Phi Divisions of the Square:
A Categorization of Composition Strategies

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Abstract

Artists and architects from ancient times up to the modern period have employed proportions based on Phi, also known as the Golden Ratio, as a system of measures for compositions. While Phi-based compositions typically are employed within an overall Golden Rectangle format, Phi divisions of the square format are far less common, despite the rich compositional possibilities. This paper examines Phi divisions of the square format and a method for deriving a Phi-division pattern for compositional purposes. The author identifies first-order and second-order characteristics of the Phi-division pattern, and how those give rise to four distinct compositional strategies in his square-format painting and digital prints.

Introduction

The Golden Ratio, $\frac{1+\sqrt{5}}{2}$ = 1.618+, usually represented by the Greek letter Phi (Φ), has over the years gained a mystical, nearly magical status in aesthetic interpretation. Indeed, it is usually referred to by one of its more lyrical titles: *Golden Section*, *Golden Ratio*, *Golden Rectangle*, or *Divine Proportion*, and it figures large in studies of *Sacred Geometry*. Such a mystical aura has grown up around Phi that it can cloud its instrumental value for creating dynamically ordered and perceptually engaging compositions. This mystique can lead some idealists to an oversimplified belief that merely the presence of Phi imbues art with an elevated aesthetic beauty; conversely, such metaphysical assumptions may be so off-putting to some realists that they might dismiss Phi or any other geometric system as a basis for making art. In this paper, I shall avoid metaphysics, choosing instead to examine: (1) some geometric characteristics of Phi divisions of the square, (2) compositional potentials inherent in those divisions, and (3) examples of Phi-based compositions in a selection of my square-format paintings.

I shall employ the term *Phi* instead of either numerical expressions or the above-mentioned lyrical terms; similarly, *Phi rectangle* will substitute for Golden Rectangle. *Phi diagonal* will refer to either of the two possible corner-to-corner diagonals of a Phi rectangle, and *Phi ellipse* will refer to the ellipse inscribed in a Phi rectangle (I shall be referring to *quarter-Phi-ellipse* curves). The complete diagram of geometric divisions, whose construction is explained below and which I employ in the development of my compositions, will be called the *Phi-division pattern* (Figure 8). The definition of *grid* will be restricted to a pattern of vertical and horizontal division lines and the corner-to-corner diagonals of the resulting square grid-units; this sense of *grid*, often employed by artists, is limited to four sets of parallel and equally-spaced lines (Figure 1). I shall employ *square format* to distinguish the largest square that is the compositional limit (the “canvas”) from squares that are within a composition. Although my studio work includes both acrylic paintings and large-format digital prints, I shall often refer to them collectively as *paintings*. For full comprehension of color and composition in the illustrated paintings, the reader is encouraged to consult the electronic version of this paper.

Phi and the Square in Art – A Brief Overview

By many scholars' estimations, Phi geometry as a basis for composition in the visual arts has a history that extends back to Classical Greece, and possibly even to Old Kingdom Egypt [1]. Given the dramatic shifts in the purposes and styles of art across these millennia, it is remarkable that Phi geometry has persisted as a creative force right up to the modern period. Although never a widespread phenomenon in modern art, Phi geometry nevertheless was employed by some of the most consequential artists and architects of the 20th century. Perhaps the most recognizable among them was Le Corbusier, whose *Modulor* is a comprehensive measurement system for architectural design, a marriage of the proportions of the human body and Phi by means of Fibonacci numbers. Le Corbusier, a prolific painter as well as an architect, also employed Phi/Fibonacci ratios in his paintings. Juan Gris, the Spanish cubist, was arguably the most inventive painter in the modern period to use Phi geometry. Partly from Gris' influence grew a short-lived movement dedicated to geometric composition in general and Phi in particular, called *La Section d'Or* (The Golden Section), which, in addition to Gris, included such artists as Duchamp, Gleizes, Leger, Lhote, Metzinger, Picabia, and Villon [2].

