Tiled Torus Quilt with changing tiles

Elaine Krajenke Ellison
5739 Wilena Place
Sarasota, Florida 34238
eellisonelaine@yahoo.com
www.mathematicalquilts.com

John Sharp
London Knowledge Lab,
London, UK
Sliceforms@yahoo.co.uk

Abstract

The quilter Elaine Ellison heard papers at Bridges in Leeuwarden in 2008 and Banff in 2009 about tessellations where the tiles change. This gave her the idea to create a quilt in which the shape of the polygons of the tiles change. Also in Leeuwarden and afterwards, she discussed with John Sharp his work in the area of tile changing, which he calls "morphing tilings". Many beautiful patterns can be created using these morphing techniques. The result of explorations with them is the quilt **Tiled Torus**, which also incorporates colour changes which is the subject of this paper.

Introduction - Elaine Ellison

The idea for creating a quilt where the shape of the polygons of the tiles change was a fascinating one that arose out of papers I saw presented at Bridges in Leeuwarden in 2008 and Banff in 2009. I also had a discussion with John Sharp at Leeuwarden and afterwards about his work in this area which he called morphing tilings. I then thought what about creating a morphed tiled surface that was also a properly coloured map? The famous four colour theorem states that, for a plane map, a maximum of four colours is required. I soon discovered that colouring a morphing tiling was not easy. As the design progressed I used four colours, but I found that the colour patterns were too confusing for the eye to appreciate.

The alternative I came up with was to colour the tiles that are congruent with the same colour—only using a variety of values ranging from dark values on the “edges” of the quilt, and light values towards the center of the quilt. Out of this strategy emerged a quilt that emulates a wavy design. This design was perfect for the eye, when the fabrics began to glow as the eye moved across the pattern in a seamless manner. It is possible that with a design that has fewer tiles, a properly coloured map would be pleasing and interesting.

History

Throughout history tiling patterns consist of a small number of tiles to give a repeating pattern. In the middle of the twentieth century, two artists developed tiles which morph or transform. The most famous of these is, of course, the Dutch artist M. C. Escher, and the other is William Huff, a professor of architecture at the State University of New York at Buffalo. It is surprising that we only know of one quilt artist who has published work in this area, Jinny Beyer [6].

Escher’s famous woodcut, *Liberation 1955,* is a classic example of a continuous deformation. *Metamorphosis 1968, Square Limit 1964,* and *Designs for Inlaid Panels in the Town Hall Leiden 1940-1942,* are other examples of transformed morphed tiles. All of these art works tell an ingenious story in a creative way. In *Tiled Torus* quilt story-telling we are able to see how a square in the upper left-hand corner evolves into a square-within-a-square towards the end of its pattern change. Each polygon as it is being transformed takes on new unique geometry to delight the eye. Adding colour to the polygon adds more interest to the design.

Huff used the idea which he called "parquet deformation" as an exercise to develop visual thinking in his students. He was inspired by Escher's woodcut *Night and Day*. In July 1983 Douglas Hofstadter, writing about these ideas in his Scientific American column [3], explained the basics of one-dimensional parquet deformation. He subsequently published his column articles in book form [4] and they form chapter 10 there. John Sharp produced many designs at this time as he describes below. The Hofstadter article also inspired Craig S. Kaplan who included it in his paper titled *Metamorphosis in Escher's Art* at the 2008 Bridges Conference in Leeuwarden, Netherlands. Robert Bosch and Andrew Pike also presented a paper *Map-Coloured Mosaics* at the 2009 Bridges Conference in Banff, Canada which refers to Huff's work through Hofstadter's book.

