Performing Mathematical Magic

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Abstract

The performance of mathematics on stage is not very common although many mathematicians are amateur magicians. In this paper we discuss mathematical magic as a performing art, given some tips on presentation as well as the description of mathematical tricks that work fine in a mathematical magic show. Everything is contrasted by personal experience performed in different contexts.

Mathematics and Magic

The relation between mathematics and magic lasts for more than 500 years. From Luca Pacioli's manuscript *De Viribus Quantitatis*, where it is described the first card magic trick in the literature, to the last column of Colm Mulcahy in his *Card Colm* section in the MAA webpage (December 2010, at the time of writing this paper), there has been a lot of work showing how mathematical ideas can be applied to different types of magic tricks, involving quick number magic for children, geometrical and topological ideas for stage magic and, of course, card tricks. Among magic writers there are two names that should be especially highlighted for being also well known in the mathematical world. One of them is Girolamo Cardano, who wrote, in *The Subtilitate rerum,* the first description in a printed book of a card trick (is it by mere chance that the first appearences of card tricks in the literature in both manuscript and printed were written by mathematicians?) and the other one is Martin Gardner, recently deceased, who left us lots of material to study.

Most people fond of recreational mathematics know Martin Gardner because of his Mathematical Games column in Scientific American. In almost each one of his books on recreational mathematics there is something related to mathematical magic. The surprising fact is the influence he had in both communities: the mathematical and the magical. In the latter, he is regarded as a magical creator, not only a mathematical magic creator. Although his role as inventor of magic tricks, Gardner tells he was not a performer and that he performed a magic show just once, when he was a student, and in his own words "that was the first time I realized that you're really not doing a magic trick well until you've done it in front of an audience about a hundred times" [2]. To introduce mathematics in culture and to reach the general public it is necessary to use a format similar to the cultural scheme people are used to. In this sense, mathematical sculptures or mathematical photographs are regarded as sculptures or pictures, removing the adjective "mathematical" that frightens some citizens (and we don't know why!). Consequently a mathematical magic show should be similar to a magic show: an amusing performance, enjoyed by the public, with participation of the audience and where the trick is not revealed. It is very difficult to do it since we mathematicians always intend to explain why things happen. In any case, there are always methods for educate without breaking the magic. One of the alternatives is just making a

break, changing your clothes and taking the role of a teacher for explaining or giving hints on some of the previous tricks. Finally, and this is very important, you should make a final great trick.

As well as there are rules of logic and axioms in mathematics, magicians are entitled to follow some rules. There are lots of books on the theory of magic, but we point out reader's attention to classical books written by Henning Nelms, Juan Tamariz and Darwin Ortiz. In those books they approach the theory of magic and showmanship, but in each one of them, in some places, it is possible to find mathematical ideas. In this sense, Ortiz [15] explains that "a magical effect may be very puzzling but it's not a puzzle". He also prevents the magicians to behave as "amateur mathematicians whose hobby is magic". Ortiz's advice is even more difficult to achieve when performing mathematical magic, because we are always tempted to give explanations but it is very important to remember that we are making a show and not a lecture. The other quoted books offer, in between magical theory, some tips for performing the tricks. For instance, in [14] Nelms proposes the use of a real speech on the Monte-Carlo Method as a patter for a magical effect. Tamariz suggests mathematical tricks for the beginners: since they are self-working, the performer only should concentrate on the patter and the presentation of the magical effect. As an example, he proposes the well known "piano trick" [19], in which the magic is made with the words "odd" and "even". His *five points in magic* are useful not only to the performance of magic but to communication in general, even these principles can be applied in the classroom. He recommends paying attention to the way we look at the audience, how we modulate our voice, the movement of the hands, the body expression and the feet position.

The main principles that a magician should remember are *not announcing the effect* and *not revealing the secret.* A mathemagician sometimes will break this rules, but he should be aware of that. Of course they are not universal rules and, in many occasions, the mathematical fact underlying a magic trick is as puzzling as the trick itself.

