Soccer Balls

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Abstract

The paper presents variations on the soccer ball: from the standard truncated icosahedron to hyperbolic soccer balls. For the latter, proposed by Keith Henderson, shapes are given with vertices on a hyperboloid of one sheet. Also, soccer balls can be stacked in DNA-like spirals, which can be executed with Zome tool – an inspiration for an artwork in countries with soccer in their genes?

Mathematical Soccer

The truncated icosahedron is the classical model for a soccer ball: blowing it up to a spherical shape turns the polyhedron into a traditional soccer ball. It is an Archimedean solid with 12 pentagonal and 20 hexagonal faces, and all of its vertices lay on a sphere. A similar procedure could also be followed using a truncated octahedron or a truncated tetrahedron, with fewer hexagonal faces, and with squares, or, respectively, equilateral triangles, instead of pentagons. However, these equivalents have not been seen on any soccer field (yet).

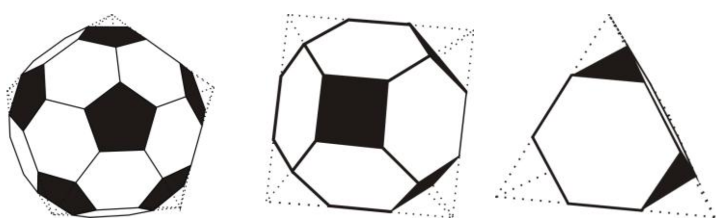

Figure 1: *The truncated icosahedron (left), octahedron (middle) and tetrahedron (right).*

Instead of considering 5, 4, or 3 triangles meeting in one point (for the cases of icosahedron, octahedron, and tetrahedron), we can consider 6, 7, or 8 triangles. The pentagons are replaced by 6-, 7- or 8-gons, but the latter are not necessarily flat polygons. Six triangles tiling the plain are transformed into flat hexagons and this is the kind of soccer bees play. However, when stacked in a near-cylinder, the truncated figures are curved hexagons.

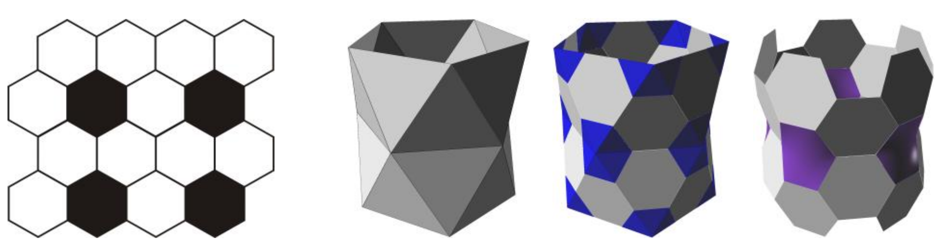

Figure 2: *An apicultural soccer ball (left) and a truncated cylinder (middle images and right).*

The heptagonal case was proposed by Keith Henderson, who called his invention a 'hyperbolic' soccer ball. This model with curved heptagons is useful as an illustration of Thurston's representation of the hyperbolic plane (see [1] and [2]).

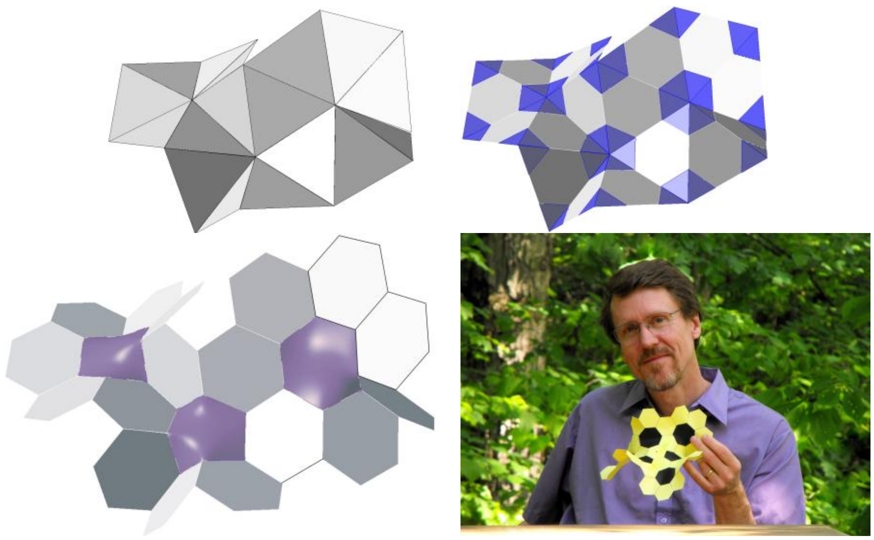

Figure 3: *Construction of a heptagonal soccer ball (top and left), and held by Keith Henderson*

Inspired by the word "hyperbolic", we can try to place the triangles on a hyperboloid of one sheet. Indeed, as the classical soccer ball is an approximation of a sphere, we can try to see if the hyperbolic model could fit to a hyperboloid. However, the curvature of a hyperboloid is not constant (as is the case for a sphere), and as a hyperboloid becomes almost flat far from its center, only the central part will be of interest (why the hyperboloid was chosen and not the pseudo-sphere with its constant negative curvature is explained in [2]). When 7 triangles meet in one point, a construction of triangles where 12 vertices lay on a hyperboloid can be obtained (the top vertices being slight off the surface; see [3]). Truncating this polygonal surface yields a figure where in the middle the truncated octahedron appears gain.