At about the same time as some modern artists adapted traditional Phi proportions to a new language of art, the square emerged as an important new format for paintings. Although there have been occasional examples of square paintings in Western art history, most paintings up until the 20th century were rectangular in format. By the 19th century in France, canvases were standardized in a series of rectangles of fixed measurements, each associated with standard subject categories: various horizontals for landscapes and marine scenes, various verticals for portraiture and figures. Indeed, many 20th century abstract artists likely adopted the square format to escape the subject associations of horizontal and vertical rectangles. The 1 : 1 format, oriented either on its side as a square or on its corner as a "diamond", was adopted by many pioneering abstract artists, including Malevich, Mondrian, van Doesburg, Albers, Bill, Lohse, and Reinhardt, and it has continued to be employed by hundreds of abstract artists in succeeding generations [3].

From these histories, two points emerge: (1) modern artists who employ Phi geometry do not, as a rule, work within the square format; and (2) modern artists who work within the square format and employ compositional geometry almost always use grids. Grid-based compositions, with their equally spaced divisions, parallel lines, and identical units (Figure 1), are easily distinguished from Phi-based compositions, with their converging diagonals, unequally spaced divisions, and variably sized and shaped units (Figure 2; see also Figure 3 for the initial construction procedure). Many abstract artists who work within the square format, including some of those listed above, prefer grid geometry because the whole-number proportions facilitate easy recognition of quantitative compositional relationships, such as equalities and simple multiples/fractions of areas and lengths. Perhaps even more important, the equal spacing and parallel lines of grid geometry emphasize the two-dimensionality of the picture-plane and de-emphasize the illusion of three-dimensional space, important to the purposes of many abstract artists seeking to escape the representational history of painting.

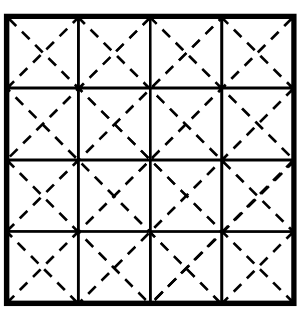


Figure 1: *Grid geometry.*

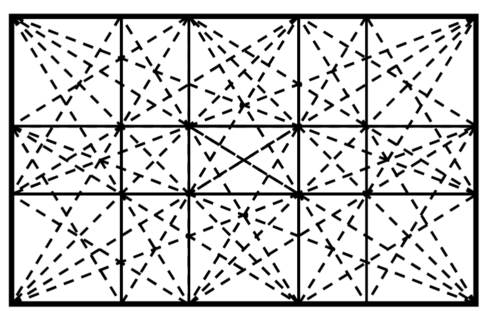


Figure 2: *Phi geometry.*

The square format and Phi proportions almost never converged in the work of modern painters, despite the important fact that Phi rectangles and squares can be intricately linked. Perhaps the most common method for drawing the Phi ratio is illustrated by Figure 3: the area added to the square creates an overall Phi rectangle, and the additional area is itself a smaller Phi rectangle, which can also be divided into a square and a yet-smaller Phi rectangle. Thus, the square is an inherent part of each Phi rectangle, at many different scales. If a square may grow outwardly to become a Phi rectangle, which itself contains component squares and Phi rectangles, then it seems reasonable to divide the square inwardly by Phi.

Phi Divisions in the Square

I first encountered the construction of Phi divisions of the square in Hambidge's *Dynamic Symmetry* [4]. Up to that point, I was familiar only with generating the Phi rectangle by the method of pivoting the diagonal of the half-square outwardly, adding to the dimensions of the square (Figure 3). Hambidge's diagram (Figure 4a) appealed to me as a painter not only because it provides a method for inwardly dividing the edges of a square canvas by Phi ratio, but also because the division could be accomplished with compass and straightedge, obviating the need for calculating in metric or royal units of measure. In this procedure, the half-square diagonal *AB* is cut at *D*, such that *BD* = *BC* (half the length of the square's base); the remaining length of the diagonal (*AD*) is transferred to square's edge, such that *AE* = *AD*, marking a Phi division (*E*) of the square. The rectangle *AEFC* is a Phi rectangle. A subsequent diagram in Hambidge (Figure 4b) shows the process carried out on all four edges, yielding a square divided by four perpendicular lines, each an edge of a large Phi rectangle whose other three edges lie on the perimeter of the square format. These four Phi rectangles overlap and divide one another into smaller squares and Phi rectangles, showing the capability of Phi divisions to generate further Phi ratios. But Hambidge's diagram is cursory, with only two vertical and two horizontal divisions; for my compositional purposes, I needed additional vertical, horizontal, and diagonal division lines.