Huff begins with a single tile which then forms an underlying framework grid. (We will do this with the quilt design using a square as the basis of our starting grid.) He says that the lines and vertices of the design are manipulated relative to their positions within the cells of the grid. The typical ways they are changed, to quote from Douglas Hofstadter's article/book [3,4], are:

- lengthening or shortening a line;
- rotating a line;
- introducing a "hinge" somewhere inside a line segment so that it can "flex";
- introducing a "bump" or "pimple" or "tooth" (a small protrusion or extrusion having a simple shape) in the middle of a line or at a vertex;
- shifting, rotating, expanding, or contracting a group of lines that form a natural subunit; or variations on any of these.

Since some of the square copies may be at 90 degrees (or some other angle) with respect to the master cell, one locally innocent-looking change may induce changes at corresponding spots resulting in unexpected interactions whose visual consequences can be quite exciting. After a line is deformed and all the other lines so respond, the tiles in the new zone of figures remain congruent with one another. Huff feels that it is this congruence of tiles that makes them appealing both from the standpoint of design and mathematics.

John Sharp's webpage, as he describes below, simplified these ideas with many examples. Elaine Ellison used these to design tiles that would change gradually enough so the image was pleasant and interesting to the eye, rather than being chaotic and unrelated.

Escher's Transitions

Kaplan in his Bridges presentation [2], *Metamorphosis in Escher's Art*, addresses the six categories of transition in the work of M. C. Escher. The following are Kaplan's categories for classifying the changes in Escher's work:

1. Realization: A geometric pattern is elaborated into a landscape or other concrete scene.
2. Crossfade: Two designs with compatible symmetries are overlaid, with one fading into the other.
3. Abutment: Two distinct tilings are abruptly spliced together along a shared curve.
4. Growth: Motifs gradually grow to fill the negative space in a field of pre-existing motifs, resulting in a multihedral tiling.

5. Sky-and-Water: This sort of transition starts with copies of some realistic shape A, ends in copies of another realistic shape B, and moves between them by passing through a tiling from two shapes that resemble A and B.
6. Interpolation: A tiling evolves into another tiling by smoothly deforming the shapes of tiles.

Kaplan's examples show that much of Escher's work uses the Sky-and-Water idea along with the Interpolation strategy. Kaplan has some cases where the Huff method which John Sharp has worked with are not followed. Kaplan has also started with an initial tile which he then transforms into a final tile without using the reference to an underlying regular grid. This causes matching problems. Each of the tiles in the quilt design evolves into another tile by small changes. This idea illustrates the strategy of Interpolation.

Design Beginnings - Elaine Ellison

For the design, I followed the Huff/Sharp method. I chose two motifs in a square (see John Sharp's explanation below). The underlying grid was a square one of isohedral tiles; the underlying pattern is a 4-4-4-4 one , that is to say a grid of 400 squares (20 by 20) became the net of the design. The vertices of the grid remained fixed throughout the transforming process. As the square's life evolves, the following sequence of transitions are used to show its life story. The square's story evolves left to right and top to bottom. The motifs have lines which always meet at the corners of a square and so, as they change, always tile. The first motif has rotational symmetry. It consists of a central square with corners joined to the corners of the square of the base tiling; the extremes of the transformation shown below go from the square shrinking to a point and expanding to the base square. The second motif consists of a central line parallel to the side of the base square with the ends joined to the corners of the base square; the extremes of the transformation are that the line expands to join the midpoint of the square sides giving an H on its side, to the other extreme of a point when the joins to the corner of the base square give an X. The symmetry of the motif allows it to be used twice with 90° rotation to give a pentagonal tiling without any translation.

