Geometric Structures and Forms as Visual Art Compositions

Daniela Velichová
Department of Mathematics
Slovak University of Technology in Bratislava
Námestie slobody 17
Bratislava, 812 31, Slovakia, EU
E-mail: daniela.velichova@stuba.sk

Abstract

This paper deals with relations of mathematics and fine art with a focus on geometric structures captured as visual art compositions. It aims to show similarities between mathematical and artistic solutions applied in the creation of a piece of work – artistic masterpiece or mathematical assumption. Some ideas are brought about their common underlying principles and motifs together with the confrontation of scientific and artistic views on the mission of creative processes in mathematics and visual art.

Prologue

What do mathematics and visual arts have in common? Is there anything more than the fact that both work with purely mathematical concepts as structures and compositions, transformations and mappings, and they use objects of various forms and shapes as models? A mathematical proposition is, the same way as any piece of art, the bearer of certain visual information, an artistic work with its own logical structure, meaning and its own unique composition. Abstract mathematical concepts and ideas have special visual representations; they can be geometrically interpreted and their relevance can be documented by the existence of a model, often purely geometrical. Imagination and imaginativeness are abilities supporting creativity and mathematical intuition, artistic compositions and harmonization of shapes cultivate sense of proportionality, arrangement, estimation. Good orientation and spatial abilities are preconditions of success in considerations about more dimensional spaces and abstract mathematical structures. Colours are carriers of additional information, enabling visualization of more dimensional data and complex structures. A little is known about general connections between different abilities of the human brain to perceive pieces of art and to understand mathematical formulas. Is the first one not an illustrative emotional visualisation and model of the latter one? Are they interfering and complementary or passing neglecting each other as not affected parallels? How can we make these two abstract abilities to influence each other utilising thus the synergetic effect of human imagination resulting from their interrelation?

Mathematical Structures in Visual Arts

Mathematics and art are two different languages, which can be used to express the same ideas. Symmetry is for example a concept frequently appearing in both mathematics and visual arts. We are able to perceive signs of symmetry almost intuitively, and these demonstrably play an important role in perceiving beauty, harmony, arrangement and reflection. Symmetry is not just the property of an object or a specific geometric transformation. This concept opens the door to understanding the essence of many abstract mathematical structures that are not necessarily geometric concepts. Commutativity, reflection or transitivity, properties of arithmetic operations and relations, or properties of abstract algebraic concepts as e.g. monoids, semigroups or groups are also closely related to and work with the concept of symmetry.

A visual artwork is related to mathematics if it encodes some mathematical structure or visualizes relations of mathematical entities and concepts, maps views of geometric structures, or embodies mathematical equation determining form of presented geometric figures like curves, surfaces or solids. Algorithmic artwork is recursive, symmetric or periodical. It can be generated by a computer program, but the essence of its creation remains intellectual. Programs generating computer mathematical art are in forms that capture this algorithmic nature of the artwork as an executable specification by means of repeating transformation process. “The beauty of algorithmic artwork lies in its inherent beauty of the algorithmic pattern or symmetry, and its creation process should be done that way. It is a pursuit of elegance that captures essence, and gives us a precise insight on relations.”, as stated in [1].

The Slovak painter Lýdia Jergušová expressed in [3] her own considerations accompanying the process of visualization of her imaginations: “I was not able to answer the question to what extent should be the stylization of presented objects, bird, apple or head in Figure 1, adapted to the ordinary human seeing and perception, and whether it has to be unambiguously readable for observer, or rather ciphered to different selected geometric shapes evoking cognitive secret, mystery and ambiguity. I have chosen the second possibility from the following reasons: presented object should not be similar to the usual logo – pattern common in advertisement and on information boards. And also because the object itself is still unknown, and enriched by a hidden mystery of its unique existence. There exist no two exactly identical apples or birds; none is the exact replica of the other one, each is an original and a unique existence. I had invented a straightforward construction plan, geometrically visualized in the artwork and unique.”