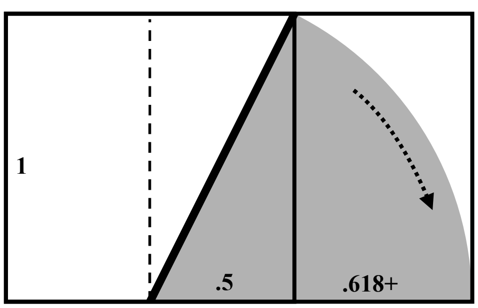
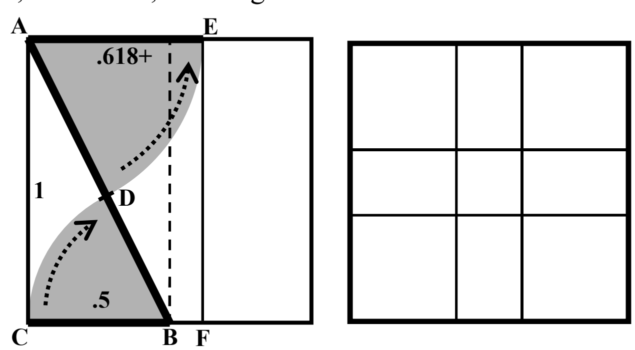


Figure 3: *Phi rectangle by outward extension of the square.*



Figures 4a and 4b: *Phi division of the square format; four Phi divisions (after Hambidge).*

To make more Phi divisions, one could of course repeat the divisional process of Figure 4a on the smaller squares in Figure 4b using compass and straightedge, but that seemed unnecessarily cumbersome. Laying aside the compass, I used the following method employing only the straightedge (for clarity here, the following steps and the associated diagrams are limited to one corner of the square format; each step should be carried out at all four corners): (1) Starting from Hambidge's original divisions (Figure 4b), draw two Phi diagonals in each of the largest Phi rectangles (Figure 5, rectangles *MNOP* and *PQRS*). Two of the diagonals (Figure 5, lines *a* & *b*) intersect at the same point (Figure 5, point *A*) at which Hambidge's two original divisions intersect, forming a smaller square inside the larger square and located again in the corner of the square format. At this stage the Phi diagonals offer nothing new, because the vertical and horizontal divisions were already established by the initial compass and straightedge procedures; but from this point forward, Phi diagonals will provide a method for deriving new vertical and horizontal divisions without the compass. Each corner of the square format may be seen to hold two additional overlapping Phi rectangles (Figure 6, *TUVW* and *WXYZ*), exactly as before but at a smaller scale. (2) Draw two Phi diagonals (Figure 6, lines *c* & *d*) in each of the new Phi rectangles. These diagonals intersect (Figure 6, point *B*) in the same manner as step 1, but again at a smaller scale. (3) Draw vertical and horizontal lines (Figure 7, lines *e* & *f*) through the new intersection, and extend the lines to the perimeter of the square format. Once again, two smaller overlapping Phi rectangles are formed at the corner of the square format. (It should be noted here that lines *e* & *f* could be drawn without lines *c* & *d*, since the diagonals of the largest Phi rectangles intersect the original Hambidge Phi divisions at the appropriate points for lines *e* & *f* ; lines *c* & *d* are included here to continue to develop the full array of diagonals desired for the Phi-division pattern.) (4) Continue the procedures of steps 2 and 3, drawing Phi diagonals for each of the new Phi rectangles and connecting the new Phi-diagonal intersections with vertical and horizontal lines extended to the perimeter of the square format. One can continue this procedure recursively to the level of detail required of the composition. Carried out at all four corners of the square format, the procedure yields the Phi-division pattern (Figure 8), which is the basic metric for the compositional categories and the paintings discussed in the next section.

