Art in Shadows of the Six-dimensional Cube

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Abstract

The three-dimensional model of more-dimensional cubes can be constructed on a rotational axis and on the joining central point in symmetrical form, based on a regular polygon. An orthogonal projection of this kind of model of the six-dimensional cube shows an image, like the projection of the cube in the direction of its diagonal, perpendicularly to the plane of the image. The projection of any derived (6> j >2)-dimensional solid fits to the network of triangles joined by their sides in this method. The hull of the 6-cube's 3-model may be the Archimedean truncated octahedron as well and the top view of the 3-model of a derivative 3-cube shows a special shadow casted by parallel beam of light. Based on all this, a reconstruction maintaining the topology of the forms made up of cubes, like hinted by the pictures for instance of V. Vasarely and T. F. Farkas, is possible. These hold latent unit mosaics of tessellations and in this manner may inspire to construct geometrical structures of further creations.

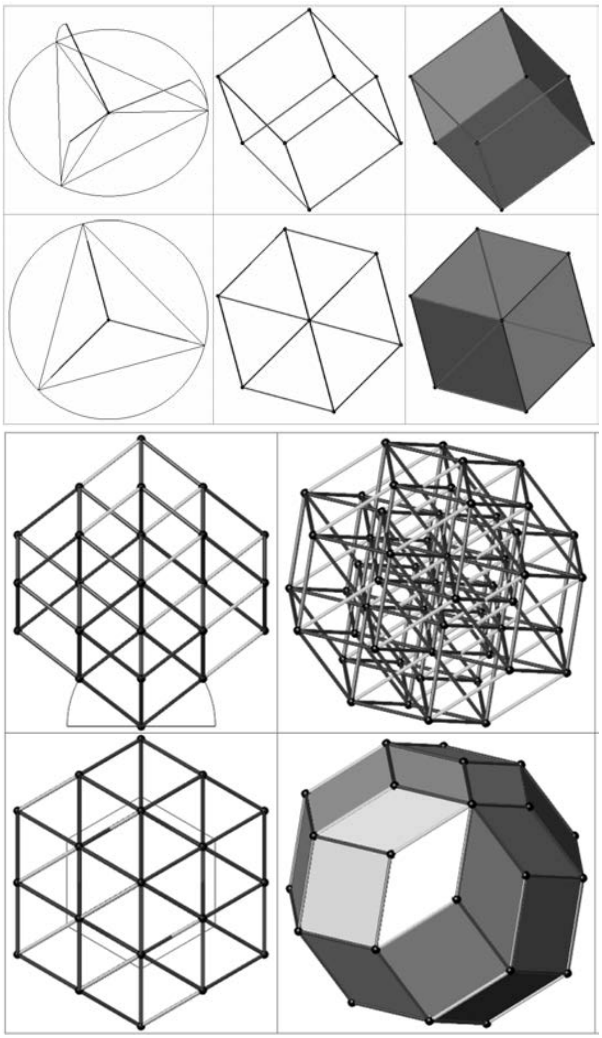
Three-dimensional Models of More-dimensional Cubes and Shadows of Derived Cubes