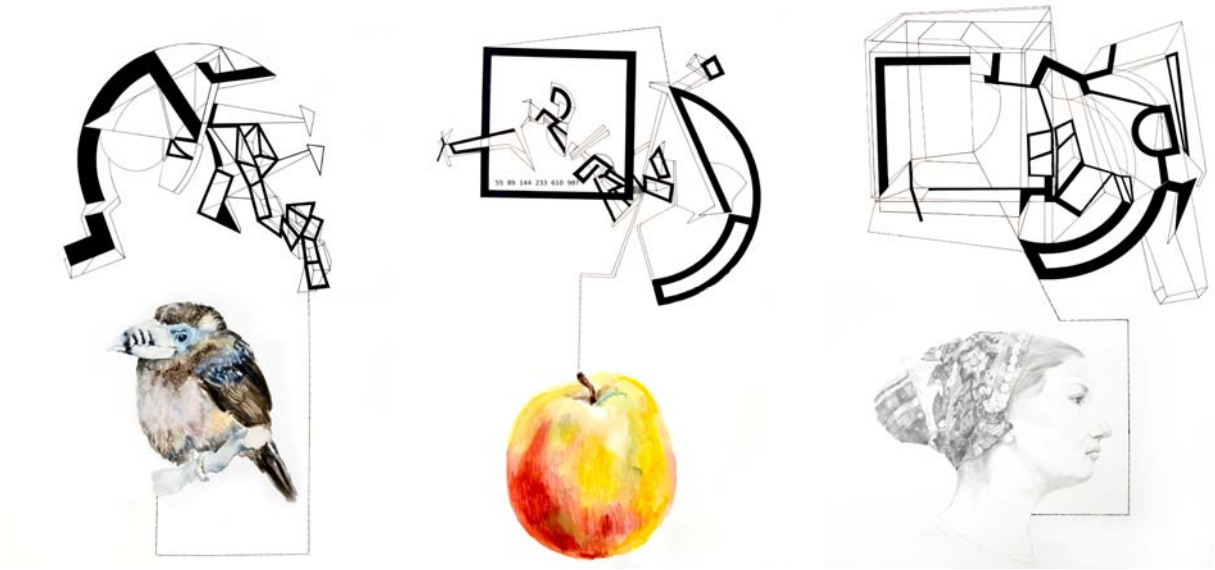

Figure 1: *Lýdia Jergušová: Bird, Apple, and Head (ink drawing, pencil and aquarelle), 2011.*

Art in Geometric Structures

Geometry is part of mathematics, which is most closely connected to visual arts, design and architecture. These disciplines deal with geometric forms and create compositions attacking our senses with their visual appearance. Enjoyment from concordant synergy of elegant geometric shapes, tuning colours and appealing balanced proportions has a strong harmonizing effect on human mind and brings it into the state of consolidated consonance. Geometric compositions demonstrate calming order, proper arrangement and pleasing finalization of our efforts to understand essential principles, core characteristics and underlying relations of the substantial mechanisms controlling regular behaviour of the miraculous life equilibrium.

The idea about a construction of a new object in a geometric space is always based on certain generating principles, which when applied to a particular basic figure, generate a new, usually more complex figure in the same space. Basic figure is subdued to specific modifications articulated by certain mathematical relations, e.g. special geometric (linear) transformations or their classes and compositions, or non-linear deformations and manipulations. These modifications represent constructions in the author's mind, they are synthetic mathematical plans for the creation of an object, which is in this sense a real piece of art.

Eye-pleasing forms presented in Figures 2 and 3 are virtual geometric models of abstract algebraic set operations of Minkowski sum, Minkowski difference and Minkowski product, rendered as enlightened objects. Surface patches are determined as sum, difference and product of positioning vectors of points on two curve segments. Generated smooth Riemannian manifolds are geometric objects with an added artistic value, if particular curve segments, as ellipse, shamrock curve, versiére, asteroid or catenary are chosen. Their complex forms, symmetric shape and conformity guarantee an exceptional aesthetic and pleasing effect of the resulting products. The position of the basic curve segments in the same or in different coordinate planes also considerably influences the form of the resulting surface patches. These structures reside on the boundary between the mathematical and artistic worlds of abstract imagination and creativity though they are created on the base of strictly defined mathematical rules and correspondences.