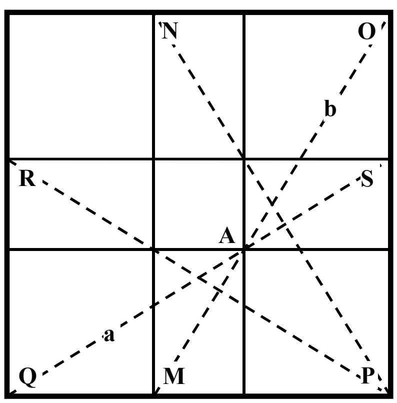


Figure 5: *Phi diagonals of perpendicular Phi rectangles.*

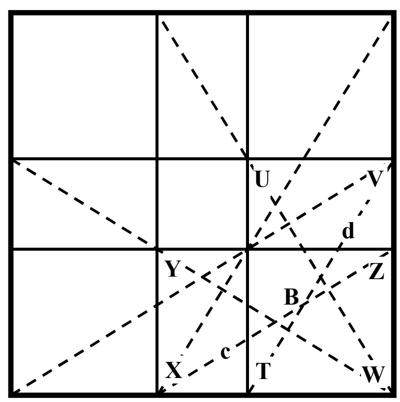


Figure 6: *Additional Phi diagonals, creating new intersection.*

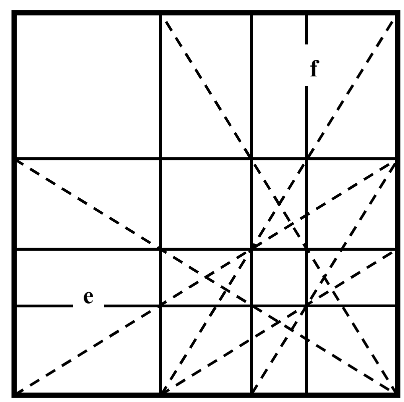


Figure 7: *New Phi division lines at intersection of Phi diagonals.*

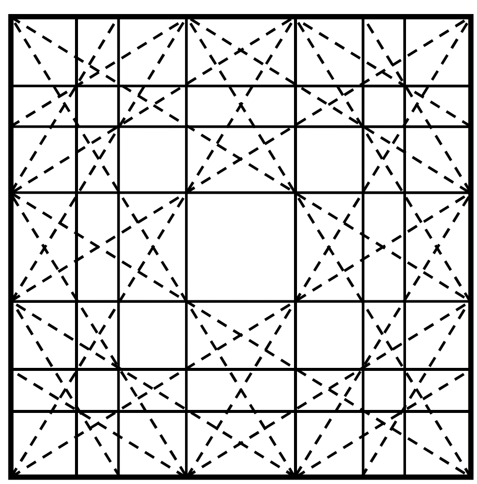


Figure 8: *Phi-division pattern of the square format, applied in all four corners.*

Although I often employ grids for compositions, the Phi-division pattern offers a variety of angles and regularly diminishing/enlarging shapes more suitable for some of my purposes than the grid. Yet that variety in the Phi-division pattern arises from only four types of lines and shapes: (1) vertical/horizontal lines at Phi-ratio divisions, (2) Phi diagonals, (3) squares, and (4) Phi rectangles. (The four longer rectangles midway between the central square and the edges of the square format are not-yet-divided combinations of Phi rectangles and squares.) I consider these four elements of the Phi-division pattern as "first-order" characteristics, the fundamental "architecture" on which my compositions are built.

The Phi-division pattern also offers "second-order" characteristics that guide my compositions. These are reflective, rotational, and translational symmetries of/in the first-order characteristics (the illustrated examples below are limited to reflective symmetry on the diagonal axis). Second-order features include: (1) symmetries without size or proportion changes (Figure 9); (2) symmetries with size changes but without proportion changes (Figure 10); (3) symmetries with size and proportion changes (Figure 11). These second-order characteristics have guided much of my composition work to date; often, my composition process is a matter of selecting sub-patterns from the overall Phi-division pattern. It should be noted here that sometimes I also employ lines and shapes in my compositions that are not explicit in the Phi-division pattern. While I endeavor to adhere closely to the Phi-division pattern, the color and composition requirements of some paintings may call for additional lines and shapes; in these cases, the additions are nevertheless governed by the features of the Phi-division pattern. Such additions might include diagonals of squares, non-Phi diagonals or curves connecting existent intersections, or smaller-scale Phi divisions. It may be useful to consider these additions as "third-order" characteristics.