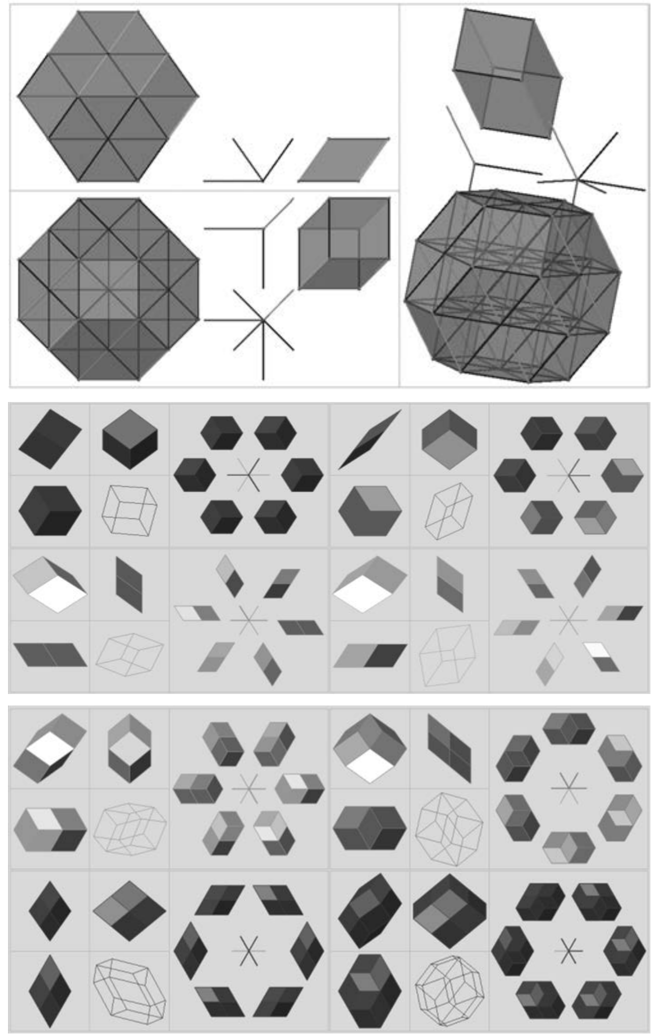
The projection of the cube in the direction of its diagonal, perpendicularly to the plane of the image, creates a special shadow. The images of the edges give the face and half-diagonals of a regular hexagon. This kind of projection of normal cubes joined at their faces fits onto a network of regular triangles joined at their sides (Figure 1).

In general: lifting the vertices of a k-sided regular polygon from their plane, perpendicularly by the same height, and joining with the center of the polygon, we get the k edges of the k-dimensional cube (k-cube) modeled in three-dimensional space (3-model). From these the 3-models or their polyhedral surface (Figures 1 and 2: top, elevation and general views) can be generated by the well known procedure of moving the lower-dimensional elements along edges parallel with the direction of the next dimension [9, 13, 17]. We may think simply about the moved vertex, edge and face of a cube and we gain the 3-model of the 4-cube by sliding the cube along new parallel model edges joining the vertices.

A shadow of this kind of model of the 6-cube shows an image, like the projection of the normal cube described in the first paragraph, independently from the angle between edges and the basic plane of the construction. (Based on the latter, one of the derived three-dimensional elements could be congruent with a simple cube, but the image of this remains unchanged.) The projection of any derived (6> j >2)-dimensional solid fits to the network of triangles joined by their sides in this method (Figures 4-5).

Former proofs show that by the parallel sliding of edges in case of three Archimedean solids, we obtain special 3- models of the 6-, 9- and 15-cubes inside these solids [12]. The hull of the 6-cube's 3-model is the Archimedean truncated octahedron. The top view of a derivative 3-cube's 3-model shows a special image. It is the same one like a shadow of a normal cube casted by parallel beam of light onto the plane of a face of the cube (Figure 3).


Figures 1-2


Figures 3-5

Connection to Arts

The wired shadows of a cube can be interpreted in two ways on behalf of the inner vertices' visibility (middle upper part of Figure 1). The lacking eighth of a cube shown in regular hexagonal shadow as a solid, is perceived either as a negative or a positive form. These phenomena are known in psychology through the experiments of Necker and Koffka [7,16]. Numerous works of art were created with this perception and interpretation phenomenon and the above discussed geometrical base structure. Two-dimensional images often show "impossible" forms, seen as non interpretable in 3 dimensions. We even tend to perceive objects that can definitely be reconstructed as "impossible", in cases of more complex geometrical structures, more so if the applied light effects and "distractional" portrayal tools encourage us to do so. Naturally, this is not always and not only the sole purpose of the work. Based on all this, a reconstruction maintaining the topology of the forms made up of cubes, like hinted by the pictures, is possible. From the transformations of the placement in space, new images can evolve. Further modifications and additional sources of light could help to perfect the illusion but our first aim is to show some examples by spatial reconstructions of Vasarely's and Farkas's pictures. Some of these or their parts are latent unit mosaics of tessellations and in this manner may inspire to construct geometrical structures of further creations. Our 3-models of the 6-cube can be the generalized base of the discussed interrelations.