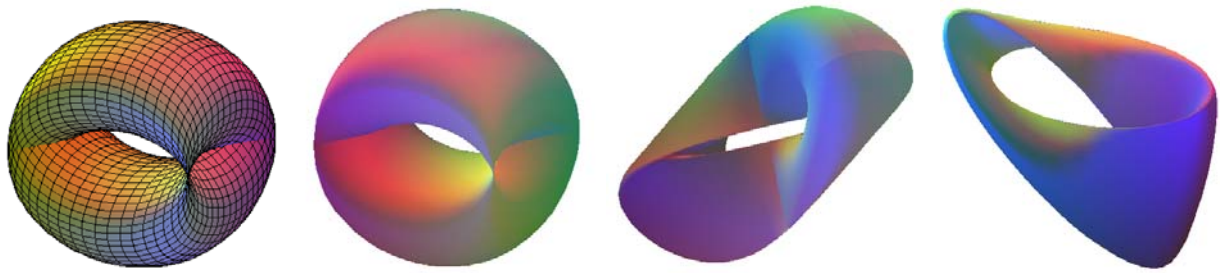

Figure 2: *Minkowski sum (left), difference (middle), product (right) of 2 ellipses in perpendicular planes.*

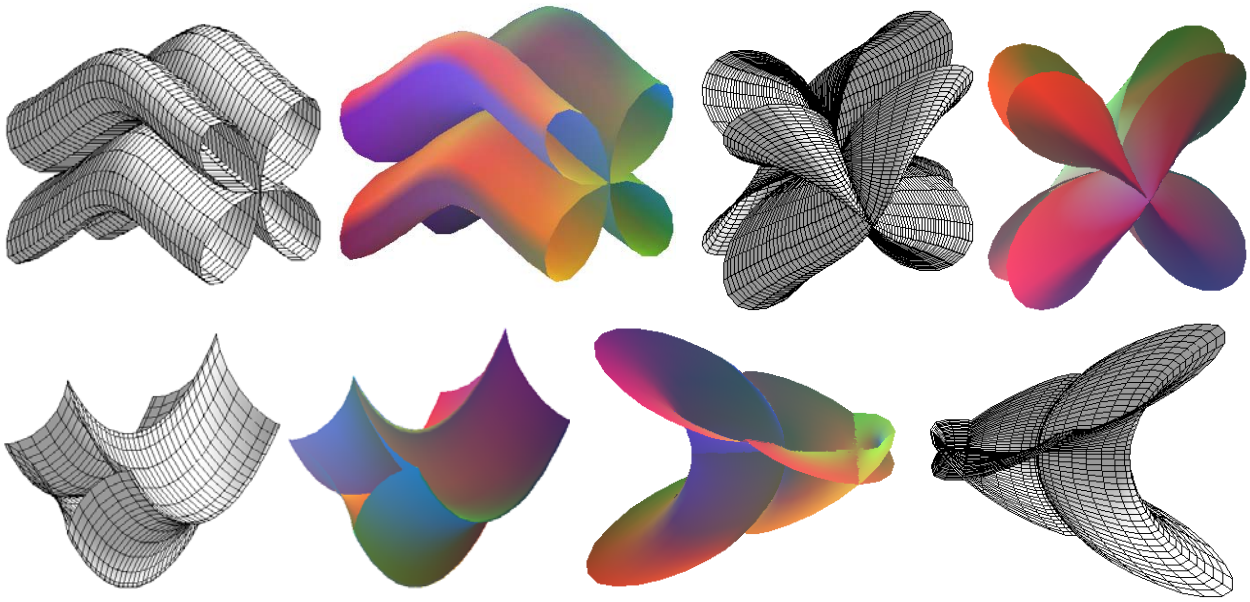

Figure 4: *Minkowski sum and product of shamrock curve and versiére* (*top*), *asteroid and catenary* (*bottom*).

Other examples of geometric compositions with an artistic touch are surfaces determined by simultaneous revolution of a specific basic curve about two axes in the space viewed in Figure 4. Cycloidal patches are generated by revolution of a line about two parallel axes, spheroids can be created by revolution of a circle about two intercept axes, and surfaces of Euler type are created by revolution of a basic curve about two skew axes, as introduced in [2]. Presented surfaces demonstrate powerful underlying construction rules, elegance of the synthetic geometric reasoning and intuition, and visualise abstract relations and connotations in form of virtual models living in the digital environment of computer algebraic systems.