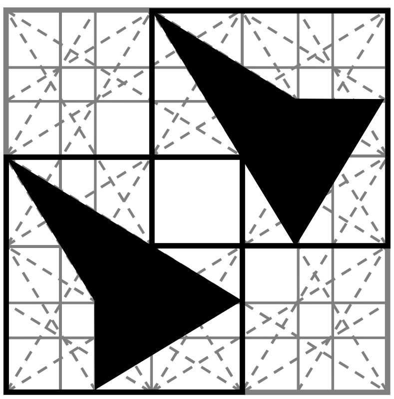


Figure 9: *Reflective symmetry, size and proportions unchanged.*

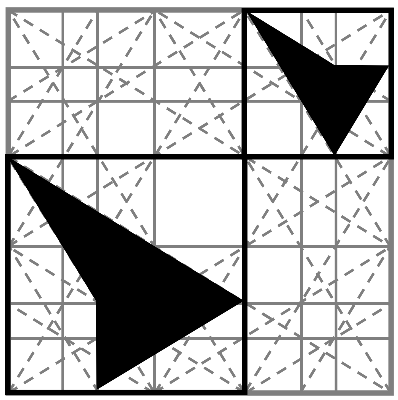


Figure 10: *Reflective symmetry, size changed, proportions unchanged.*

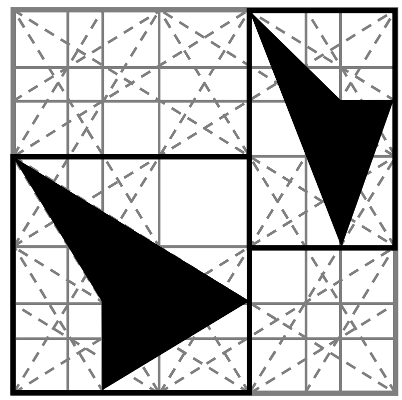


Figure 11: *Reflective symmetry, size and proportions changed.*

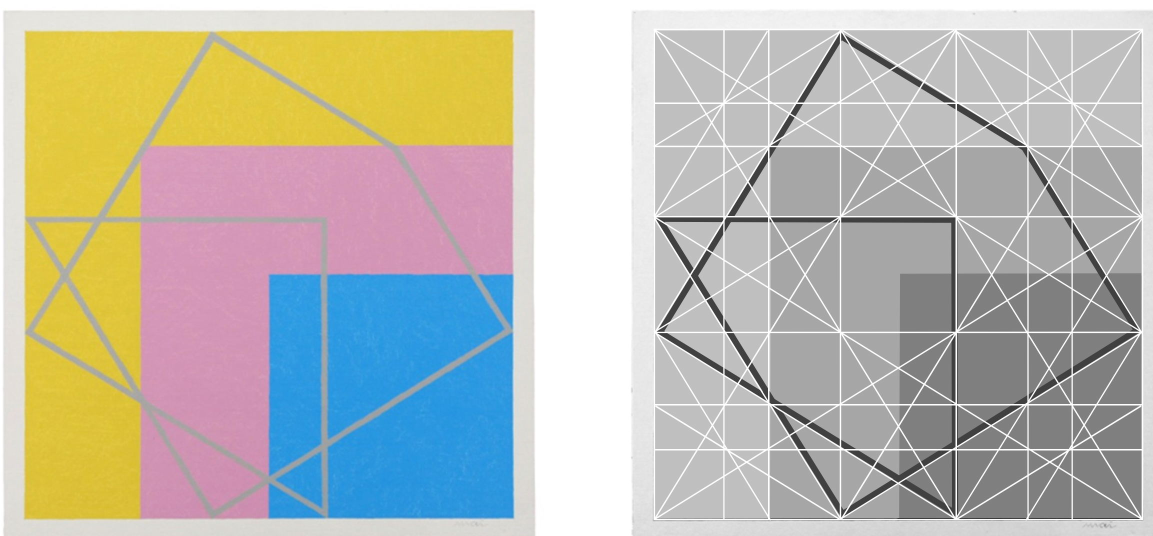
Phi Composition Strategies in Square-Format Paintings

Many of my paintings over the years have been investigations of simultaneous color contrasts (illusory changes of physically constant colors). These color-motivated paintings necessitate specific edge-contacts and proportions among shapes (see [5] for details). The Phi-division pattern has offered effective and fertile possibilities for those compositions. The second-order relationships invite the eye to compare and to discover relationships among its parts, and between the parts and the whole; these comparisons of shapes can further stimulate comparisons of colors and discoveries of their context-specific illusions. The visual discovery of self-similar shapes and proportions throughout a Phi-based composition is something analogous to the experience of rhyme and meter in poetry. Ordinary language, whether verbal or visual, is made extraordinary by the intricacies of enfolded order.

