A Mad Weave Tetrahedron

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Abstract

Mad weave (anyam gila) is a type of basketry originating in Indonesian area. There is very little literature on the technique, and it is not widely used, but it produces a very pleasing fabric, with a symmetry (p6, or **632** in orbifold notation) that makes it suitable for the construction of polyhedra with triangular and hexagonal faces. Unlike baskets (which cannot be used if they have no opening) woven polyhedra are closed structures, and the weaving elements form closed loops. If the polyhedron is woven on the skew to the edges of the faces the weaving elements in general follow complicated paths that are difficult to predict, but on the tetrahedron they are quite straightforward. A skew mad weave tetrahedron with a non-trivial colour pattern is described.

Mad Weave

Mad weave is a basketry technique with weaving elements in three directions (at 60° to each other) woven close together so that an almost continuous surface is produced. Technically it is a twill weave since for any pair of directions the weaving elements in one direction go under 1 over 2 (those in the other direction go over 1 under 2). The weaving pattern is cyclically related: A goes under 1 over 2 Bs (so B goes over 1 under 2 As), B goes under 1 over 2 Cs, C goes under 1 over 2 As (figure 1). The resulting structure has p6 (**632**) symmetry, with small holes occurring at the centres of sixfold and threefold symmetry. It is important to remember the distinction between the two types of hole and the chirality of the weave when trying to understand how it works, especially when trying to create it practically.

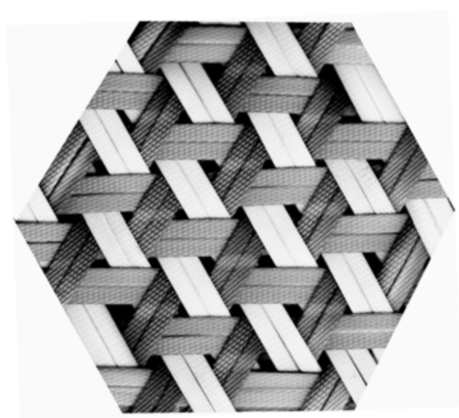

Figure 1: *A sample of mad weave.*

The technique seems to have originated some time before nineteenth century, probably in Eastern Indonesia, and an early account [1] goes into some detail about the preparation of the traditional material (pandanus leaf) and the method of working. The basket starts with six strands around one of the sixfold centres of symmetry, and is built up by working in three directions, braiding, not weaving. This seems to have been the only primary information available because the detailed technical description that follows was worked out by American craft-workers.

Shereen LaPlantz's Method

Probably the most influential description of mad weave [2] is rather difficult to obtain, but it has been reprinted by The Caning Shop (www.caning.com). The author was very well-known as an artist and teacher, and she set out to find an easy way to make mad weave that is accessible to ordinary craftspeople. Her method minimises the many potential sources of confusion by setting down rules that are quite easy to remember, and will guarantee success if they are followed consistently. She begins by weaving the base of her basket, initially as an over 1 under 2 twill in two of the three directions. Her rules rely on strong visual cues, and she is very specific about which directions to use so that the intermediate stages have the right appearance. She then laces-in the elements in the third direction, guided by cues in the existing framework.

The corners of a mad weave structure are the greatest potential source of difficulties. LaPlantz's rules specify exactly how to continue weaving a twill at the corners, and then all that is needed is to complete the fabric by lacing-in the remaining elements.

This method is undoubtedly a very successful way to create mad weave baskets, but the inflexible rules can become a limitation, and they provide little insight into the underlying geometry of the structure. It is also difficult to use with less flexible materials, especially at corners.

Richard Ahrens's Method

Richard Ahrens has attended several Bridges conferences, and many of his works have been displayed in the conference art exhibition. His "Genus 1 doughnut" and "Genus 2 doughnut" (http://www.bridgesmathart.org/art-exhibits/bridges06/ahrens.html) are coloured in a way that illustrates his method of creating mad weave. He begins with the much more common open hexagonal plaiting [3] (figure 2 left), then fills-in the open hexagons.

Figure 2 illustrates how mad weave actually consists of three interlaced open hexagonal structures. The central image shows two of them, and that on the right the complete mad weave. Notice that open hexagonal plaiting consists of open hexagons surrounded by apparently closed triangles. Interlacing another such layer breaks up half of the triangles, creating the characteristic sixfold centres of mad weave, and introduces new ones into the open hexagons. If at each point of sixfold symmetry of the completed mad weave the two interlaced open hexagonal layers are separated in a consistent way so that A always lies above B, B above C and C above A, three interpenetrating surfaces (each defined by open hexagonal plaiting) corresponding to an arrangement described by Rinus Roelofs [4] is produced. In fact it is just mad weave again from another point of view.

