Resilient Knots and Links as Form-Finding Structures

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Abstract

The paper explores knots made of resilient filaments. The elastic energy in the cyclic periodic knots causes them to turn into flat symmetric woven patterns. The sequential development of the simplest cyclic knots of resilient material turns them into kinetic structures. The structures can transform into a spatial state and take on the shape of different surfaces. This method of form finding may have application in many practical fields including art, design and architecture.

1. Cyclic Shape of Resilient Knots

From the mathematical point of view, a knot is a one-dimensional curve placed in ordinary three-dimensional space so that it begins and ends at the same point and does not intersect itself. Plane projections or diagrams of a spatial knotted curve may take different shapes with the same number of crossings. A piece of knotted string with its ends joined together is a common physical model of the abstract mathematical knot. The physical properties of the string are not very important. The string must be just long and flexible enough to allow the tying of the knot.

Knots tied with resilient filamentous materials, like steel wire or fishing-line, cannot take random shapes. In this case, the elastic energy of the bent closed filaments tends to take its minimum value and twists the knots in the space forcing them to take energetically balanced shapes. The bending distributes tension forces along the knots, smoothing the loops and causing them to take equal sizes. As a result, the crossings become contact points and knots tend to coincide with the plane.

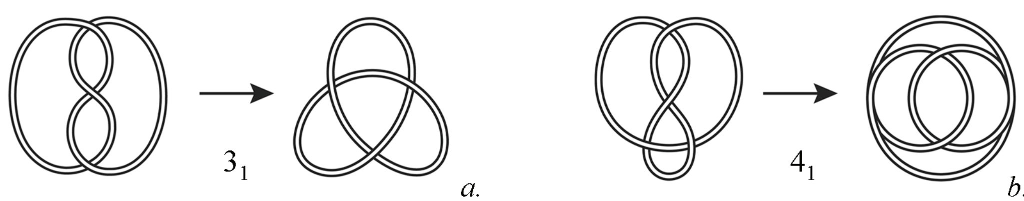

Figure 1: *(a) Trefoil and (b) figure eight knots made of soft (left) and resilient (right) materials.*

The energetic stability of the knotted resilient filaments are closely associated with the values of symmetry of their diagrams. For example, the trefoil of resilient materials with D_2 symmetry tends to turn into the trefoil with D_3 symmetry (Fig. 1 *a*). The figure eight knot with D_1 symmetry made of a resilient material turns into the shape with D_2 symmetry (Fig. 1 *b*). The cyclic shape of the loops and the absence of sharp bends provide the best distribution of the elastic energy in the knots.

The symmetric knots, including torus ones, relate to the periodic type [1]. The alternating cyclic knots have a general name “Turk’s-Heads” and are classified in accordance with the number of their bights (petals) and leads (coils). The symmetry of Turk’s-Heads is the reason for their aesthetic value and application as decorative knots. The application of Turk's-Head knots for braiding of cylindrical, toroidal or sometimes spherical surfaces shows their potential ability to function as form-finding structures. The regular weavings of Turk's-Heads made of soft materials just duplicate the solid 3D surfaces with their contact crossings, but do not function as independent structures.

Turk's-Heads have two basic variants that differ as to their structures. The first one includes the torus-like Turk's-Heads like the trefoil and the figure eight knots, in which loops embrace the central facets of their diagrams. The second variant includes the knots known as chain Turk's-Heads [2], in which loops do not embrace their central facets. Good examples of these two variants of Turk's-Heads are the alternating knots 9_{40} and 9_{41} (Fig. 2 *a*, *b*). In modern knot tables, these knots are depicted as non-periodic diagrams. However, in the knot table by K. Reidemeister (1932) knot 9_{40}, also known as Chinese Button Knot, is represented in the Turk's-Head shape [3].