Three Examples Based on Victor Vasarely's Pictures

The first picture indicates two different oriented, alternating spatial interpretations (Figure 6). Thus we think first of an impossible three-dimensional shape. The first reconstruction is based on models of two derived 3- and 4-dimensional parts of the 3-model of the 6-cube. With our solution, we can create a tessellation as well (Figures 7-9). After further investigation, it turned out, we can have a reconstruction with the same parallelepiped or simple cubic form (Figure 10). It can be the model of the derived 3-dimensional element of the above model of the 6-cube. We can see an example for tessellation with the gained spatial unit in figures 10-12. Other possible arrangements with the same unit can result in statically better solutions to create real two sided frieze- or relief-like products from stones detached by faces.

Figure 6: *Victor Vasarely: Gestalt bleu, 1969 Photo: Xavier Zimbardo [2]*

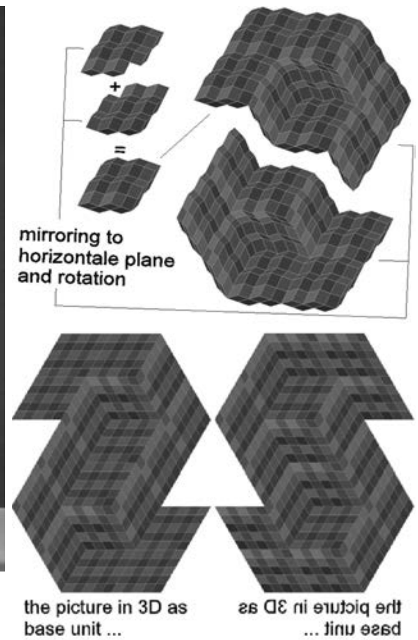


Figure 7-8

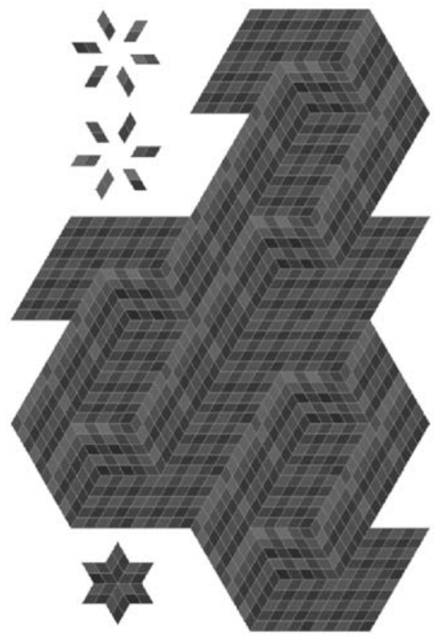


Figure 9: *Tessellation*

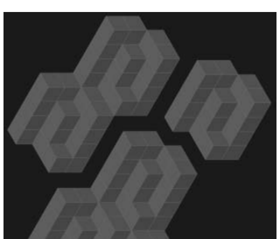


Fig. 10: *Spatial unit*

Fig. 11: *Tessellation in frontal view*

Fig. 12: *Tessellation in backside view*

The 3-dimensional reconstruction of the next picture is constructed from our first model of the 6-cube and from derived elements of this (Figure 13). Taking a wider frame, the gained centrally symmetric mosaic may be repeated periodically by mirrors on the border lines (Figures 14-15).

The hull of the 6-cube's 3-model can be the Archimedean truncated octahedron as well. The top view of the 3-model of a derivative 3-cube is a special shadow described above (Fig. 3). We can build the spatial reconstruction of the third work of Victor Vasarely with this base element (Fig. 16).