Richard Ahrens normally uses plastic strapping, which is quite stiff and very suitable for open hexagonal plaiting, but it is quite difficult to use for mad weave, especially at corners. By first creating a supporting open hexagonal framework his method avoids the serious practical difficulties that Shereen LaPlantz's has with stiff materials. Its major disadvantage is that it can be quite confusing in the intermediate stages, and it is not always very obvious whether to weave a strand over or under. It is fairly straightforward if the original open hexagonal plaiting is obvious (do the opposite of the neighbouring strand in the original structure), which could be why his two mad weave pieces are coloured the way they are. If a different colouring scheme is used it can be far from obvious which strands belong to the original structure except at the start, and other guides must be used. Near the end it is quite easy to recognise the visual cues that LaPlantz uses, but otherwise it is all too easy to go wrong.

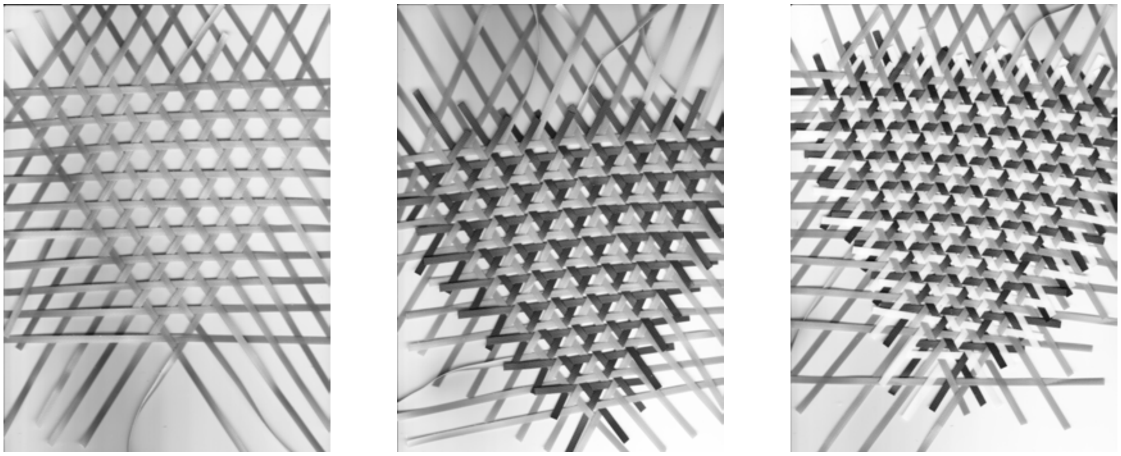

Figure 2: *Filling an open hexagonal structure to produce mad weave.*

Colour Patterns

One of the great attractions of twill is the wide variety of colouring effects that it can generate. Weavers are restricted to a warp and weft at right angles, but nevertheless they can create a huge array of patterns which are also available to basket makers [5] [6]. Mad weave provides further possibilities. LaPlantz [2] considers some with two colours, and David Fielker has considered what might be possible with three colours [7]. There does not seem to be any theory that relates the sequence of colours in the elements to the appearance of the resulting pattern, even in the simpler case of right-angle weaves, and the usual approach seems to be, “try it and see”. In fact beginner-weavers often produce samplers of patterns [8] for their future reference, and in order to develop some intuition for the way things work. There are further complications with mad weave because the structure has a period of three in each direction, allowing more variation in the relative phases.

Generally colour patterns change at corners, but in a few cases it is possible to create a basket that is coloured consistently. For example an open hexagonal basket can be converted to mad weave using a contrasting colour, resulting in a consistent pattern, as in the Richard Ahrens pieces already cited and figure 2. Shereen LaPlantz considers this pattern, and indicates that it is not consistent: her illustrations show that a base with this pattern will give sides that are different. The reason is that it will be consistent only if all the corners have the same position relative to the open hexagons. In other words an edge of the base must have the number of elements in any direction a multiple of 3, so she must have derived her diagrams from baskets of a different size.

Polyhedra

Although most traditional basketry products are open there are a few, typically for use in games, that are topological spheres. Cubes are probably the most obvious, and examples exist with weaving elements parallel to the edges, but they can also be created by plaiting diagonally to the edges [9]. The commonest basketry balls consist of six loops lying in diametral planes, corresponding to the edges of a (more or less spherical) icosidodecahedron.