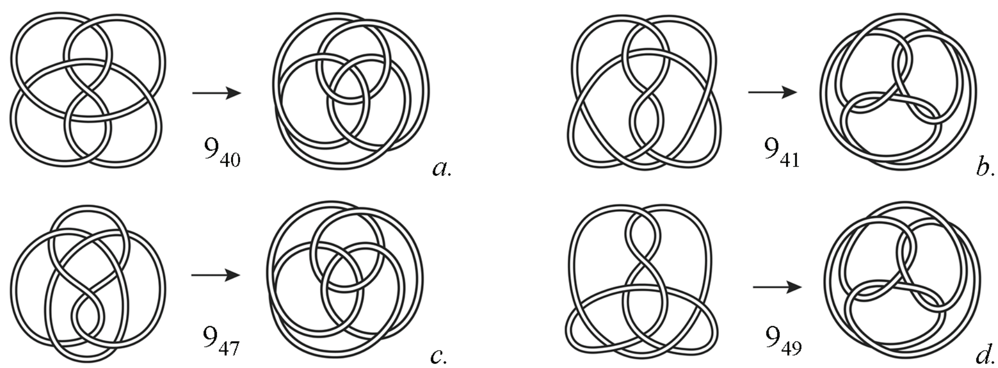

Figure 2: *Turk's-Heads with nine crossings (a), (b) and corresponding non-alternating knots (c), (d).*

The non-alternating periodic knots 9_{47} and 9_{49} (Fig. 2 *c*, *d*) have the same D_3 symmetry as the knots 9_{40} and 9_{41} respectively. A comparison of these two pairs of similar knots made of a resilient material shows that the alternating knots have more contact crossings and form a more stable structure then the non-alternating ones. The chain-like knots (Fig. 2 *b*, *d*) keep the shape of their flat structure much better than the torus-like ones. This is because each of the loops of the chain-like knots tends to stay separated from its neighboring ones, as opposed to the torus-like knots, which tend to distribute the elastic energy between the loops and take the position at the central lines of their ring-shaped regions with a minimum of energy. The cyclic alternating knots 9_{40} and 9_{41} of resilient materials may take spatial positions but they do not have enough contact crossings to stabilize the forms.

2. Development of Cyclic Knots

The force of friction in the contact crossings of the cyclic knots must be sufficient to maintain connection, but not so strong as to allow them to slide along each other. The balance of these two demands is important to turn the flat cyclic knots into kinetic transformable structures. At the same time, the quantity of contact crossings must be large enough to form a fragment of the point surface with uniformly distributed friction forces. The principle of periodicity allows us to go from the simplest cyclic knots to the more complicated ones by means of their development.

To develop a cyclic knot with a small number of crossings, it is necessary to identify the shaping principles of its structure in order to repeat them sequentially. For example, each of the next cyclic knots in the sequence started from the trefoil, is derived by adding one bight and one lead to the previous one (Fig. 3). The quantity of bights and leads grow according to the linear sequences $a_n = n + 2$

(3, 4, 5, 6, 7, 8 …) and $a_n = n + 1$ (2, 3, 4, 5, 6, 7 …) respectively, but the corresponding quantity of the crossings grows as a quadratic sequence $a_n = n\,(n + 2)$ (3, 8, 15, 24, 35, 48 …). Similar methods of knot development are typical for decorative knots and knotted ornaments [4].

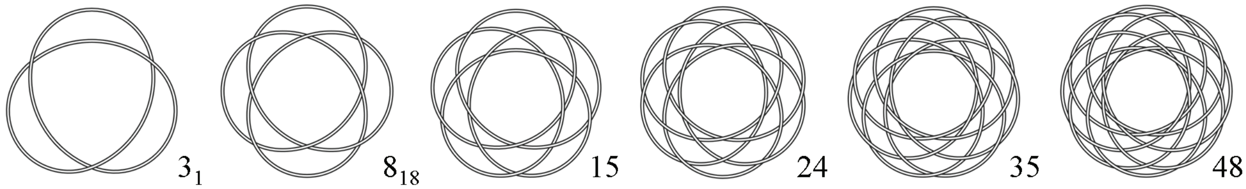

Figure 3: *Sequence of cyclic knots with common structural principle.*

The total quantity of crossings in cyclic knots depends on the number of bights and leads. At the same time, the quantity of *physically contacting* crossings depends on the number of waves on the elastic closed filaments. The number of waves corresponds to the bridge number of knots. A bridge is a linear fragment of a knot that starts from under one crossing, goes over its other linear fragments, and ends by passing under another crossing. The waves play the leading role in strengthening the structure of cyclic knots and the distribution of friction forces. They stretch the fragments of knotted filaments between the neighboring contact crossings of the same sign (the over or under crossings). As a result, the knots work as structures, which are both stretching inside the bridges and compressing at the contact crossings.

