At What Level Do Mathematical References Act In Contemporary Architecture?

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Abstract

We examine some of the basic ideas underlying the theoretical body of non-linear complex systems, non-Euclidean spaces, topology and the possible meanings of the migrations of these concepts into architectural practice.

Introduction

The history of science is full of boundary crossings, which strongly suggest that interdisciplinarity is a recent concept for an old *modus faciendi*. In a certain sense, scientists understand nature largely through metaphorical constructs, or models, that they transport from a known world to the explanation of a new one [1]. Such models range from the atomic level, to models in chemistry and biology — as protein folding, chaperone proteins — and global warming. Although some authors point out that these metaphors and analogies can represent "bad science" because of their misdirected thought [7], by pursuing this approach a great number of benefits have come and continue to come to science.

The alliance between architecture and mathematics has a long history. Ever since building began, architecture has relied on mathematics to achieve visual harmony, and structural robustness. For centuries architects have applied the principles of Euclidean geometry, modelling space with lines, triangles, circles, regular polygons, polyhedra and conic sections. Over the last century mathematics has seen new developments in non-linear systems, chaos theory, fractals, topology, and non-Euclidean geometries. The new concepts arising from these developments, associated with the emerging digital technologies and massive computer processing power, produce shapes that are suggestive, provoking and unusual. These shapes are a challenge for architects as they represent a promise of new spatial relationships and configurations. New concepts in non-linear complex systems, non-Euclidean spaces and topology have been invoked as justifying the genesis of certain architectural forms [2-4], [8] [9]. Information fluxes between different fields can be a source of inspiration, having only a symbolic function, as traditional culture represented in vernacular architecture, biological images in organic architecture, or the history of architecture in postmodernism. The question then arises: is the large use of scientific and particularly mathematical references simply rhetorical, representing only a greater permissiveness of form, or it is a shift of paradigm as pointed out by Jenks [4]? Considering some actual buildings, we examine in this paper possible meanings of the migrations of some mathematical concepts into the architectural practice.

Chaos, non-Euclidean geometries and topology

Let us consider a system describing a physical process. It is normal to expect that when using different starting values the system would produce different results, but somewhat similar qualitatively and quantitatively. However, if the system is non-linear, small perturbations in the initial data can lead to large variations in the final results. This unpredictable behaviour and sensitive dependence on the initial conditions is called chaos. For example, the motion of the planet Mercury is described by a non-linear unstable system which implies that a small error in its initial position will be amplified such that there is a limit to how far into the future we can predict its position. Mathematical chaos arises in huge complex systems, such as weather forecasts or turbulent flow, but also in very simple systems as those described by the logistic equation. The previous systems are deterministic and their future could be completely determined by the initial data. However they are not predictable, because non-linearity leads to explosive amplifications of initial perturbations.

Figure 1: *Top left and top right: Performing Arts Centre, Abu Dhabi, and Spiral Tower, Barcelona, of Zaha Hadid*[1]*; Bottom left: UFA Cinema Centre, Dresden, of CoopHimmelb(L)au*[2]*; Bottom right: Denver Art Museum of Daniel Libeskind*[3]

The chaos that occurs in non-linear physical systems is a purely deterministic phenomenon, far from the idea of disorder that its name may suggest. However architectural sensibilities that borrowed the concept pretend to materialize it through a certain random superposition of the "layers" of the project. We can mention without being exhaustive some works of Zaha Hadid, Daniel Libeskind and of the group Coop Himmelb(L)au. The superimposition of layers in an

[1] http://www.zaha-hadid.com/home
[2] http://en.wikipedia.org/wiki/File:Dresden-Kristallpalast-nigh.jpg
[3] http://www.daniel-libeskind.com/projects/show-all/extension-to-the-denver-art-museum-frederic-c-hamilton-building/

apparent random way is visible the pictures at the top of Figure 1. The two pictures at the bottom of Figure 1 illustrate the idea of unstable equilibrium. The migration of the concept of chaos appears as purely semantic, being used to evoke a certain disorder of the contemporary world and the frailty of human knowledge.

The main difference between Euclidean geometry and non-Euclidean geometries is the concept of parallelism. If in a plane two straight lines, both perpendiculars to a third line are indefinitely extended then, in the framework of Euclidean geometry, they remain at a constant distance. However within Riemannian geometry the lines approach each other and within hyperbolic geometry the distance increases as we move away from the intersection points with the common perpendicular. In Euclidean geometry the universe is represented by a flat plane whereas the universe is a sphere for Riemannian geometry and a hyperboloid for hyperbolic geometry. The concept of parallelism underlying Riemannian and hyperbolic geometries suggests the possibility of using positively curved and negatively curved walls. However this only represents non-parallel walls in a Euclidean geometry. When Zaha Hadid says[4] that "The most important thing is motion, the flux of things, a non-Euclidean geometry in which nothing repeats itself, a new order of space" or D. Liebeskind states[5] that "we're not dealing with architecture as a knowable Euclidean container or Cartesian space…a large body of architectural work is now already built on a different premise", the non-Euclidean geometry is misleadingly identified with the use of twisted and curved surfaces, which are Euclidean objects in a Euclidean space.