Compositional Categories. Over the course of making dozens of paintings using the Phi-division pattern, I have found four general categories of my compositions. Three of these four categories closely match the second-order characteristics outlined above. The compositional categories are: (1) Scale- and Proportion-Invariant Symmetries, (2) Scale-Variant and Proportion-Invariant Symmetries, (3) Scale- and

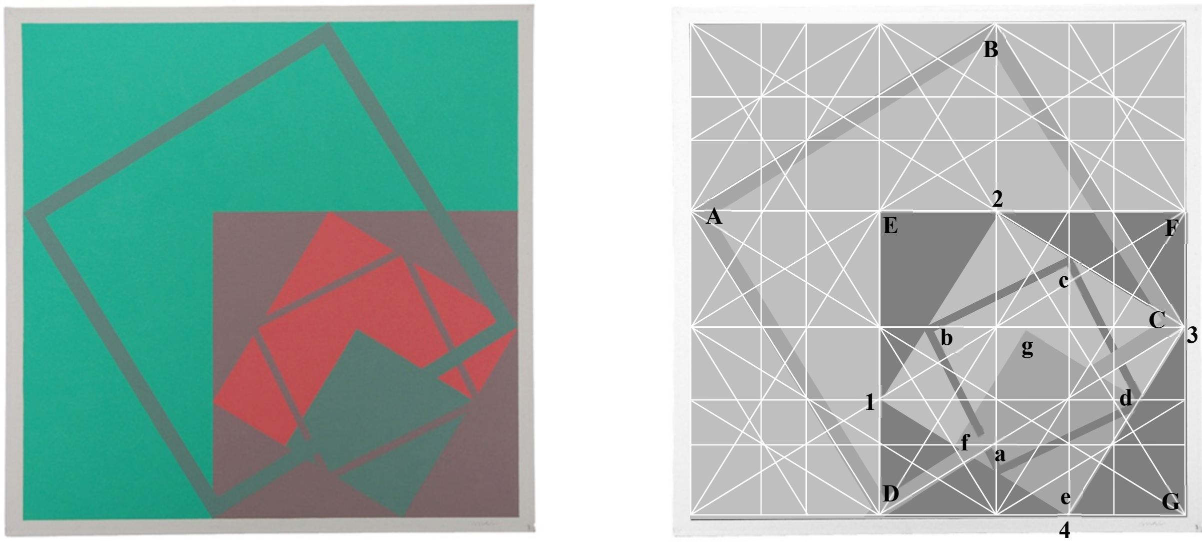
Proportion-Variant Symmetries, and (4) Asymmetrical Loops (Quarter-Circle and Quarter-Phi-Ellipse Curves). The symmetries of categories 1, 2, and 3 may occur as reflections, rotations, or translations, or some combination of those. Although I have produced many compositions from each of these four categories, limitations of space in this paper will permit discussion of one example for each. In the majority of my paintings, it is the color-lines that best display the compositional characteristics of the above categories; the color-areas might be organized by the same compositional category as the lines, or by one of the other categories, or perhaps by none of the categories. In all cases, the color-lines are closed circuits; the circuity of the lines is dictated by the demands of simultaneous color contrast [5].

1. Scale- and Proportion-Invariant Symmetry (Reflective Example). Generally, I do not employ simple reflective symmetries without scalar or proportional changes because the repetition can create a perceptual predictability, even a stasis, which is counter to my compositional purposes. However, the Phi-division pattern possesses diagonal-axis symmetries that are perceptually livelier and more challenging than vertical-axis symmetries, to which our eyes are adapted and which we are accustomed to recognize. "*Counter-Axial (Primaries)*" (Figure 12a), displays a line that is reflectively symmetrical along the lower-left to upper-right diagonal axis of the square format. The color-areas are reflectively symmetrical along the opposite diagonal. The combination of these two simple symmetries creates an overall asymmetry and, more importantly, permits the color-line to cross over the three color-areas for simultaneous color contrast effects. The shapes and lines derive directly from my selections of second-order bilateral symmetries on the square-format diagonals of the Phi-division pattern (Figure 12b).

