Escher-type Tessellations and Pull-up Polyhedra: Creative Learning for the Classroom

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Abstract

In this practical workshop participants will gain an appreciation of how Dutch artist M.C. Escher created his intriguing tessellations through the distortion of simple shapes and the application of geometric transformations. Participants will be guided through the creation of their own Escher-type tessellations and these ideas will be extended into three-dimensions with pull-up Platonic solids patterned with Escher's designs.

Introduction

Dutch artist Maurits Cornelius Escher (1898 – 1972) is renowned for his beguiling *regular divisions of the plane*. Also known as *tessellations*, these are created by arrangements of closed shapes that completely cover the plane without gap or overlap. Escher used regular and irregular tessellations to create optical illusions and combined shapes to produce what he called '*metamorphoses*' in which the shapes changed and interacted with each other, sometimes even breaking free of the two-dimensional plane. Escher's work captures the imagination and his tiling patterns come alive: through the manipulation of the basic shapes in his tessellations, applying *symmetry operations* or *transformations*, Escher was able to render the shapes into animals, birds, and other figures. The effects are astounding.

This workshop has been designed to use the creative process encapsulated in Escher's art as the stimulus for cross-curricular work, linking studies in the visual arts and mathematics for classroom activities. After viewing a short animation on the creation of Escher-type tessellations, participants will have the opportunity to produce their own repeating tessellations, exploring the manipulation of shape and the application of transformations. These methods will act as the incentive for designing other patterns and lead to the exploration of three-dimensional shapes and tessellations also investigated by Escher.

During the workshop participants will be encouraged to reflect on their own teaching which, in turn, will allow them to introduce new teaching strategies in the classroom that help to:

- make cross-curricular connections between art and mathematics
- develop two- and three-dimensional spatial awareness
- broaden students' creative outlook and develop practical skills
- increase students' motivation and enjoyment of mathematics

Creating Escher-type Tessellations

Escher's repeating designs were produced predominantly through the use of the geometric transformations of translation and rotation. Reflection tends only to be present when it is used as a mirror line within a motif such as a line of bilateral symmetry (for example, see the tessellation used on net for octahedron in Figure 3c). See [1] for the definitive resource on the work of Escher with extensive illustrative materials and discussion. Two simple processes for designing Escher-type tessellations using the simple transformations of translation and rotation are outlined below.

Cut and Slide.