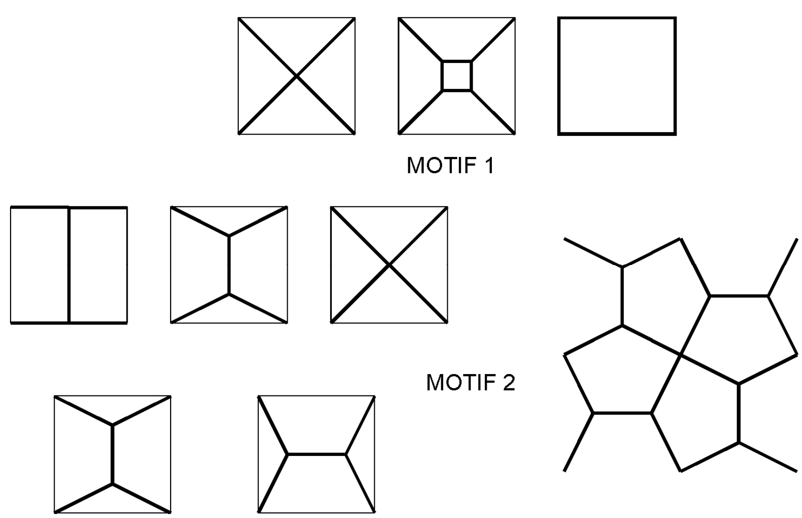

Figure 1: *The two motif variations*

Note how both motifs end in an X, so they can merge into one another. At the other extreme, motif 2 does not strictly go to a square but does merge quite well. Figure 2 shows the transformations in the complete design using 20 by 20 base squares. What emerged from this pattern was that square 20 would loop back to square 1 and the pattern would be continuous. In essence, the pattern loops left to right plus the pattern loops top to bottom, thereby being an unfolded torus! This was an interesting discovery for me in the process of designing the quilt.

Colour has also been important in the design. Medium values were chosen to construct most of the quilt. Very light values of colours were centered in the quilt to create a warm sparkly look. Yellow was used quite liberally to help achieve this impression. Other considerations were the black and gold squares as I did not want them to dominate the pattern. Eventually, the black and gold squares in the lower right hand side of the pattern did help to balance the strong central black and gold squares. In the final stage of machine quilting, gold metallic thread helped to mute the strong contrast of these tiles. The quilt took on a shimmery effect after the quilting was complete.