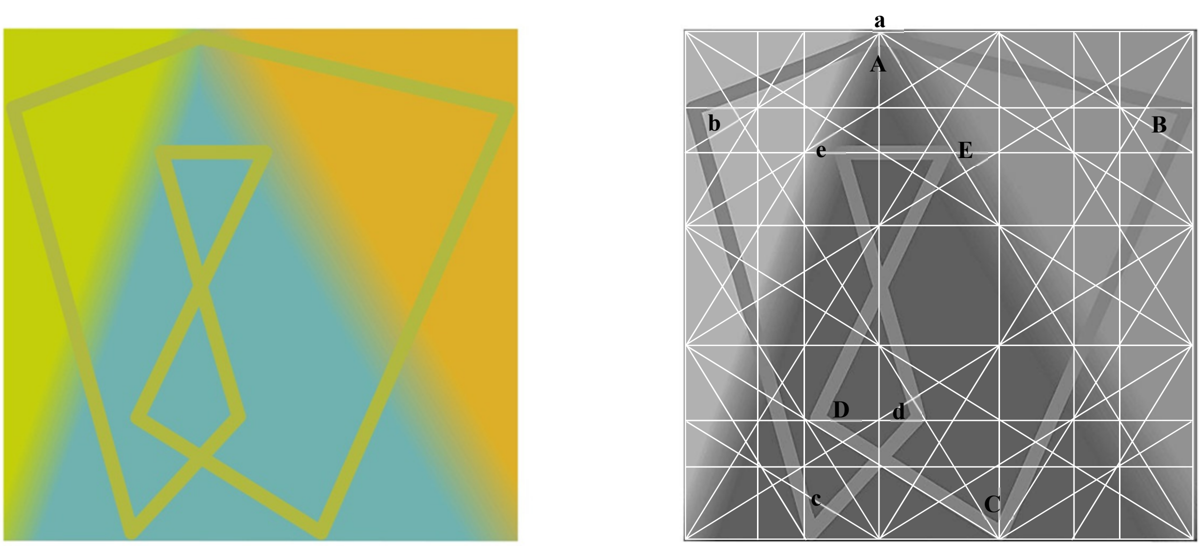


Figures 12a and 12b: *"Counter-Axial (Primaries)" acrylic on hardboard, 24x24 inches.*

2. Scale-Variant and Proportion-Invariant Symmetry (Reflective and Rotational Example). The painting, "*Convergence (Red and Green)*" (Figure 13a) displays a large, L-shaped area of bright green at the upper left corner of the square format, occupied by a dull-green square line (Figure 13b, *ABCD*) that touches all four edges of the square format at Phi divisions. In the lower right corner of the square format is a dull-red square area (*DEFG*) occupied by a smaller, tilted square area (*1234*) made of an L-shaped area of bright red and a yet-smaller square area of dull-green (*defg*); inside that red and dull-green square is a dull-red square line (*abcd*). This smaller square (*1234*), with all of its components, is the same shape configuration as the square format itself, a "microcosm"; compared to the square format, the square *1234* is (1) smaller by Phi ratios, (2) symmetrically reflected on the vertical axis, and (3) rotated counter-clockwise along Phi-diagonals (the vertexes of square *1234* contact the edges of square *DEFG* at Phi divisions). This Phi-based compositional organization permits different but equivalent color effects to operate at larger and smaller scales, while maintaining the same proportions of color quantities across those different scales; that is, the larger and smaller color-lines are in the same proportional relationships to their respective L-shaped and square color-areas. These self-similar relationships are well suited to the proportional constancies necessary for simultaneous color contrasts [5].



