Geometric Forms that Persist in Art and Architecture

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Abstract

Our mind tends to recognize shapes and forms in the world. Geometric shapes persist in Art and Architecture from Prehistory to Modern Age. Here we discuss some examples of this "persistence" (sinusoids, catenaries, helicoids). Examples are chosen from Mesopotamian, Gothic, Islamic, Baroque and Modern Art and Architecture.

1 Introduction

Our mind tends to recognize shapes and forms in the world: perception is transformed into proportions and geometrical shapes, that artists and architects have continuously used to produce their artworks, from cavern paintings to modern Art. Forms have been recognized in Nature, used because of their aesthetic value or functionality. They have been first understood at the emotional level and later elaborated more consciously. Some of them, more easily reproducible and recognizable, have pervaded the visual symbolic communication and have been linked to Semantics [1]. A potentially infinite family of "geometrical shapes" have crossed the ages and the cultures, giving rise to the «persistence of forms» (see [1],[2],[3]), about which we have a larger project. We shall here shortly discuss three emblematic shapes: i) catenaries (and catenoids); ii) sinusoids (and sinusoidal concoids); iii) helices (and helicoids). We have chosen these non-algebraic curves since they create surfaces in various ways (ruled, by revolution or minimal). Their non-algebraic character is visually clear, although only the algebraic methods of Cartesian coordinates has allowed us to study them as solutions loci of suitable (non-algebraic) equations. Eventually, the methods of Infinitesimal Calculus have provided us the tools that allow us to calculate their "curvature" and to interpret them also as curves of motion generated by dynamical processes [4].

2 Catenaries

A "catenary" (from the Latin word *catena*) is a curve in Euclidean plane representing the shape of a hanging chain supported at its fixed ends, under the pull of Galilean gravitation. Its U-like shape is governed by the Cartesian equation $y = a \cosh (x/a) = 1/2[\exp(x/a) + \exp(-x/a)]$ where a is a real parameter (the ratio between the horizontal component of the tension and the linear density of the chain). It is roughly similar to a parabola if limited to a small portion. The history of this curve (that provides an instructive example of the interaction between experiment and theory) is thoroughly discussed in [1]. First defined by Guidobaldo Del Monte (1545-1607) and later discussed by his pupil Galileo, they both incurred in errors that were later corrected by Joachim Jungius (1587–1657) - published posthumously in 1669. The correct equation was eventually derived in 1691 by Gottfried Whilelm von Leibniz, Christiaan Huygens and Johann Bernoulli. Huygens was the first to use the term *catenaria* in a letter to Leibniz (1690). The catenary is rather important in Architecture. Early used in the construction of arches as an "inversion of the hanging chain around a horizontal plane", the curvature of the "inverted catenary" was soon understood to be provide stable arches and vaults [5]. Examples of well approximated catenaries can beare found in Taq-i Kisra (Ctesiphon, Mesopotamia – Fig. 1a).

Fig. 1 – *(a) catenary arch, Ctesiphon VI B.C.; (b) study model by chains for "Sagrada Família"; (c) model for construction in Museum (photo Lorenzi); (d) A. Siza Expo 1998, Lisbon, Portugal Pavillion*

It somehow remained in Islamic Architecture but was forgotten in Europe for long time. Its modern "rediscovery" seems to be due to Robert Hooke during the rebuilding of St Paul's Cathedral in London: in 1671 Hooke announced to "have solved the problem of the optimal shape of an arch" [6] by inversion in the horizontal plane. The surface of revolution of a catenary is called "catenoid"; as a "minimal surface" can be obtained as the shape of a soap film bounded by two parallel circles. Modern Architecture has re-discovered the catenary as the ideal curve for an arch that supports only its own weight (provided the material allows just compression and no torsional moments occur inside it [1]). Antoni Gaudí (1852-1926) – who left an impressive mark on the role that Mathematics plays in Architecture – applied catenaries in the design of "La Sagrada Família" [7], by developing a new method of structural calculation based on models that involved ropes and small but heavy sacks (Fig. 1b,c); according to the Calculus of Variations the ropes formed catenaric arches. Photographs of the resulting plastic model, appropriately turned upside-down, indicated the pressure lines of the structure envisaged. Catenary shapes were used by Gaudí in most of his architectural work, as in the crypt of the Church of Colònia Güell [7]. The use of Catenaries in Architecture has been further incremented by the use of concrete, which allows an unprecedented levity [8]. As an example, we mention the work of Alvaro Siza who designed the Portugal Pavilion for Expo 1998 (Fig. 1d), where levity is realized by hanging chains filled by a thin layer of (white) concrete. An area of 65x58 meters is thence topped by a geometrical shape molded in only one piece 20 cm thick; the orthogonal independent sections form a family of (parallel) catenaries and straight lines, respectively, so that the resulting Gaussian curvature vanishes [1].