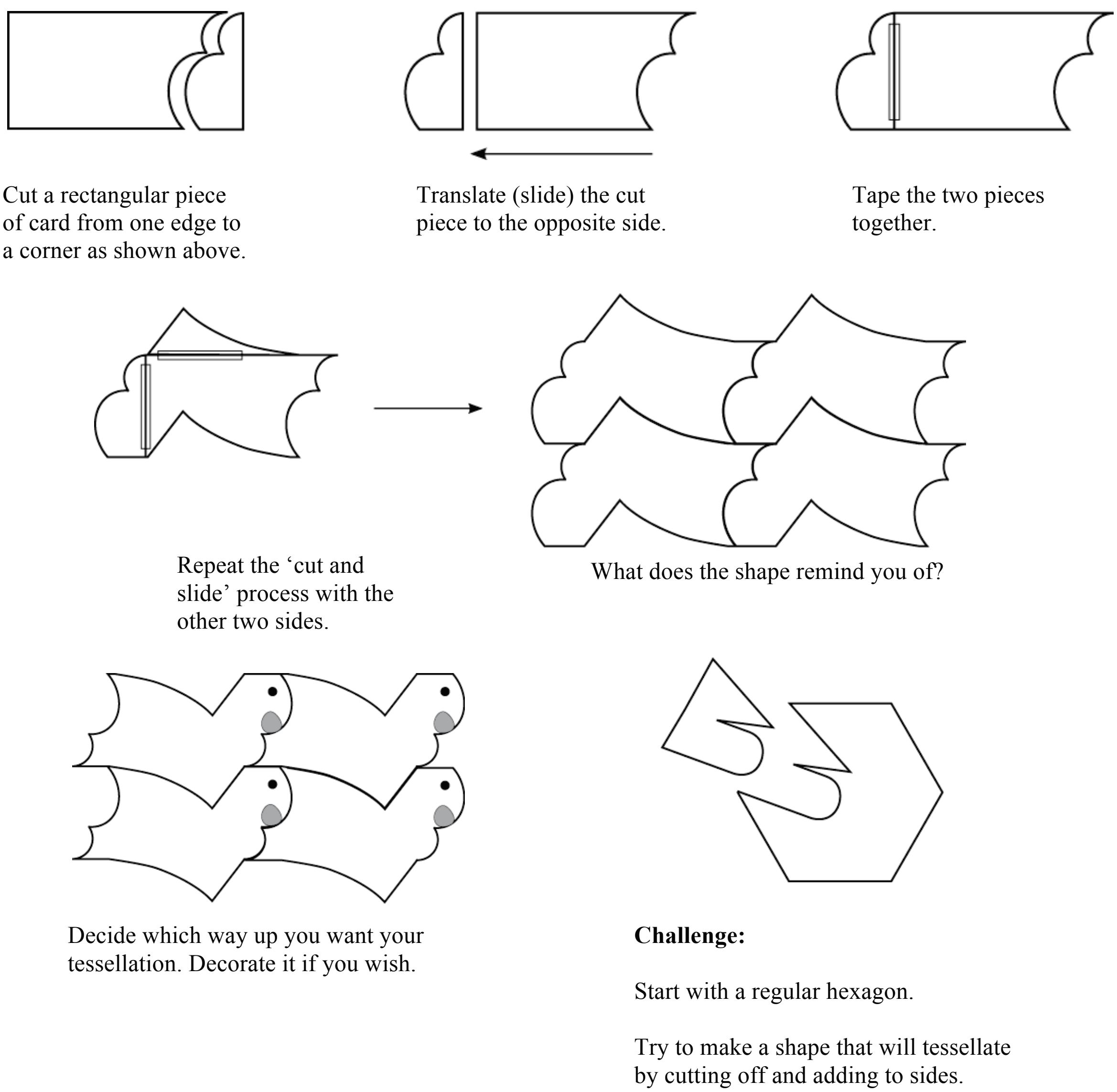
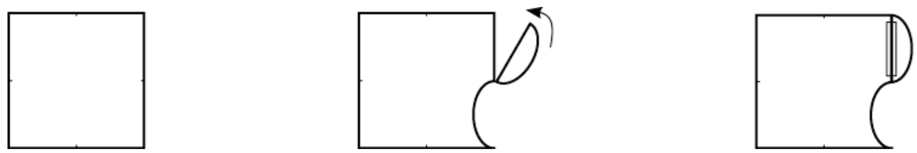


Figure 1: *Instructions for the 'cut and slide' method of producing an Escher-type tessellation.*

Cut and rotate.

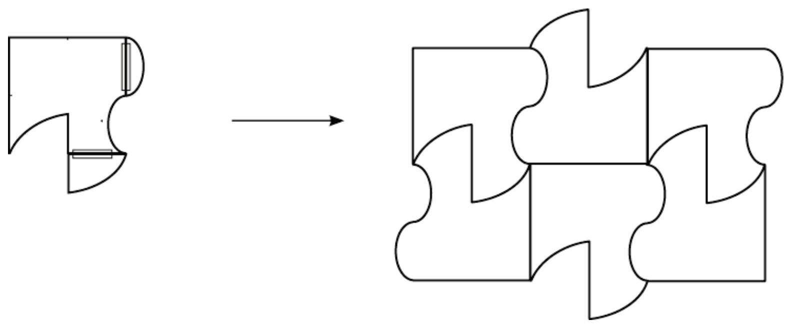


Mark accurately the mid-point of each side of a card square.

Cut out a piece from half of one side and rotate it about the mid-point.