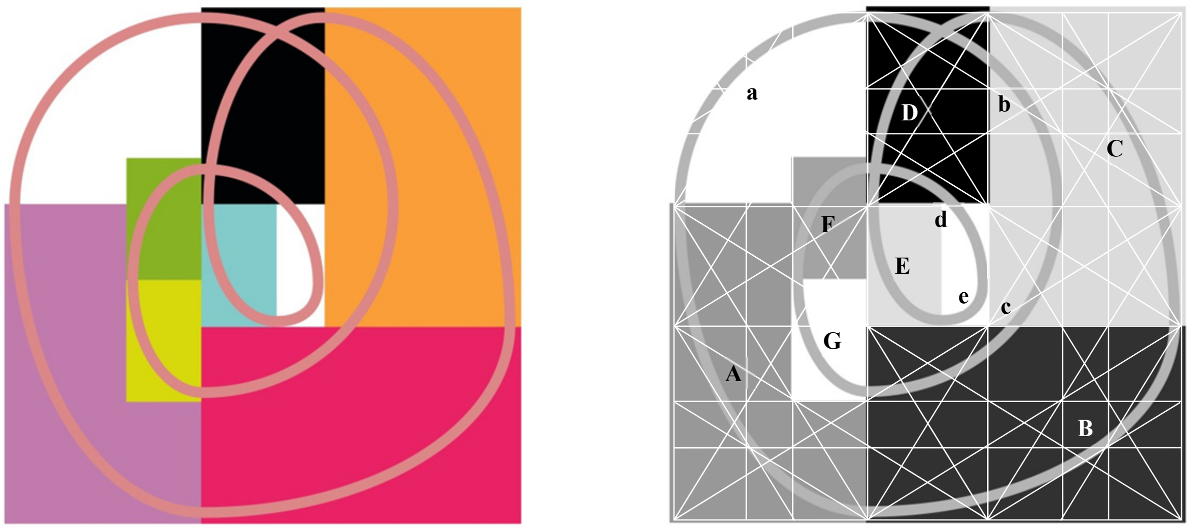
Figures 13a and 13b: *"Convergence (Red and Green)" acrylic on canvas, 32x32 inches.*



Figures 14a and 14b: *"Mirrored (Yellow-Green)" digital print, 28x28 inches.*

3. Scale- and Proportion-Variant Symmetry (Reflective Example). *"Mirrored (Yellow-Green)"* (Figure 14a) displays an angular, yellow-green, closed line, whose left portion (Figure 14b, *abcde*) is a symmetrically reflected and horizontally "squeezed" transformation of the right portion (*ABCDE*). The scale change occurs in the horizontal dimension, while the vertical scaling is unchanged; that is, the vertical locations of vertexes exactly correspond in the left and right portions of the yellow-green line, while the horizontal distances are smaller at left, yielding a related but narrower configuration of lines and angles. The line can be visualized as two overlapping, open quadrangles joined into a closed circuit at two locations: at a vertex at the top edge of the square format (*a/A*), and at the short, horizontal line segment (*eE*) immediately below the top vertex. The top vertex, the horizontal segment, and the two overlap-intersections of this "angular loop" lie on the vertical axis of reflective symmetry, which is located on the Phi division of the square format. The three triangular areas of color in the background share two diagonal edges that extend from the top Phi division to the bottom corners of the square format.

4. Asymmetrical Loop (Quarter-Circle and Quarter-Phi-Ellipse Curves). "*Circuitous (Red)*" (Figure 15a) displays a curvilinear, closed loop made from quarter-segments of both circles (Figure 15b, *a,b,c,d,e*) and Phi-ellipses (*A,B,C,D,E,F,G*). Each curved segment of the color-line is derived from a square or a Phi rectangle in the Phi-division pattern. In this composition, the color-areas behind the color-lines are Phi rectangles of varying sizes (at lower left, the L-shaped area is a larger Phi rectangle overlapped by two smaller Phi rectangles). In other, related paintings, the color-areas are squares or some combination of squares and Phi rectangles. In this and the other paintings from this series, I select combinations of curves that will permit closure of the line as a continuous loop. So far, this process has yielded asymmetrical loops; symmetrical and systematic combinations are still to be developed.



Figures 15a and 15b: *"Circuitous (Red)" digital print, 28x28 inches.*

Conclusion

The paintings and digital prints illustrated in this paper are only a few samplings of the many square-format compositions I have developed from the Phi-division pattern. The process over the years has been more inductive than deductive; that is, the compositional categories discussed in this paper have emerged from, more than they have been *a priori* to, the paintings. There are likely more categories that can be defined in my previous work, and very likely there will be more categories to be developed in future work.

References

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[5] J. Mai, *Compositional Constraints of Simultaneous Color Contrast: Toward a Classification of Types*, Bridges Proceedings 2010, Pécs, pp. 59-66.