2 Sinusoids

Fig. 2 – *(a) Augusta Raurica, Switzerland, II Century AD; (b) Basilica of San Clemente, Roma*

The sinusoid is another geometrical shape that has crossed the Centuries; after XVIII Century sinusoids were so common that William Hogarth took them as prototypes of elegance in his influential book "The Analysis of Beauty" [9]. Its symbology is related on one hand with the notion of "cyclicity" (i.e., periodicity) and on the other with an idea of "flowing" and "waving". Oscillating forms are in fact ubiquitous in the antiquity; in ancient times when they were defined and used for astronomical measures, models and predictions. Oscillations – that are currently used in signal processing and time series – have been formalized in XVIII Century, through the pioneering works of Euler [10]. In [1] we have discussed their presence in Roman mosaic floors of Imperial age and in XVI Century (when they acquire 3-dimensionality). The sinusoidal motives that pervade, as "frameworks", several Roman imperial floors are usually called “*can corrente*” (i.e., “running dog”). A *can corrente* is characterized by an infinite group generated by one translation and perceptively reminds the motion along the translational direction. They may either express an explicit connection with water, both at symbolic and effective level (e.g., in thermal baths), or the appearance of a “braid”, i.e. an implicit relationships with the third dimension (“above/under”) in which the 2-dimensional line is embedded. Waving motives persist in heraldic representations (“*ondato*”), as well as in Church floors that so-called “*Cosmati*” constructed in XI Century, mostly in central Italy [1]. In this last case the motive is more properly called a “*guilloche*” and has a high perceptive impact that emphasizes its role in liturgical itineraries (see Fig.s 2a,b). In modern mathematical terms we can say that believers stand in rectangles filled by multicolor marbles and characterized by a finite group. The deacons advance in the middle of the church along the *guilloche* and stop at so-called “*quinconci”* [1] or where another *guilloche* is intersected [11].

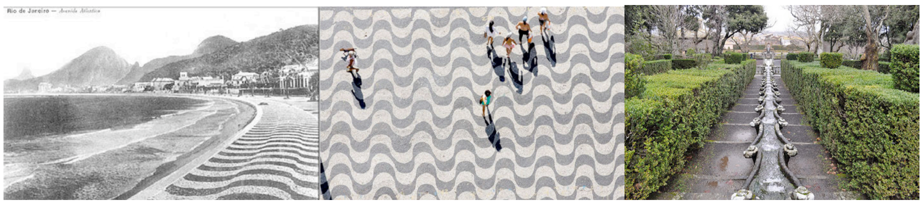

Fig. 3 – (a) Avenida Atlantica, Rio de Janeiro, 1930 (b) idem today, promenade by Burle Marx; (c) “water chain” Cordonata del Gambero, Villa Lante, Viterbo

Undulated motives are found in gardens and in modern “landscape architecture”. An emblematic example is the black and white wavy motive that fills the Brazilian promenade of Copacabana (1961), which in turn reminds the pavement of the famous Rossio square in Lisbon (Portugal). This pavement makes part of a larger project by Roberto Burle Marx (1909-1994); the black and white motive follows in fact traditional techniques of Portuguese *calçadas* (formed by square stones in pavements of public places that have to resist heavy use). As we said, sinusoids may be related with the presence of water (as, e.g., in front of the Copacabana Palace, where in the 20’s of XX Century Burle Marx has used them along the whole sea front; Fig. 3a,b). Another example resides in so-called “Italian Gardens” of XVI Century, usually known as “water chains” (*catene d’acqua* - Fig. 3c) that border water flows. Looking at surfaces, we see that also sinusoids have been used in Architecture to allow levity. As a further example of Gaudì’s geometric creativity, we mention the “Schools” of “Sagrada Família” (Barcelona, 1909) where he used small straight segments to construct a curved roof in the form of a conoid with sinusoidal section (a ruled surface), sustained by thin walls (9 cm for a 5 meters building of 10x20 meters – [1],[12],[13]).