Outline of a performance

The performance of magic as an art, not only as a curiosity or puzzle, requires knowledge of different techniques. Magic is also called "the queen of arts" as well as mathematics is regarded as "the queen of sciences", so there is another common point between the two disciplines. In our mathematical magic show we must decide whether we prefer giving some explanation of the effects we are performing or not: telling the secret of the magic trick can break the magic, so it is not recommended. But if you choose to explain the secret you can not do it immediately, at least you should give some time before the explanation, in order to let the audience understand what happened. Magic needs its time and we should follow its rules, even when we make a mathematical magic show. I like using slides in the show, presenting the creators of the tricks, aspects of everyday mathematics or bibliographical references. In that way, people interested in your performance can study in depth the subject and the other attendants, at least, will have enjoyed your show.

There is a common advice for a magician when he is about to design his show: "Choose your best trick and your second best trick in your repertoire. Make your second best trick in the opening and finish the performance with your best trick. It does not matter what you are doing in the middle". After more than five years performing regularly in Science Museums and similar places, I think that this advice is essentially right, but it does matter what happens in the middle and I like alternating numerical and geometrical tricks. Usually a venue lasts for 45 to 60 minutes and I prefer involving as many people as I can. Professional magicians often make the same show but they change the public. I prefer changing the tricks from time to time, since if you get bored of performing always the same effects the audience will notice that. Emotions are very important in magic, in the same way they matter in other artistic disciplines

so, when magical literature is read it is useful to imagine the effect and different people will look to the same trick on different ways, according to their own psychology. The best advice is practice, practice and practice.

Without intention to be dogmatic, I present some suggestions for the choice of the tricks in a mathematical magic venue:

1.- Sum Prediction. Six people from the audience are asked to go to the stage. The magician gives a chalk to one of them and deals 4 cards to each one of the other five people. He asks everyone to shuffle the cards and, by turns, asks for one to each person, and the assistant with the chalk writes down its value composing a five digit number. This procedure is repeated three more times. Finally all the four numbers written are summed up. The result of the addition coincides with the prediction made by the magician.

2.- Ring and Rope. Three people are asked to go to the stage. The magician gives them a rope and asks them to examine it. He also gives them a ring and asks to check that it is not broken. The magician ties the ends of the rope to make something homothopic to a circle and hangs it in the arms of two of the assistants. Later the magician inserts the ring in the rope (passing the rope through the ring hole) and tells that the ring will be released in a magical way. He ties even more the arm of one of the assistants, holds the hands of both to prevent the ring going out and finally he asks the third person to hold the ring. Magically it is released when secret words are pronounced.

3.- Change of Color. The magician asks a person from the public to help him. He gives the spectator a piece of paper with sides of different colors. The magician asks the helper to close his eyes. The magician folds and unfolds the paper in a special way. When the person from the public opens his eyes, the colors have changed from one side to the other.

4.- Blind divination. This time the magician chooses a person for helping him and writes the digits from 0 to 9 in a blackboard. The magician keeps his eyes closed for the rest of the show. Later he asks the volunteer to form two five-digit numbers, involving all the ten digits written on the blackboard. Next he orders to sum up those five-digit numbers. Finally the mathemagician asks the assistant to choose one digit in the result, to surround it with a circle, and to tell him every other digit (not the chosen one) in the sum. The circled digit is guessed.

5.- Birthday paradox. The magician ask for six volunteers. He states that he does not know them and that he feels one of those people has something in common with another spectator in the room. He asks, the first volunteers about his birthday. Next he asks if anyone in the room celebrates his birthday in that date. In the affirmative, the trick stops here. In the negative he makes the same procedure with the following volunteer. Surprisingly, usually, one person from the audience celebrates his birthday the same day as one of the volunteers.

6.- Probability. The magician asks two people to go to the stage. He gives them a deck of cards and asks them to shuffle it. They talk about probability and the magician deals some cards to each one. The magician always guesses the color of the cards the spectators are holding. Later he is able to predict even the suits. After that part, he deals more cards and summing up all the values of all the cards in the hand of one of the volunteers it coincides with a prediction. Finally he guesses the cards that the other spectator is holding.

7.- Butterfly cube. The mathemagician asks for 15 volunteers. He gives an origami polyhedron to each one and announces the polyhedra will be transformed into butterflies. The polyhedra are thrown up and hit in their way down. All of the volunteers are able to make the conversion.