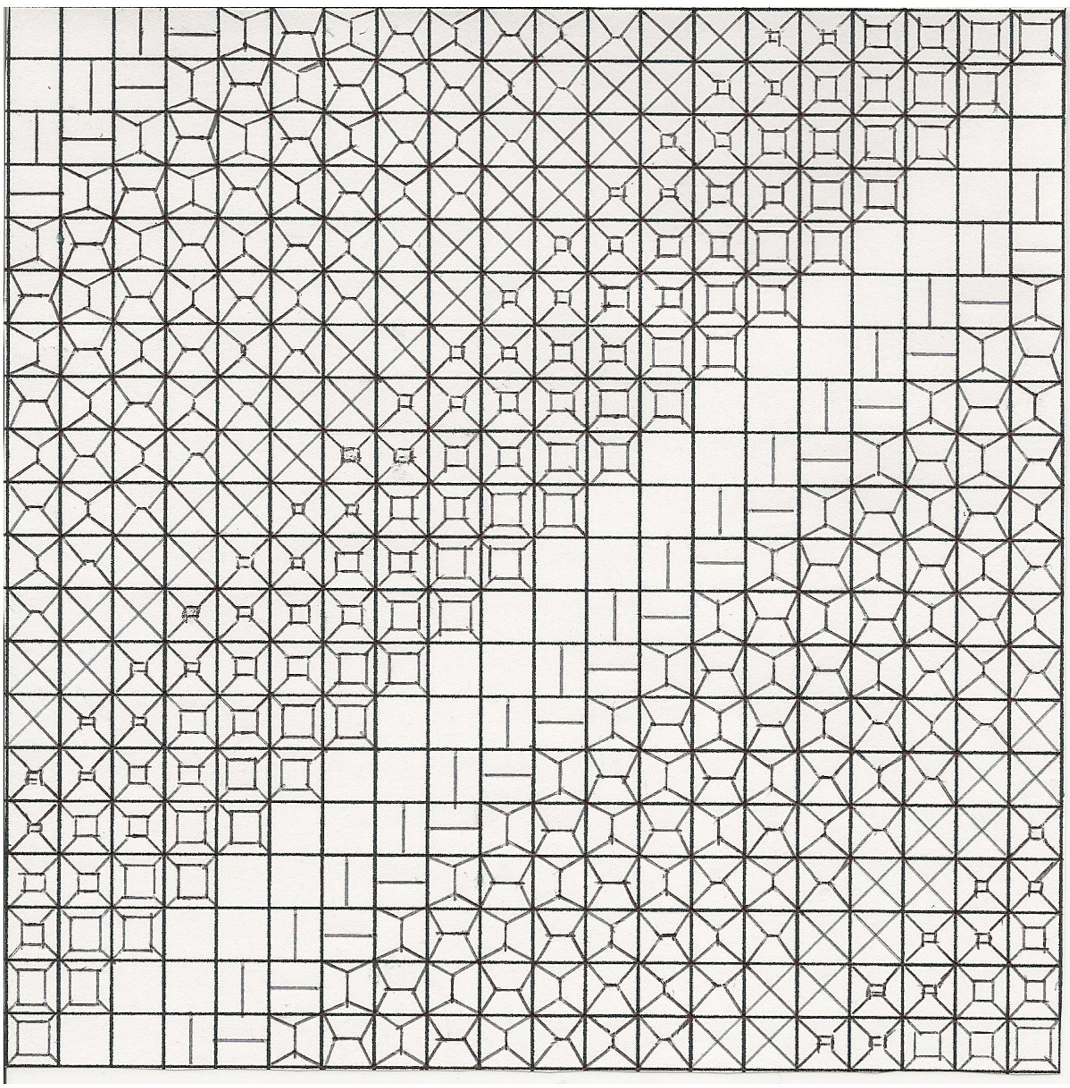

Figure 2: *The net and structure of the quilt*

Making the quilt - Elaine Ellison

I created a paper pattern in order to view the quilt at full size. Each individual tile was cut out with scissors and pinned on to the paper pattern. Colours could be adjusted at this point. Viewing the quilt from at least 4 feet with a large viewing angle verified that the colouring was a good one. The construction of a quilt is always an adventure. The order of sewing is crucial - much like structuring a mathematical proof. How seams are constructed and pressed are also a concern. For the *Tile Torus* quilt, the seams were pressed towards the previously sewn polygons.

The actual sewing demanded a variety of techniques. What began as a piecing project quickly developed into an appliqué project. The reason for the change in techniques was to accentuate the vertices and edges of the polygons as they developed. A piecing technique did not show the actual vertices and edges clearly. The gold "hypercube" sections were created by appliquéing black material onto the gold lame material. Further, the free-motion quilting using gold lame thread emphasized the shimmery look to this quilt.

Using a diagonal approach to piecing gave the quilt stability. Since there was no material stabilizer to attach the pieces to, too much play in the fabric was an issue. Sewing the tiles together in a straight line might work if a stabilizer were used. Most quilts with this amount of detail and hand work will easily take 300 hours or more to create accurately.

Figures 3 to 5 show the quilt in progress at various stages.

Figure 3: *After 36 hours work*

Figure 4: *After 115 hours work*

Figure 5:, *The completed* **Tiled Torus** *quilt after 310 hours*

Conclusion

The purpose of this paper is to show how mathematics can help invent innovative artwork. As each polygon was added to the previous polygon of this quilt, a developing story of a square's journey came to life. It is hoped that this paper stimulates new research in the construction of parquet deformation/morphing tilings.