Triangles and hexagons are the most natural polygons produced with mad weave (and open hexagonal plaiting) although 60° rhombi and rectangles built from modules with √3 proportions are also possible. It is possible to make three Platonic polyhedra (tetrahedron, octahedron and icosahedron) along with two Archimedeans (truncated tetrahedron and hexagonal antiprism) [10]. Richard Ahrens's method is the most convenient, starting from open hexagonal forms. Corners are achieved by reducing the number of elements in a hexagon by doubling a strand for a 5-edge vertex, doubling two for a 4-edge vertex, such as in an octahedron, and doubling three for a 3-edge vertex, so that each strand does a 180° U-turn. The same method works with mad weave, but it will only work at points of sixfold symmetry, not at the threefold points.

Figure 3 shows an open hexagon icosahedron, which can be seen as the traditional six loop icosidodecahedron with some added elements, although there would be problems constructing it in this way because of the difficulty in keeping the lengths right. The existing structure keeps everything in place if it is built using open hexagonal plaiting.

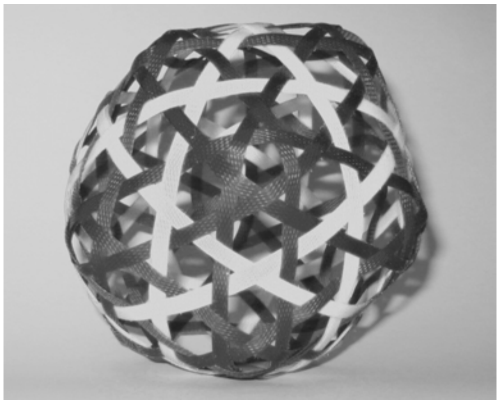

Figure 3: *An open hexagonal plaited icosahedron.*

All of the Platonic polyhedra produced in this way have diametral loops (the tetrahedron has three loops in planes mutually at right angles and the octahedron four) with smaller loops in parallel planes, so that many small lengths are used in the construction. The hexagonal antiprism is rather more interesting, and the simplest open hexagonal construction needs only two strands, so that six are needed to make the mad weave equivalent (figure 4). The open hexagonal framework has strands of opposite colour, and the mad weave is completed by adding in pairs of strands that are opposite in colour to the parallel strand in the base framework. The chiral appearance of the finished polyhedron results from the chirality of the structure.

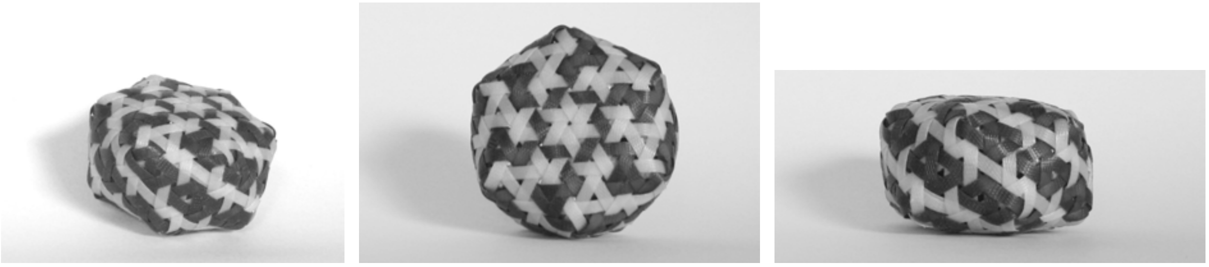

Figure 4: *Three views of a mad weave hexagonal antiprism.*

Skew Weaving

If the principal directions of a weave are skew to the edges of a polyhedron then in general each strand follows a complicated closed path around the surface. Felicity Wood has explored many of the simpler possibilities on a cube [11], and been surprised by the range of different configurations she has found. There is a similar situation with open hexagonal and mad weave polyhedra with the exception of the tetrahedron.

One way of trying to understand polyhedra woven on the skew is to imagine that the weaving is unwrapped by rolling the polyhedron around the appropriate edges in turn on a plane. The weaving element will end up along a line in the plane, and the faces of the polyhedron can be imagined stamping their images onto the plane. The line will pass through every image on the plane. For most polyhedra, if they are rolled around their edges on a plane the face of the polyhedron in contact with the plane at any given position on depends on the path taken. This is because if it is rolled around a fixed vertex, when it comes to its starting position a different face is on the bottom. For example three faces of a cube meet at a vertex but it needs to roll through four squares on the plane to complete a circuit. The tetrahedron is an exception because it turns through two circuits for one circuit of a point in the plane, so that if each of the faces of the tetrahedron were given a different colour, with the colour is transferred to the plane, a uniform colouring of the tessellation 3^6 is produced as the tetrahedron rolls around. Kodi Husimi has observed that the Japanese pattern Kagomé (essentially open hexagonal plaiting) can be produced by rolling a tetrahedron and printing from it the projection of an octahedron [12].

This means that if a tetrahedron is woven on the skew no weaving element changes its direction as it covers the surface, so it is possible to produce a mad weave tetrahedron with exactly three strands (figure 5).