3. Transformation of Developed Cyclic Knots

To transform a structure of the developed cyclic knot it is necessary to reduce the size of its perimeter while keeping the size of the center fixed. The distributed forces applied to the peripheral points of the flat structure (Fig. 4, *a*) cause the increase in elastic energy in the knotted resilient filaments and the structure takes the form of a spherical segment (Fig. 4, *b*).

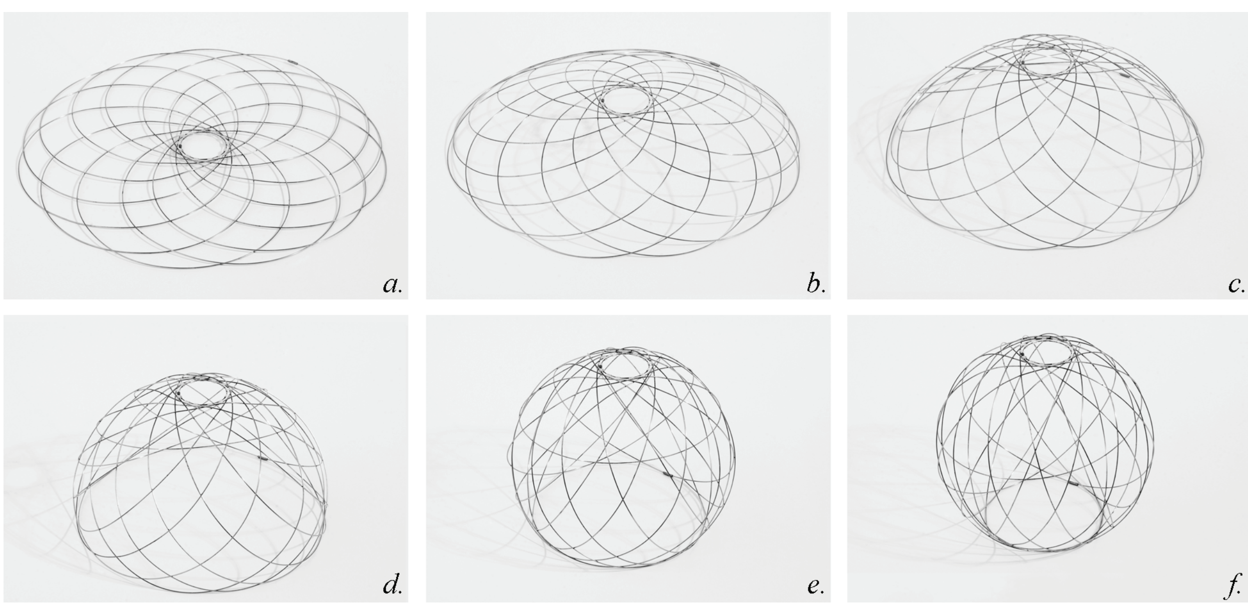

Figure 4: *Stages of transformation of complicated cyclic knot.*

If the forcing is continued, the structure further transforms (Fig. 4, *c*) and successively takes the forms of a hemisphere (Fig. 4, *d*), a truncated sphere (Fig. 4, *e, f*) and finally a sphere, that is the position then the size of the peripheral circumference equals the central one. The structures of chain type cyclic knots have a fixed radius to their ring-shaped areas. In contrast to them, the structures of torus type cyclic knots are not stable and need supplemental fixings, such as rings for their central polygonal facets.

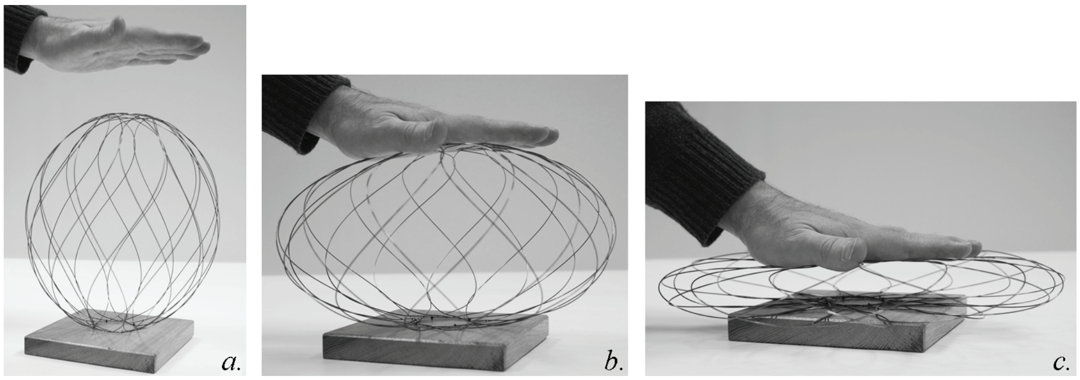

Figure 5: *Flattening of NODUS structure*.