Appendix - Design possibilities - John Sharp

When I first saw the Hofstadter article [3], I felt it was exciting, but not very well explained for anyone wanting to construct morphed tiles. However, taking the examples in [3] or [4] offers excellent scope for teaching analytical skills. I then realised that what was happening was that you take a basic tiling, like one of the three regular ones (or even a semiregular one) and put a motif in the unit (or more than one for semi-regular). The motif could be transformed in some way, but it was based on lines that went to the vertices (or it could be a fixed point on the sides) so that the tiling always joined up. The year 2000 in the UK was designated MathsYear2000 by the government's education department. There were many activities and a website of resources. I wrote a number of subject areas for this. As often happens, the government changed, money ran out to maintain this project; then it passed into other hands and re-emerged as Counton.org and has since mostly been lost. However, there is a websource called *the Wayback Machine* which has been archiving the material, which can be found at [1], if you go to the *Explorer* section in the Counton pages.

So the basic idea as described above is to take a motif that you can transform. This transformation can occur in one direction or in two. The stages can be linear, so that for example the central line in motif 2 (figure 1) can steadily go from a point to a line, or the change can be according to some simple function. Depending on the symmetry, a motif can be rotated or reflected. These few possibilities soon yield many variations for a single motif. I have books of tilings to show this. A simple program allows them to be turned out at an alarming rate. The creativity is in designing the motifs. As Elaine Ellison has shown, it is also possible to mix motifs in a number of ways; she has two separate ones and also has used the orientation of motif 2 to create something different.

From figures 1 and 2, it is easy to see the transformations. When you are presented with a completed result the decoding often happens suddenly, but it does not detract from the aesthetic view.

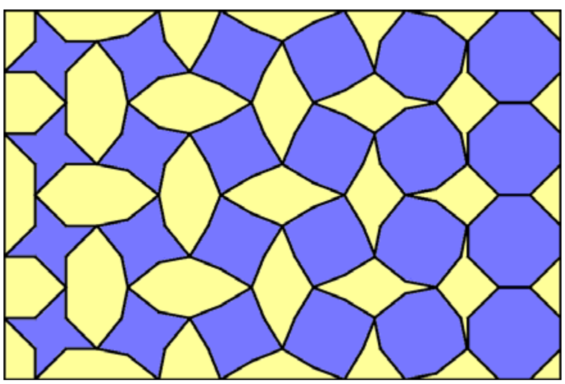

Figure 6: T*iling based on Islamic motif*

So in figure 6, you may recognise part of an Islamic tiling at the left with the motif being a square with triangles added to the side. The reflections result in an elongated hexagon in the original.

The transformation is to flatten the triangles to form a square and then continue the movement outwards which eventually results in an octagon.

Artists tend to look at the holes in between, so the hexagon/octagon changes in a way you might not have expected.

I happen to be colourblind, so do not have the skill that Elaine Ellison does to add the extra dimension of colour. But even black and white can add some interesting new results. Figure 7 shows a morphing tiling which has been coloured black and white. Amazingly, this gives rise to an illusion of curved lines which it is impossible to see in the original.