3 Helices

Fig. 4 – (a) colonne tortili in Santa Caterina church , Palermo (1566-1596); (b) helical stair Tomar, Portugal (1160-1400); (c) F. Borromini, helical stair in San Carlino church, Roma, Italy, 1640

Helices and spirals are a further common geometrical shape. We speak of a “spiral” whenever – in polar coordinates – we have an open curve that “spirals” around a center in a plane, usually progressing towards the exterior according to some specific law. Many spirals are well known, among which the most celebrated one is the "Archimedes spiral" related with Golden Mean [14]. Another "spiraling" effect is related with the "helix", i.e. a spiral that evolves steadily along a cylinder (or a cone). A "cylindrical (regular) spiral" is the composition of a rigid uniform rotation in a horizontal plane and a uniform translation along an orthogonal axis. Helices and sinusoids can be combined together providing nice artistic shapes. Sinusoids can in fact be used to design the "spiral columns" (or *colonne tortili* - which have a sinusoidal profile) much used throughout history and places, by Islamic and Gothic architects as well as in Baroque (Fig. 4a,b,c). A helicoid is instead a ruled surface obtained from a helix (as a minimal surface with helical border). Generally speaking, the term "spiral stairs" defines a staircase that gradually climbs up, revolving around a central axis; in XVI Century helical stairs were much in use and some examples can be used to point their 3-dimensional static stability (Fig.s 5 a,b). Notice first that the central axis might be solid or virtual. We shall give a few examples of helical stairs with "virtual axis". As we remarked in [1], these nice helical stairs rise around a pole without a column in the centre: the central support is made by a helix and this leads to a perceptual effect of emptying. Whence stairs do not have a conventional straight central support, the tightly wound "inner stringer" functions here as one; this twisted central support is built in fact like a giant spring. If the central helix is close enough, one can perceive a *colonna tortile* in the middle, instead of the cavity. The upward rotation provides a sense of elegance and can be used in narrow spaces. Such structures are so stable that might not be anchored at walls either (see the Louvre stairs of Fig. 5a). Accordingly, the helical stairs can be roundly released from walls and other supports like central columns or pilaster, returning in this way – again - an image of levity (inspiring values of the Italian architectural school of Rationalism - Fig. 5c,d) – see also [15].

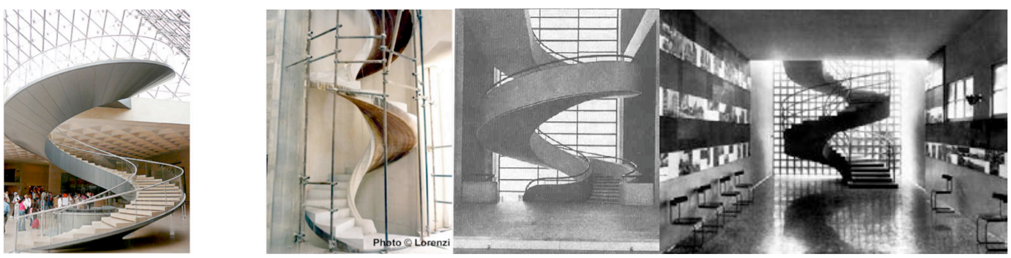

Fig. 5 – (a) I.M. Pei helical stair, Louvre, 1990; (b) Gaudì stair (1920), Barcelona (photo Lorenzi); (c) Pagano - Buzzi helical stair, VI Triennale Milano, 1936; (d) L. Moretti stair, GIL, Roma (1936)

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