The spatial transformation of cyclic knots consists in the sliding of the contact crossings along the resilient filaments and in the twisting of the filaments around their central axis. This is because the cross-sections of the filaments must be round. The waves on the filaments move and change their lengths to adapt to the current disposition of the contact crossings. The developed transformable cyclic knots created out of resilient materials I labeled "*NODUS*" structures ("*nodus*" meaning "*a knot*" in Latin) [5]. In contrast to the solid models of surfaces, the point surfaces of *NODUS* structures can change their Gaussian curvatures without breaks or folds (Fig. 5 *a – c*).

4. Combinations of *NODUS* Modules

Two chain type *NODUS* structures joined at their perimeters form an elliptic surface (Fig. 5, 6, *a*) and joined at their centers form a hyperbolic one (Fig. 6, *b*). These structures can be transformed one into another due to the phenomenon of torus rotation. A torus can rotate around its central circle axis, interchanging the outer and the inner parts of the surface. The *NODUS* structure, as the outer (elliptic) or the inner (hyperbolic) parts of the torus, can also rotate around the circle axis of an imaginary torus. The structure can turn inside out and take the forms of the outer and the inner parts of the torus surface.

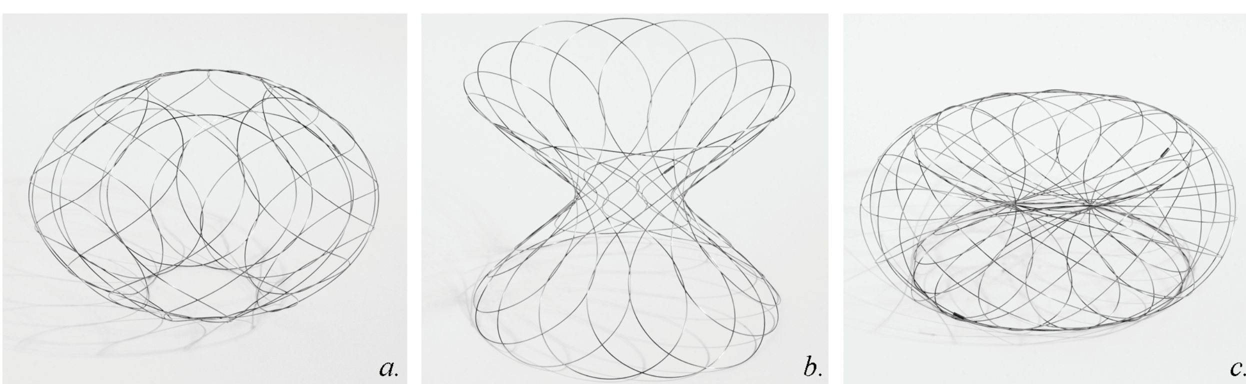

Figure 6: *NODUS structures of elliptic (a), hyperbolic (b) and combined (c) curvature*.

The complete surface of a torus is the linkage of two *NODUS* modules of the elliptic and hyperbolic curvatures (Fig. 6, *c*). Another method of making a torus is to join two torus type *NODUS* structures by both their central and peripheral circumferences. The received structure consists of two (or any other even number) linked mirror torus knots (Fig. 7, *a*). The volume increases by compression of the inner hyperbolic part of the structure (Fig. 7, *b*).

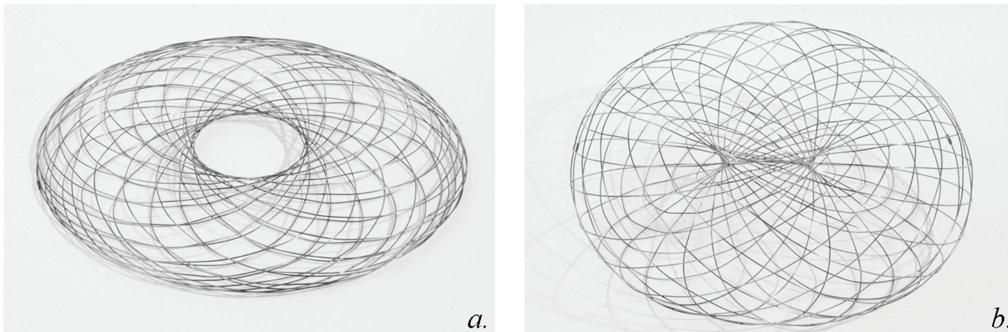

Figure 7: *Torus surface as a linkage of two torus knots*.