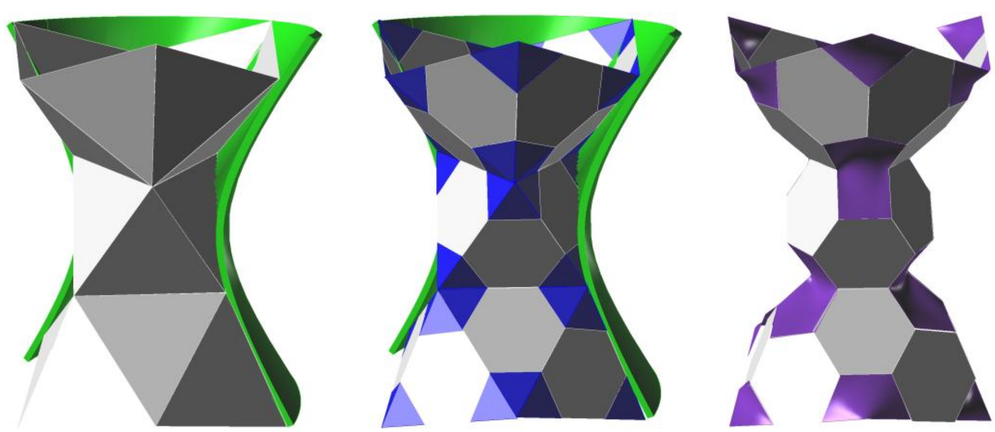

Figure 4: *A heptagonal hyperbolic soccer ball; the top and bottom vertices are not on the hyperbola.*

The case where 8 triangles meet in a vertex is more elegant: a polygonal surface can be constructed where the vertices of 18 triangles all lay on a hyperboloid. Moreover, the hyperboloid is equilateral, and thus closer to being the counterpart of a sphere (the equation of a circle is $x^2 + y^2 =1$, that of an equilateral hyperbola $x^2 - y^2 =1$).

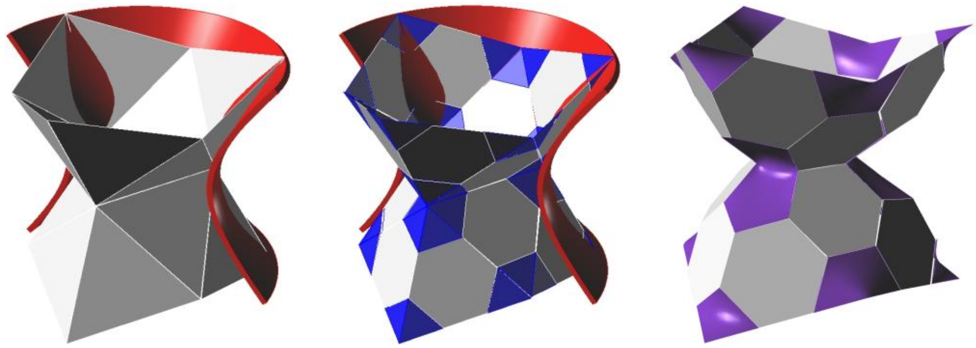

Figure 5: *An octagonal hyperbolic soccer ball. The vertices of the triangles on the left are all on the hyperboloid, though not necessarily the vertices of the hexagons.*

Stacking Soccer Balls

The soccer ball considerations raise an interesting question for users of the Zome tool. Indeed, dodecahedrons are fun to stack, and even give rise to beautiful artworks such as Jon Barlow Hudson's 'Double Helix: Flowing Balance' at the Wright State University in Dayton, Ohio (VS).

Figure 6: *Stacking dodecahedrons: Jon Barlow Hudson's 'Double Helix: Flowing Balance' and an artwork for children for climb on, in Bruges (Belgium).*

However, dodecahedrons cannot be stacked face to face in Zome (as confirmed by Paul Hildebrandt himself!), and so this construction is out of reach for the Zome adept (see [4]). A construction using inscribed icosahedrons is possible but would not be very stable, as a single vertex would serve as link. Here, the soccer ball offers a solution: they can be stacked face to face in Zome (hexagon to hexagon).