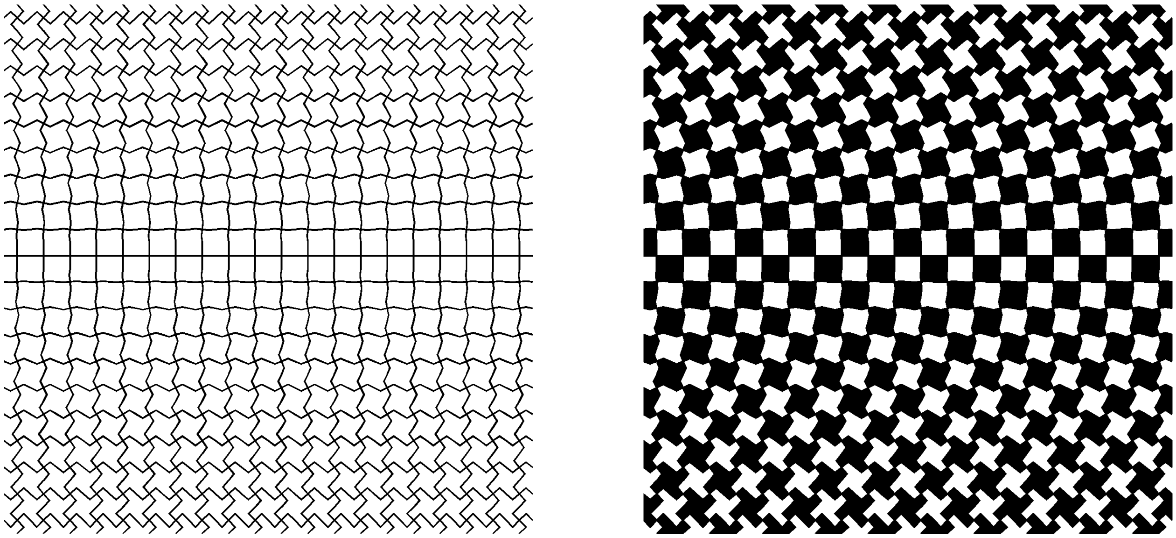

Figure 7,: C*reating an illusion*

Another possibility that has not been explored is to use circular/polar grids. The examples in figure 8 have also been coloured black and white and give rise to spiral effects which are not there.

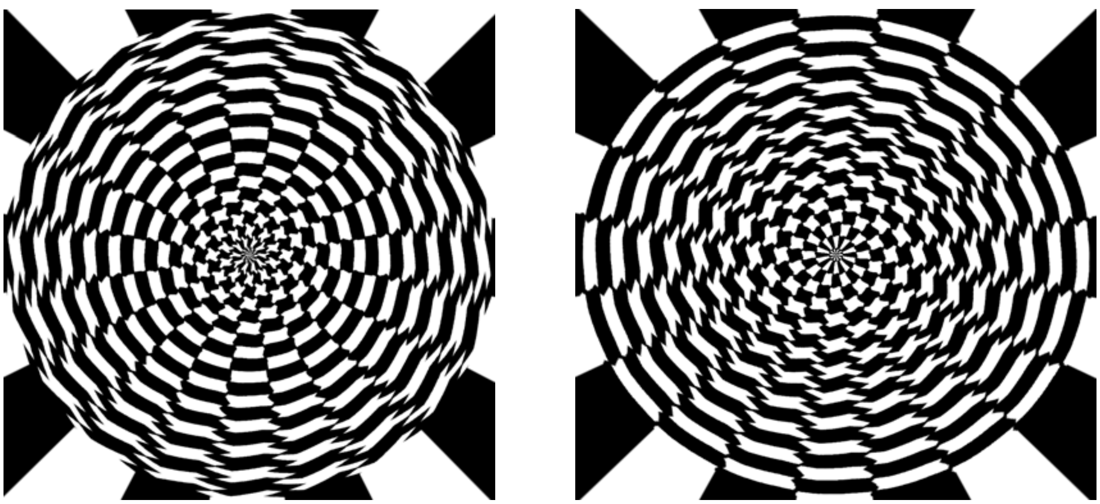

Figure 8: M*orphing tilings on polar grids*

References

[1] John Sharp, Morphing Tiling, http://web.archive.org/web/2002-2003re_/http://www.counton.org.
[2] Craig S. Kaplan, *Metamorphosis in Escher's Art*, Bridges Leeuwarden, Proceedings 2008.
[3] Douglas Hofstadter, *Parquet Deformations: Patterns that Shift Gradually in One Dimension*, Scientific American Magazine, July 1983 pages 14-20.
[4] Douglas Hofstadter, Metamagical Themas *Questing for the Essence of Mind and Pattern*, Basic Books NY 1985
[5] Robert Bosch and Andrew Pike, *Map-Coloured Mosaics*, Bridges, Canada, Proceedings 2009.
[6] Jinny Beyer, *Designing Tessellations: The Secrets of Interlocking Patterns*, Contemporary Books, Chicago Illinois 1999, pages 229-237