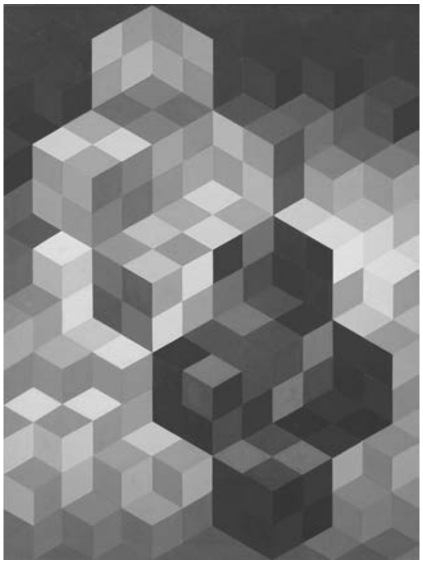


Fig. 13: *V. Vasarely, Duo-2, 1967*

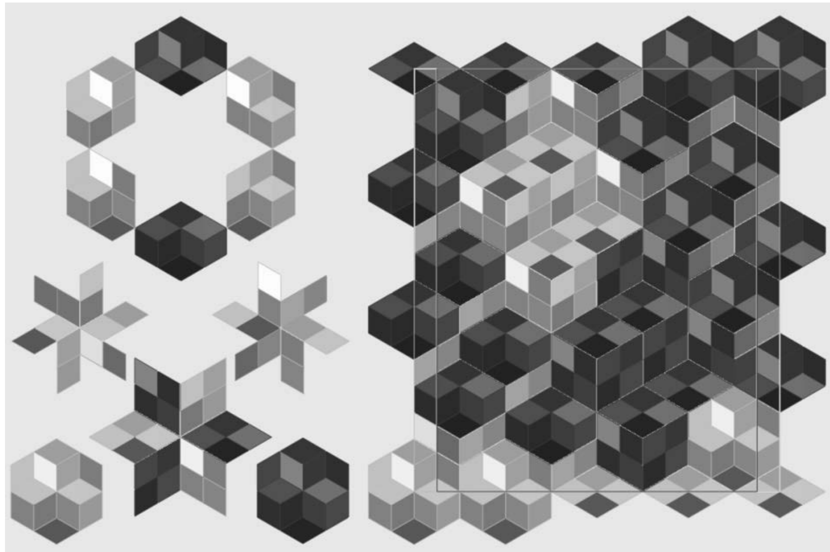


Fig. 14: *Selected elements and the reconstruction as unit mosaic*

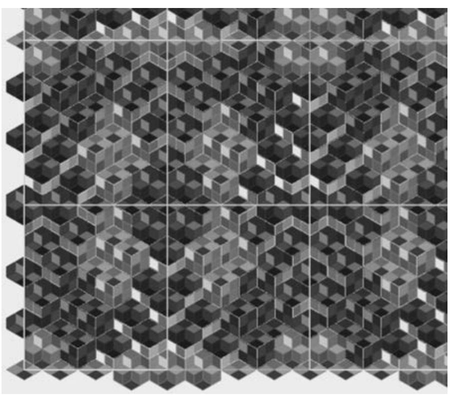


Fig 15: *Some repetitions*

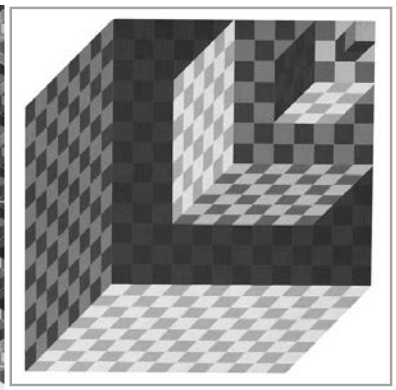


Fig. 16: *V. Vasarely, Felhoe, 1989*

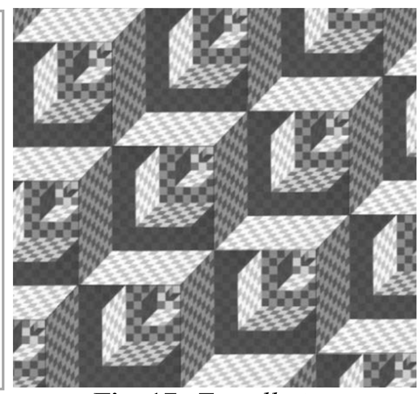
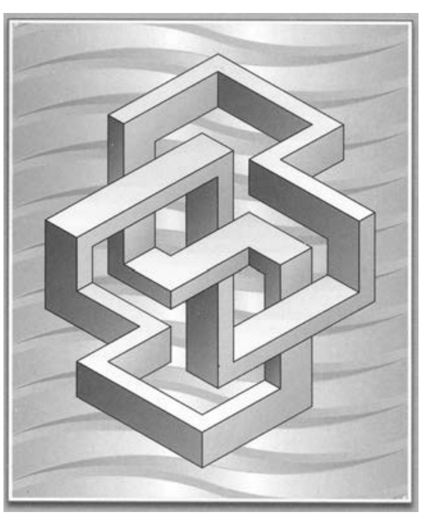