Some mathemagical words

The tricks presented in the previous section are only a suggestion following the idea of involving as many people as possible on the stage. Also, that list combines geometrical with numerical tricks. The final one (the "butterfly cube") is a tip. The performance should finish with the best trick and, usually, it is a card trick or a rope trick. In card magic there are really good effects based on hidden mathematical principles. If we are performing mathematical magic it is desired a balance between show and maths. Usually we are interested in the fact that the attendants recognize the mathematical foundation of the tricks. There are a lot of good magic books that use mathematics but keep it hidden (see [9,16,17]). Of course, you can find other mathematical tricks for preparing your show in classical references [5,9,18,13].

The first trick presented, not completely described, is an easy consequence of the commutative property and can be found in [11]. The second one is an adaptation of a Pacioli's puzzle presented in [16]. The idea in this old trick constitutes the basis of more advanced rope and ring, as well as rubber band, tricks. The procedure for getting the ring free is the following:

1. The magician crosses the closer part of the rope on the other part, as described in figure 1, step 1.
2. We insert the loop we are holding in our left hand inside the volunteer's left thumb (figure 1, step 2). We should not release the rope we keep in the right hand until the trick finishes.
3. We move the ring to the right and hold the rope with the left hand as described in picture 3.
4. We put the loop in our left hand in the volunteer's thumb. Releasing the rope the ring gets magically outside it.

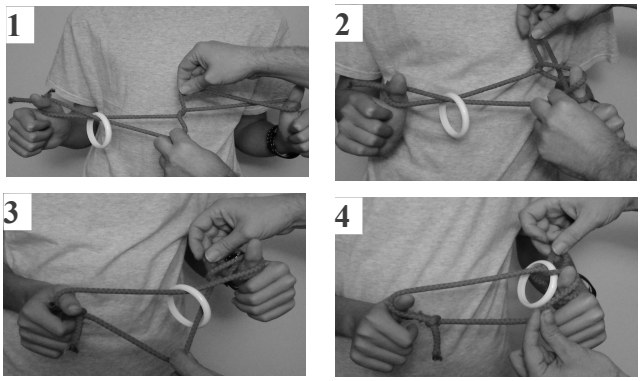

Figure 1: *Description of Pacioli's ring and rope trick.*

Looking carefully to the above procedure we find that if we release the rope we have in our right hand, it will fall from the finger. The movement we make is topologically equivalent to taking the rope outside the hand of the person located at our right.

Change of color (trick #3 in our list) is a topological puzzle, equivalent to "reversing the sweater" trick [7]. I like presenting this trick just after Pacioli's one telling that Pacioli originally posed the above trick as a puzzle, showing the relation between "serious" and "informal" mathematics and the way they have evolved side by side along the history. I also talk about Pacioli's life and his importance in mathematics and about his influence on Leonardo. If possible, it should be good using some slides with Leonardo's drawings in *De Divina Proportione.* Specially the drawing of the cuboctahedron, because the "butterfly cube" represents an origami cuboctahedron. We need a sheet of paper with different colors in each of its sides (the best way to get it is by gluing two different colored sheets). The paper is creased horizontally and vertically so that you get 16 equal rectangles. We cut along the thick lines (see figure 2) in order to make a flap on the paper. Later we make the person helping us to choose one of the colors and ask him to hold the sheet of paper by the flap, having that color towards him and the other side towards the magician.

The magician "shows" the impossibility of changing the color from one side into the other. You ask him to close his eyes and, keeping them closed, solve the puzzle at the same time you ask him to remember which color does he have. When he opens his eyes he gets surprised you were able to change the color. What really happens is something equivalent to get the other side passing through the hole in the paper. Of course, you can not go through, it is the paper who does. It is interesting to remark the change of orientation of the hole in the paper: if it is originally at our left, when folding and passing the paper through the hole it will be at our right. Deliberately we are not giving the puzzle solution here. It is not too much difficult. Try and enjoy it for yourself!