As far as topology is concerned several references have been made to its influence in architectural practice and even to the onset of a topological tendency [3]. From a historical point of view, mathematics was first concerned with the study of quantity and it has progressively evolved to the study of qualities. Topology is a branch of mathematics devoted to the study of qualities of objects that are invariant under certain kind of continuous transformations, with continuous inverse, called homeomorphisms. Examples of homeomorphisms are compression, folding or torsion, twisting and stretching. The exact geometry of the objects, their location and the details of their shape are irrelevant to the study of their topological properties. Topological properties are not linked with proportions, lengths or distances and do not rely on the exact shape of the objects. For example in R^3 a cube and a ball are topologically equivalent because there is a continuous map that transforms one to the other. In the framework of topology a certain object can have an infinity of equivalent Euclidean configurations, independently of measures or curvatures. How do architects use topology? The question can be analysed from the point of view of the objects or of the dynamic process that continuously deforms them. Let us consider a topological object, for example a Klein bottle, which is a surface that can be produced by gluing two Möbius strips together along their edges. It cannot be "embedded" in the tridimensional space without intersecting itself, but we can, however, create its projection which is called an "immersion". The geometry of a Klein bottle could be translated in an architectural language by observing that when entering a Klein bottle we enter an internal space that is exterior to its own body; afterwards we enter in an exterior space that due to its small dimensions gives the impression of an interior space; finally we enter in an interior space that feels like an interior space. We mention two examples where the geometry of a Klein bottle has been evoked: Arnhem Central of UN STUDIO, and the Klein bottle house of Charles Mc Bride (Figure 3). It seems that the underlying idea in both projects is to have a first experience of the building as an interior space of exterior type, after a feeling of an exterior space of interior type and finally an interior space. Observing Figure 3 we note an almost spiral configuration – in fact the first shape that the project assumed - that is passed through itself, unlocking a "new series of relationships and sequential spatial

[4]http://teachingcompany.12.forumer.com/a/33-art-philosophy-and-noneuclidean-geometry_post2322.html

[5]http://query.nytimes.com/gst/fullpage.html?res=940DE0DE1739F931A25755C0A96E948260&pagewanted=all

experience"[6]: a metaphor of a Klein bottle, a topological surface that in fact has no physical realization in the R^3. If references to topological objects are symbolic how do topological concepts influence a project? Simply because objects can be viewed as the result of successive transformations of previous shapes, and also as the initial state of future shapes. For topology it is the same object that evolves in time and, in this sense, architects look at space as dynamic and continuously changing.

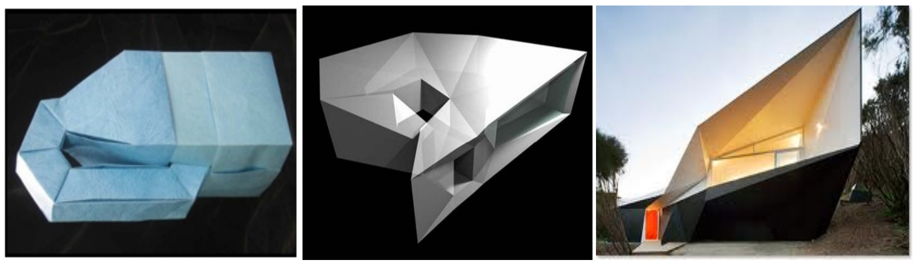

Figure 3: *Klein bottle house of Charles Mc Bride built in 2008 in Rye, Australia*

Conclusion

Since antiquity, architectural composition has always used, in an instrumental manner, proportion relations that are measurable relations. Borrowing the concepts of the so-called "new" mathematics— chaos, non-Euclidean geometries and topology— architects have gained elasticity and are continuously changing the picture of space, shifting the emphasis from an architecture of the object to an architecture of the process. In contemporary architecture such concepts seems to act mainly on a metaphorical level [5] by drastically changing the number and type of shapes in the architectural dictionary. The use of information technologies, computer-assistance in design and manufacturing, permitted the architectural object to emerge as evolutive and as the result of deformations in time. This huge change has been possible because digital media, through the use of mathematical formulae, act as a generative tool for the derivation of shape [6].

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