between programmers and artists, it was very beneficial that in our lab at the Media Research Center, Sun Yat-sen University, the two diverse professions, artists and programmers, are represented equally and they work together on a regular basis within the context of industrial projects. With the inherent order derived from the magic squares, it is difficult to produce "poor images", however, within the great mass of produced images. Some are still more appealing than others. As examples, we selected three images from three different schemes (see Fig. 6).

In order to expand magic images, we try to appreciate them from different interpretative views. One of the schemes can be interpreted as *architectural*. The height and proportions and the predominant right angle gives an impression of buildings and facades (see Fig7). Another scheme is inspired by *eight trigram*, an old Chinese image. It is a symbol of Chinese Taoism. It presents harmony and unity itself. We have expanded the *eight trigram* image into 4 x 4 magic square images (see Fig. 8).

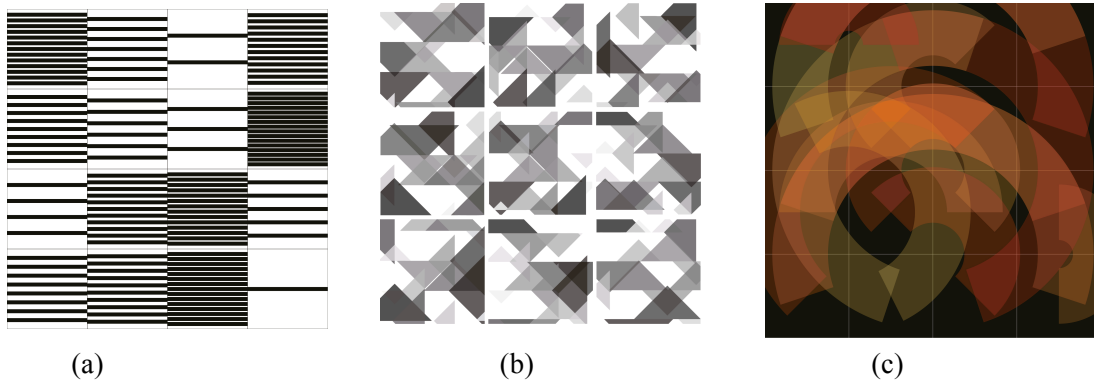


Figure 6: a) *magic square image with lines* b) *4*4 magic squares image with triangles*
 c) *image using colored arch half-ring to connect continuous numbers in imagic square like 1→2,2→3.....*



Figure 7: *Images made up from Architectural* **Figure 8:** *Images made up from Chinese Eight Trigram interpretations*

Summary

In the numerous experiments we've carried out, we have shown, that the properties of magic squares can be transferred to rule systems, which in turn will construct aesthetic events. The key issue becomes the design of *representation schemes* for which we have tried out binary representation schemes, color-shape schemes, and black-and-white-shape schemes. The aesthetic value of the images generated with these rule systems, was verified in an experiment, where the images constructed along the properties of magic squares were valued significantly over the randomly generated images among the judgments of students from major of design art.

Since the generated images are all based on the same rule system, each image sequence is a series, but diverse. This is can appropriately be applied to pattern design as a collection in a great number of other contexts.

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