The pretzel surface with two holes is a combination of two torus structures on a plane (Fig. 8, *a*, *b*) and result from a linkage of two (or any other even number) pretzel knots.

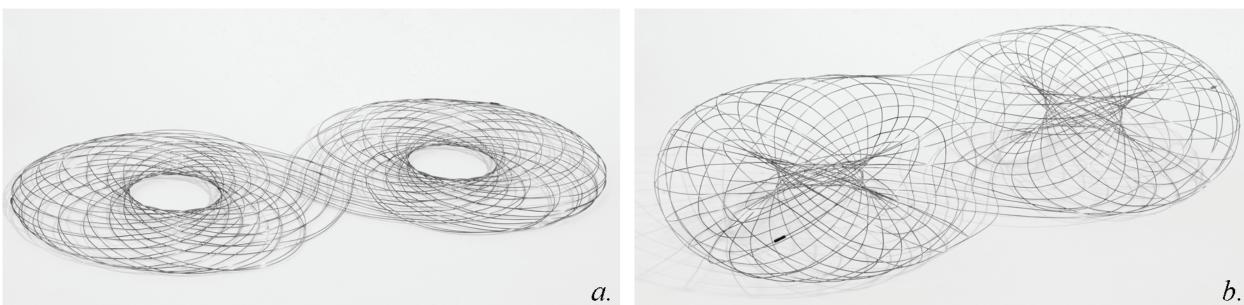

Figure 8: *Pretzel surface as a linkage of two pretzel knots*.

The method of multi-layered combinations of the *NODUS* modules make it possible to create many other forms, for example surfaces with different numbers of self-crossings (Fig. 9, *a*, *b, c*).

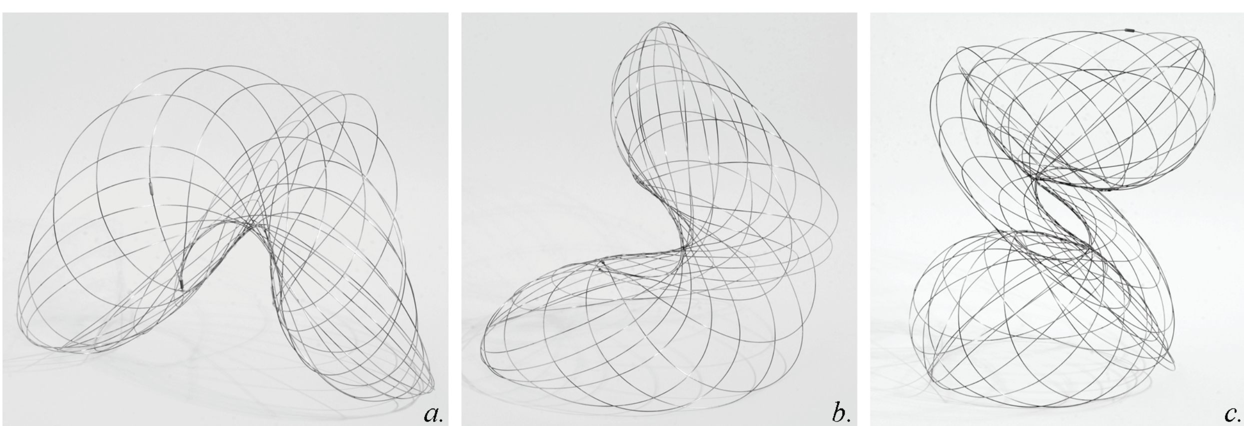

Figure 9: *Combinations of NODUS modules with one (a), (b) and two self-crossings (c)*.