Fig. 17: *Tessellation*

Two Examples Based on Tamás F. Farkas's pictures



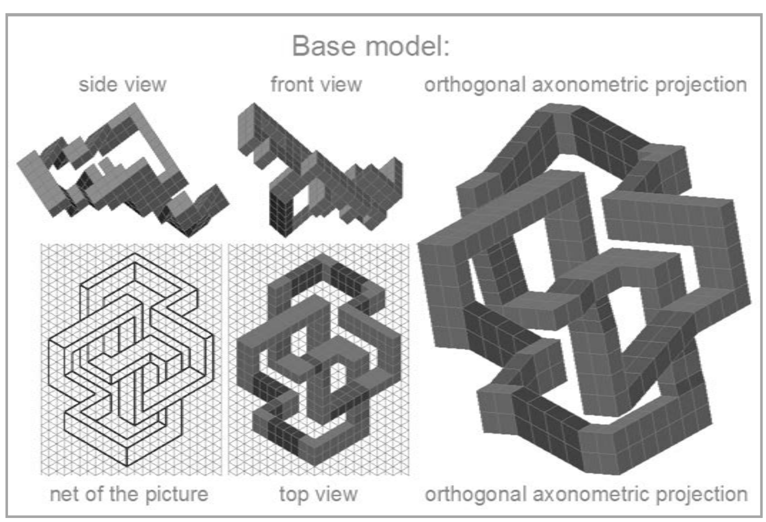


Fig. 18-19: *T. F. Farkas, Picture from a cover art [5] - Spatial reconstruction*