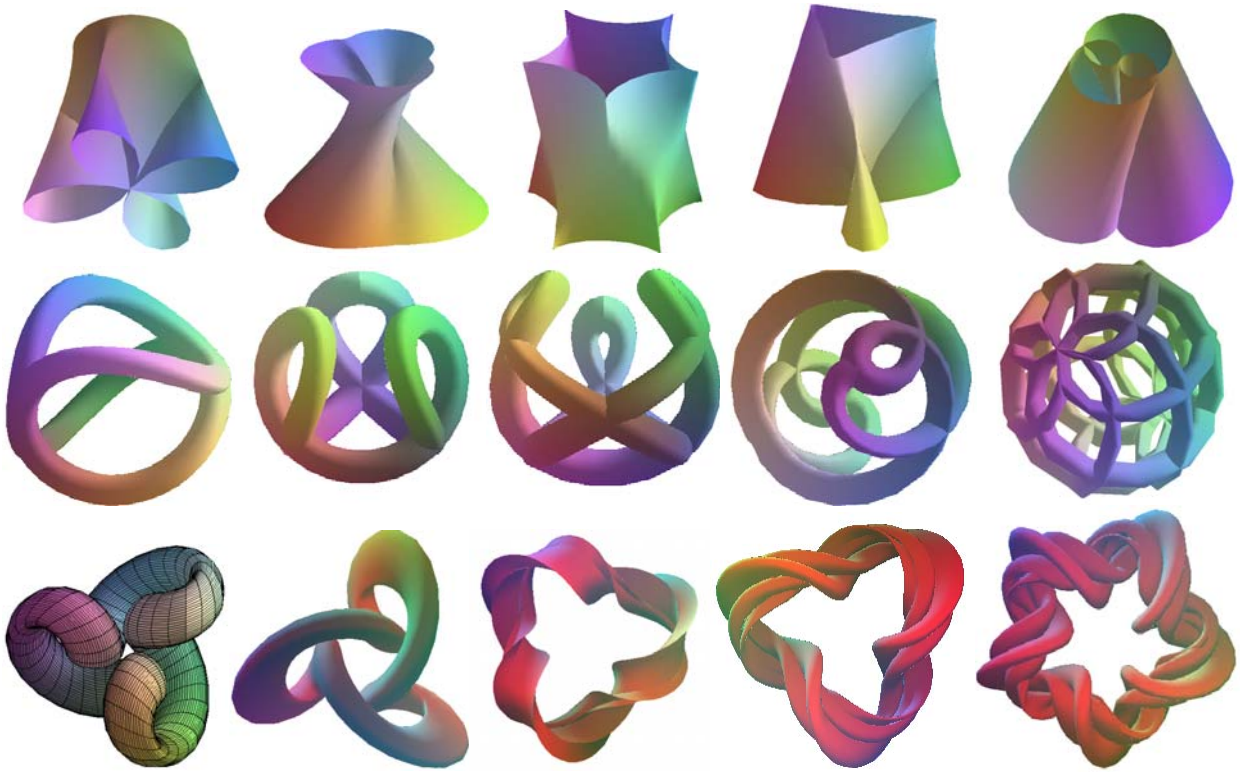

Figure 4: *Two axial surfaces of revolution: cycloidal* (*top), spheroids (middle), Euler type (bottom).*

Conclusions

Mathematical and visual art construction plans are based on similar principles and realise the beauty and elegance of the object in the sense of its uniqueness and inner determining laws. They are revealing substantial details and core characteristics of created objects, and are accessible to perceptive observers. Reward for their efforts to achieve understanding lies in the remarkable experience and delight with the achieved aesthetic impression of the piece of art supported by the underlying mathematical rule.

References

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[2] D. Velichová, *Two-axial Surfaces of Revolution*, KoG, N°9 (2005), Scientific and Professional Information Journal of Croatian Society for Constructive Geometry and Computer Graphics, Zagreb 2005, Croatia, ISSN 1331-1611, pp. 11-20.
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