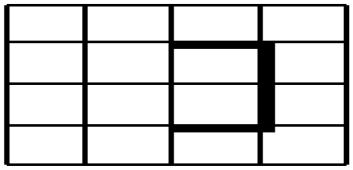

Figure 2: *Change of color.*

Blind divination is one version of a very well known trick based on divisibility by 9. The key is that, when we write two numbers involving the ten different digits and we sum them up, we always get a multiple of 9. I like combining it with the divination of the last digit of a barcode [4]. Both are related to error detecting codes and there are lots of interesting everyday events that can help us to compose the patter: parity bits, check digit in a credit card, letter in euro bank notes, ...

The birthday paradox trick is just another way to present the well known birthday paradox on stage. The surprising thing here is that, for a big audience you should take a very little sample. In a place with more than 100 attendants the probability laws explain why the trick works: the probability of a birthday coincidence under those conditions is greater than 0.84. Most people associates birthday paradox to the number 23, but this presentation improves the effect, even among people used to deal with probabilistic facts.

Probability appears again in the last and best trick in the show. It is described in [1] and it is based in the Gilbreath's principle [8] and a stacked deck following the (mathematical) ordering known as Si Stebbins, although it was first used by the portuguese mathematician Gaspar Cardoso de Sequeira in his book *Thesouro de Prudentes*, published in Coimbra in 1612. In this ordering every card is three values higher from the preceding card, and the suits rotate in clubs, hearts, spades and diamonds, so it results A♠, 4♥, 7♣, 10♦, K♠, 3♥, 6♣, 9♦,..., until we arrive to the last card, that is J♦. Paying attention to the cards sequence it is not very hard to identify the card located in the n-th position (this is used in some other tricks). An easy way to get that ordered stack consists on ordering each suit from ace to king, so we deal the first one: A♠ (the spades packet will have now K♠ as the top card), we continue with the second: 4♥. When we look for it in the hearts packet, we cut it just in the place we locate that card, so 3♥ will be the top card. In a similar way when we remove 7♣ from the ordered suit, the first card will be 7♣, so everything gets prepared for the next deal. This is because we have 4 suits and 13 cards in each suit.

There are lots of mathematical card tricks based on Gilbreath's principle. Usually they are very good tricks and some of them could be good choices for opening or closing the show. Perhaps the easiest Gilbreath's principle application can be made with a deck sorted following the pattern Red-Black-Red-Black-Red-Black... When 26 cards are dealt on the table, one on the other, we have two packets: the remaining 26 cards we have in our hand, sort as Red-Black-Red-Black... and the 26 cards that are on the table, sort in a reverse way: Black-Red-Black-Red,... When we make a riffle-shuffle with these packets, every time we take 2 cards from the top of the mixed deck, they will be of different color. The same

principle applies if the deck is originally sorted by suits or numbers. Since Si Stebbins ordering combines color, suits and numbers, we can show three different effects based on the same principle.
Another topic in mathematical card magic are faro shuffles and should be mentioned here. S. Brent Morris has dedicated his wonderful book [5] to this topic. A faro shuffle requires to split the deck in two packets of 26 cards each and, by pressing them, the cards interweave. These shuffles have important mathematical properties, such as when 8 out-faro shuffles are performed, the deck returns to its original ordering. Morris, as performer mathemagician, also gives some tips for presentation, contributing to the artistic perspective of mathematical magic.

The butterfly ball is usually performed as a tip, once the show has finished. The butterfly ball is an origami model of a cuboctahedron. It consists on 12 modules and was designed by Kenneth Kawamura [12]. It is a strong structure that crashes when it is thrown and hit. Usually I like filling it with confetti providing a nice end to the show. The bad thing is the time you need to build the 15 polyhedra. But it is good for puzzling the audience! Another ideas for origami magic can be found in [6].

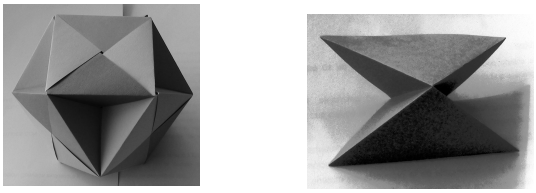

Figure 3: *Origami cuboctahedron and one of its modules*

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