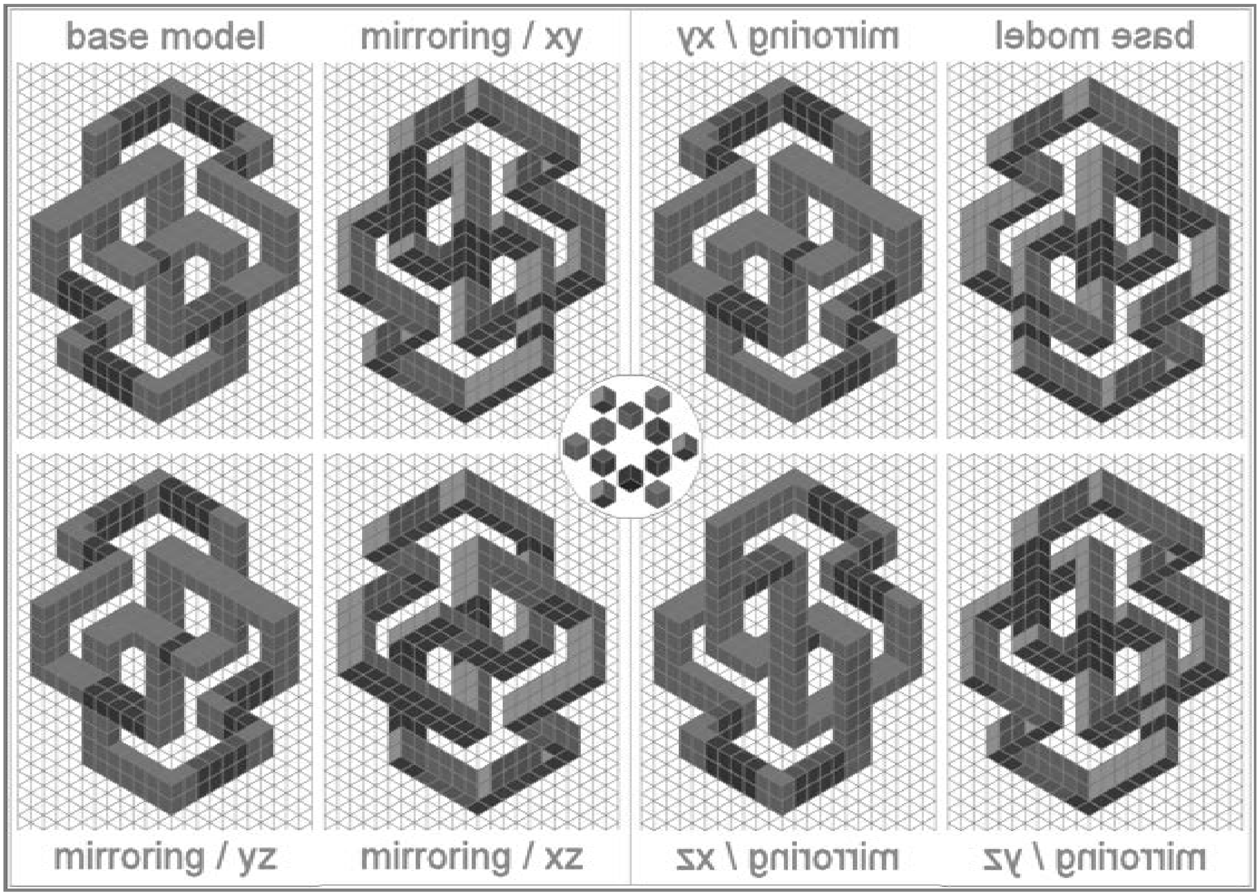


Figure 20: *Spatial transformations of the reconstruction*

We can build the spatial reconstruction of the first picture with two derived 3-dimensional elements of the 6-cube's 3-model based on the former principles. We may gain six different pictures out of the eight ones by mirroring this form to the planes of the rectangular coordinate system (Figures 18-20).

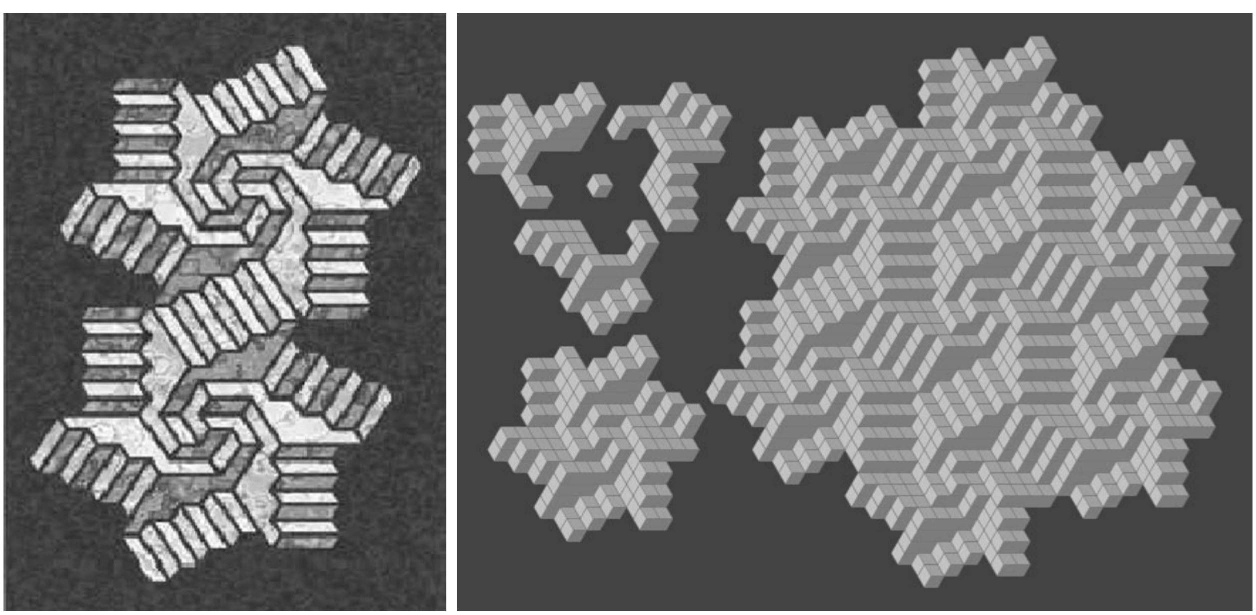


Fig. 21-22: *T. F. Farkas, Tibet I. 1997, (cut) [14] - Spatial reconstruction and tessellation*

The base of the second picture can be reconstructed from a unit mosaic built with only one derived three-dimensional element of the 6-cube's 3-model and we may continue the tessellation (Figures 21-22). The backside view shows naturally a new picture of this periodical tiling. Other possible arrangements with the same basic element can result in statically better solutions built from stones detached by faces.

Remarks

This paper describes some specialties of a wide topic that naturally could not be detailed due to the necessary limit of size. More references listed below can be reached on the internet and may help in studying the foreground by related references as well. The creation of the constructions and figures was aided by the AutoCAD program and AutoLISP routines developed by the author.

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