The knotting of the ring-shaped areas of the *NODUS* modules gives one more direction for this form-finding method. The two concentric circle edges of the ring-shaped modules turn into the two concentric knotted edges of the knot-shaped ones. Multi-layered knotted surfaces with self-crossings emerge as a result of this combination of *NODUS* modules (Fig. 10, *a*). The two concentric circle edges of the ring-shaped area may also be joined together to form a single double-covered circle that produces an edge of the one-side surface with one self-crossing (Fig. 10, *b, c*).

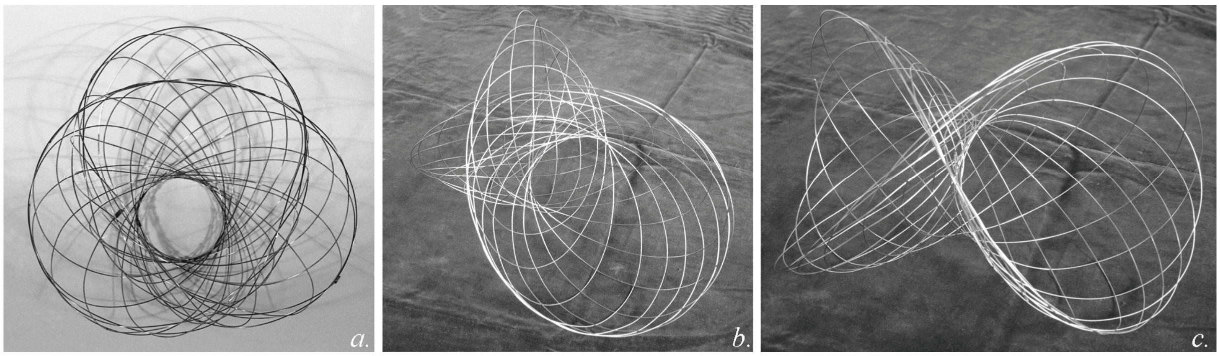

Figure 10: *Knotted surface with three self-crossings (a) and one-side surface with a self-crossing (b, c)*.

5. Large-sized *NODUS* Structures and Practical Applications

The *NODUS* structures shown in the photos above were made of spring steel wire about 1 mm in diameter. The sizes of the structures are not more then 30 cm which permits transforming them by hand. The transformation of larger structures, however, demands a regular distribution of compressed force on the peripheral points. To transform large-sized *NODUS* structures, I designed and built a special mechanism (Fig. 11 *a* - *f*). It transmits the force of the rotating horizontal spindle to the vertical jackscrew, which is connected with 18 rolls placed on the circle 1 m in diameter. Rotation of the wheel handle transforms the flat *NODUS* structure into a spherical one and undoes it, as well. Although this mechanism was initially used as just research equipment, it has also been displayed in several art exhibitions as a piece of kinetic art.

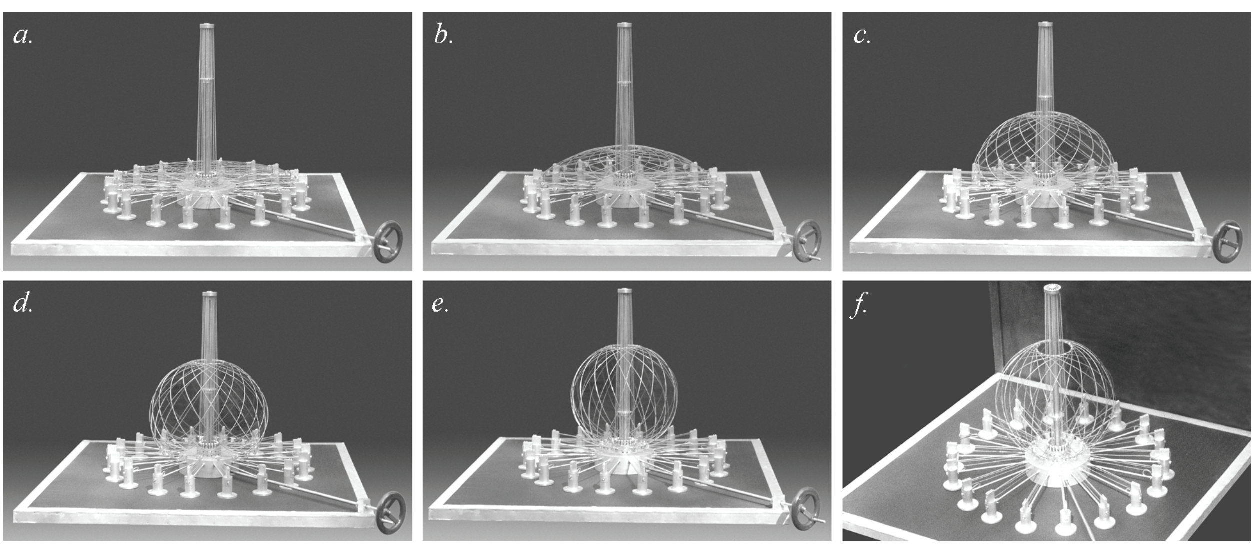

Figure 11: *Mechanism for transformation of large NODUS structures*.