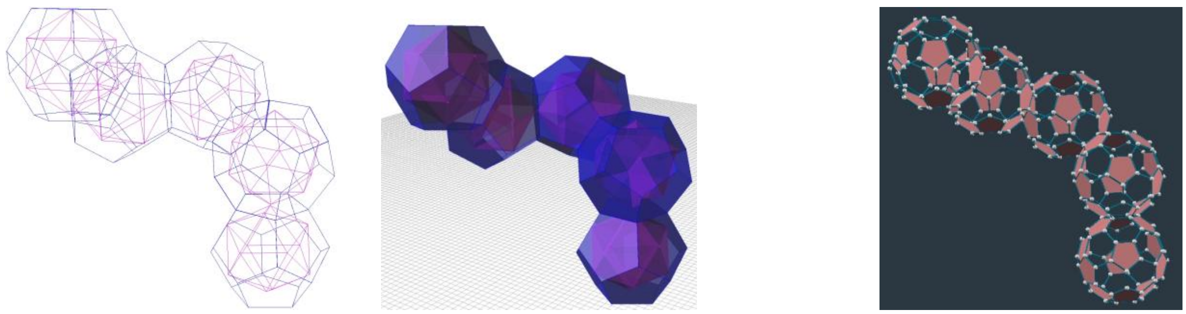

Figure 7: *Stacking dodecahedrons could be done using the inscribed icosahedrons (left, middle) but it probably is more elegant to stack Zome soccer ball (right; construction made in vZome).*

In countries where soccer seems to be in the genes of the population, a DNA-like stack of soccer balls could be a good inspiration for an artwork. The illustration shows such a double helix, where six hexagons determine the distance between two soccer balls for a pleasing (and solid) construction.

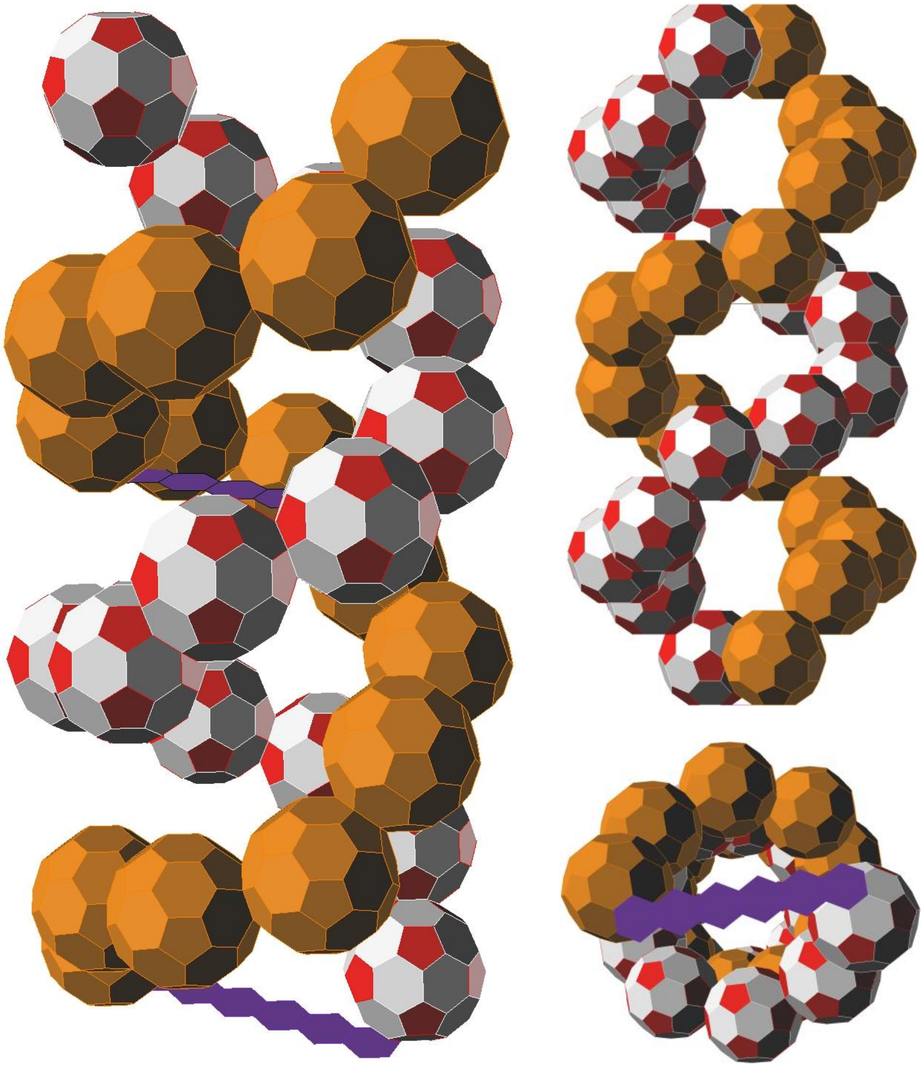

Figure 8: *Stacking soccer balls as a double helix.*

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References

[1] C. D. Bennett, B. Mellor, P. D. Shanahan, *Drawing a triangle on the Thurston Model of Hyperbolic Space*, http://myweb.lmu.edu/bmellor/ThurstonV3.pdf.
[2] D. Huylebrouck, *Polyhedra on an equilateral hyperboloid*, to appear.
[3] D. Taimina and D. Henderson, *Experiencing Geometry: Euclidean and Non-Euclidean with History*, 3rd edition, Pearson Prentice Hall, 2005.
[4] Zome tool web site: *http://www.zometool.com*