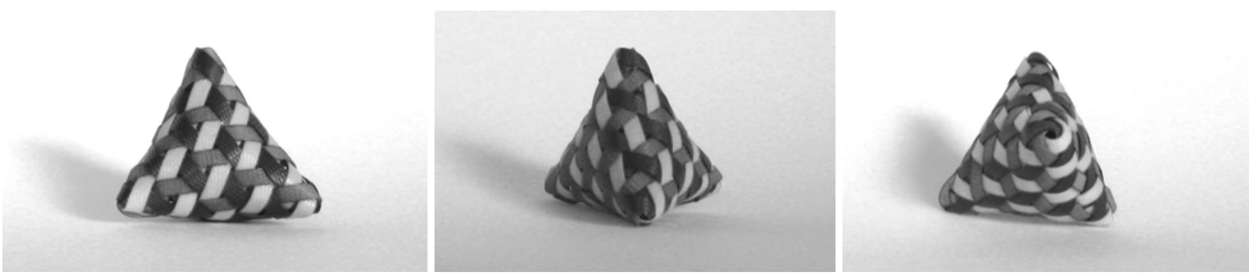

Figure 5: *Three views of a three strand mad weave tetrahedron.*

The view along a twofold axis of rotation (figure 5 centre) shows how a strand takes a more or less helical path around the tetrahedron. Since there are only three strands the only reasonably symmetrical colour pattern has them all different. The elements of symmetry of the tetrahedron either leave the colours invariant (twofold) or cyclically permute the colours (threefold).

These skew structures are quite tricky to produce in practice, and the only method I have found that works is to make a section of flat mad weave, identify the corners and weave the loose ends together, removing elements as necessary.

A Skew Mad Weave Tetrahedron with Non-trivial Colour Pattern

The next simplest possibility is to have two strands in each weaving direction (six in all). This provides some opportunity for more interesting colour patterns, but with reduced symmetry. There are two strands at each edge of the tetrahedron. Consider the three strands lying just on the inside of a face. If there are two colours (one of each at each edge) then they must be all the same if threefold symmetry is to be preserved, but there is no way to do this consistently on all four faces, and there is no way for a threefold symmetry element to interchange two colours. If there are three colours then it is possible to interchange the colours cyclically, but the fact that opposite edges must have the same weaving elements forces each of the remaining three triangles to have two edges of the same colour. The only colour symmetry is a single threefold rotation that cyclically interchanges the colours.

Figure 6 shows such a tetrahedron, and the colouring is surprisingly complex for such a simple scheme. One reason is that the loops are in effect single strands that are doubled, so the sequence of colours in any direction is AABBAABB…but the weaving pattern has an intrinsic periodicity of 3, so the repeat distance of the pattern is 12 strand widths.

Obviously there is a whole series of tetrahedra with an increasing number of elements = $3n$, and if n is divisible by 3 they can be made starting with an open weave framework. Colour schemes will have tetrahedral symmetry (in the same way as the n = 1 case) only if the pairs of elements symmetrical about an edge have the same colour (applying the same arguments as in the n = 2 case).

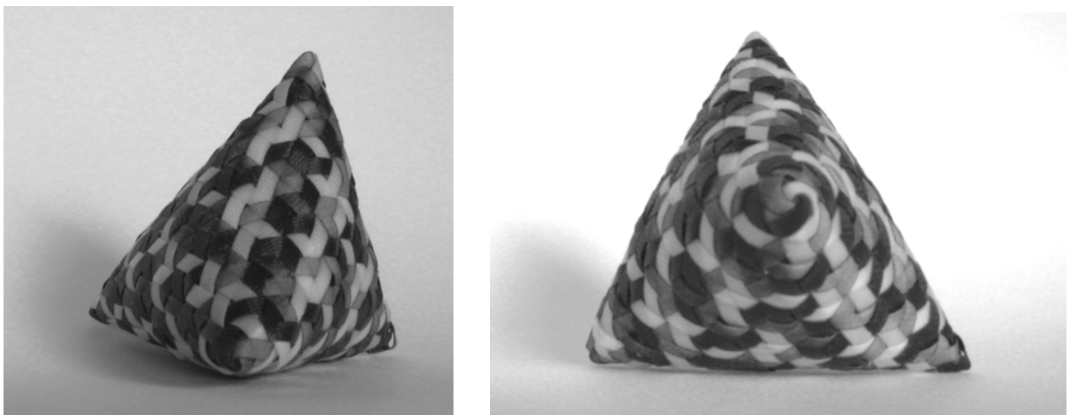

Figure 6: *A skew mad weave tetrahedron with six strands in three colours.*

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