The material of the large-sized *NODUS* structures must be resilient and light at the same time. Large structures of steel wire are too heavy to carry their own weight. It is better to use some modern resilient materials like plastics or fibro plastics. To test the possibility of assembling and erecting a large *NODUS* structure, I undertook an experiment with the structure of a torus like Turk's Head knot that was 6 m in diameter (Fig. 12 *a* – *f*). The material involved was PVC tubes, 4 m long and with a 16 mm cross section. The assembling method was the consecutive butting of the tubes beginning with the central fixing ring (Fig. 12 *a*).

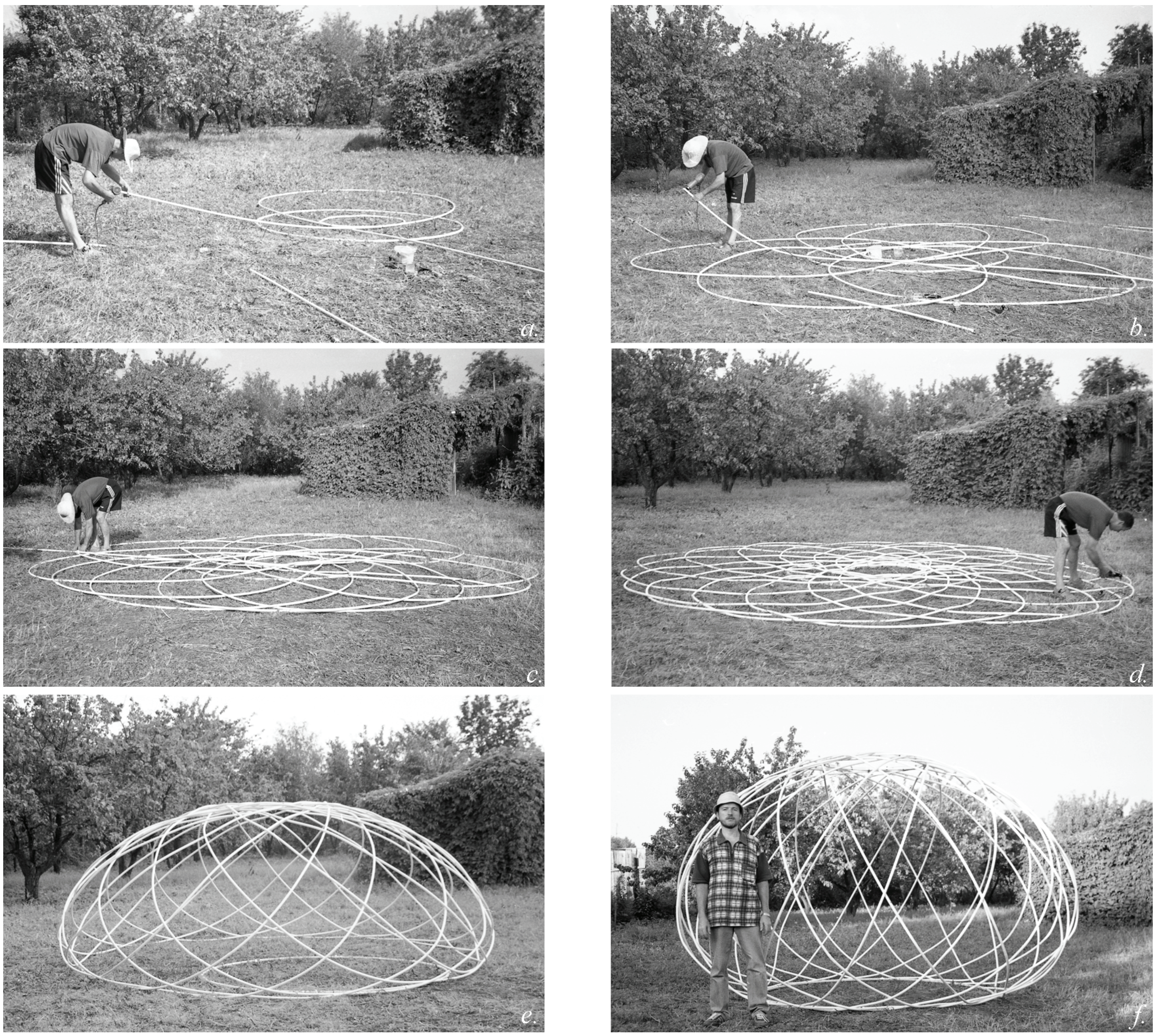

Figure 12: *Experiment with large-sized NODUS structure of PVC tubes*.

To control the correctness of the knot pattern, I used a small-scale diagram drawn on letter-sized paper. I did not draw any diagram on the ground. Then I went along the knotted line and fixed every finished loop with the central ring (Fig. 12 *b*). At the same time, I wove the tubes to make the contact crossings and the waves (Fig. 12 *c*). When the flat *NODUS* structure was completed (Fig. 12 *d*), I lifted the central rind and the structure took the shape of a deformed spherical segment (Fig. 12 *e*). As the final step, I connected the second fixing ring to the peripheral points of the structure and it became springier and took the shape of a truncated ellipsoid (Fig. 12 *f*).

The contact crossings kept the structure stable by the friction forces, without any supplemental fixing devices. I believe that the structure of more elastic materials, such as fibro plastics or titanium alloy would behave much better and take a springier shape. Another interesting variant would be to try to assemble and erect a dome-shaped structure of natural resilient materials like bamboo.

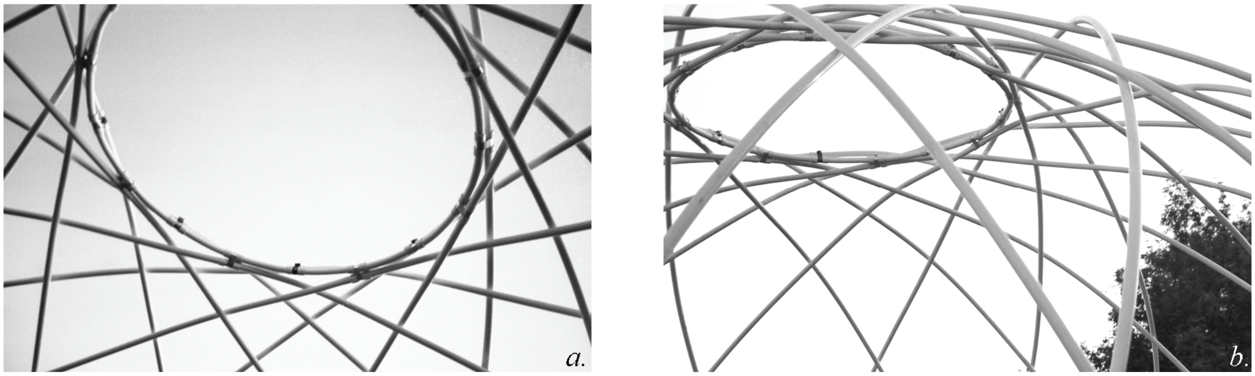

Figure 13: *Internal (a) and external (b) views of large-sized NODUS structure*.

The large-sized *NODUS* structure looked beautiful in natural surroundings. The pattern of the interwoven curve line grid offered a different aesthetic impression from the internal (Fig. 13 *a*) and the external (Fig. 13 *b*) point of view. This experiment proved that cyclic knots of resilient materials offer a new method of form finding that may find application in many practical fields including art, design and architecture.

In addition, there is a potential field of application for the small-scale form-finding knots. It is well known that, on the molecular level, long-sized filamentous objects are resilient. Many of the natural elastic filaments, such as polymeric molecules including DNA, often take circular closed forms of rings and knots either single or linked. It is possible to imagine *NODUS* structures woven of nanowires and similar nanomaterials. In the near future, nanoscale structures will constitute the basis of many industries. Perhaps some sort of nanodesign and nanoart will arise, as well. In this case, the form-finding knot structures may become a bridge not only between art and science, but also between different scalable levels of physical reality.

References

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