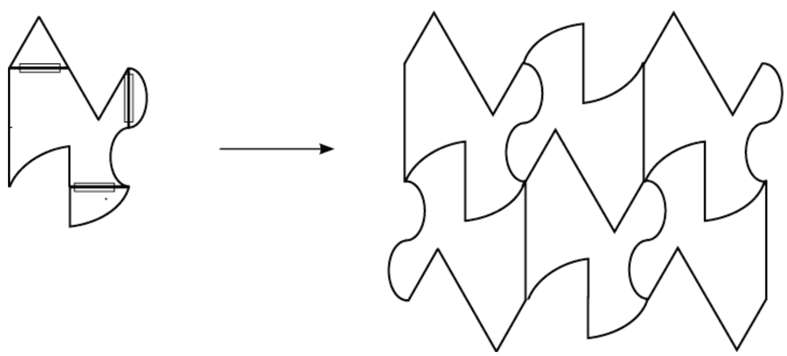
Do not reflect it!

Tape this piece to the uncut half.

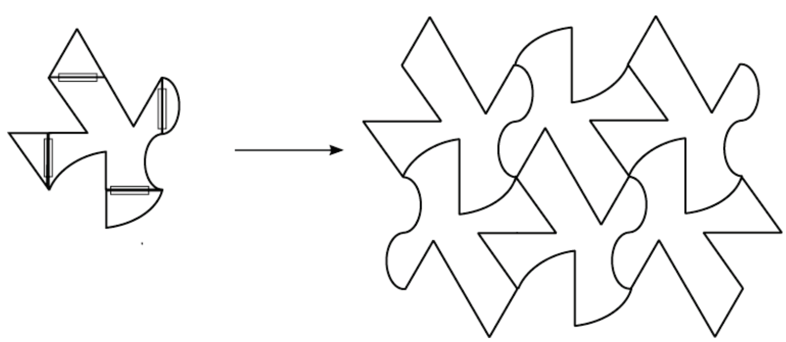


Repeat the ‘cut and rotate’ process for another side.

Show that this shape will tessellate.



Repeat the ‘cut and rotate’ process for a third side. Show that this shape will tessellate.



Repeat the ‘cut and rotate’ process for the fourth side. Show that this shape will tessellate. Decorate if required.

Figure 2: *Instructions for the ‘cut and rotate’ method of producing an Escher-type tessellation.*

Tessellations on Polyhedra

In addition to creating tessellations in the plane, Escher also experimented with covering the faces of mathematical solids with repeating designs, although only one design for a decorated icosahedron was ever produced in finished form (see [1] and [3] for further discussion). A number of carved spheres, covered with a single repeating motif, were also produced. More recently, a systematic means for applying patterns to the faces of the Platonic solids has been developed by [2]. This work identified the different pattern classes that are capable of repeating regularly across the faces of these solids, a design project that was largely inspired by the work of Schattschneider and Walker [3]. An exhibition of designs created from this work [4] led to the collaboration and subsequent development of pull-up patterned polyhedra by mathematician and educator, Meenan, and textile designer, Thomas [5].

The adapted Escher patterns on the nets of the tetrahedron, octahedron, cube and icosahedron, illustrated in Figure 3, were achieved through the straightforward extraction of the net from the plane tessellation. The patterning of the dodecahedron required a different approach due to the impossibility of a regular five-sided figure tiling the plane. Through the exploitation of the concept of duality, a method by which pattern can be applied to the dodecahedron through projection from a patterned icosahedron was utilized (see [6] for further discussion).

The problems encountered in the application of color symmetry patterns to repeat across the faces of polyhedra, focusing particular attention on the icosahedron, were discussed in [7]. Maintaining the repetition of the tessellation and its constituent coloring in exactly the same manner around the solid as in the plane was no trivial task due to the restrictions imposed by Escher's colorings of the designs, many of which no longer matched when folded into different planes. Colored nets are provided for the tetrahedron, octahedron and cube in Figure 3. The patterned nets for the dodecahedron and icosahedron are provided with adapted outline patterns to allow participants to explore ideas of color symmetry and color mapping.

Basic making instructions:

1. Carefully cut out the net for your pull-up polyhedron.
2. Use a ruler and a sharp point to score lightly along the remaining black lines.
3. Make holes at the points A, B, C, D, etc.
4. Thread and weave thin string or strong thread through the holes A, B, C, D, etc, to link the faces together.
5. Gently pull up the net to make your polyhedron.

Sharp scissors, accurate cutting and scoring of the nets are crucial for well-made pull-up polyhedra.

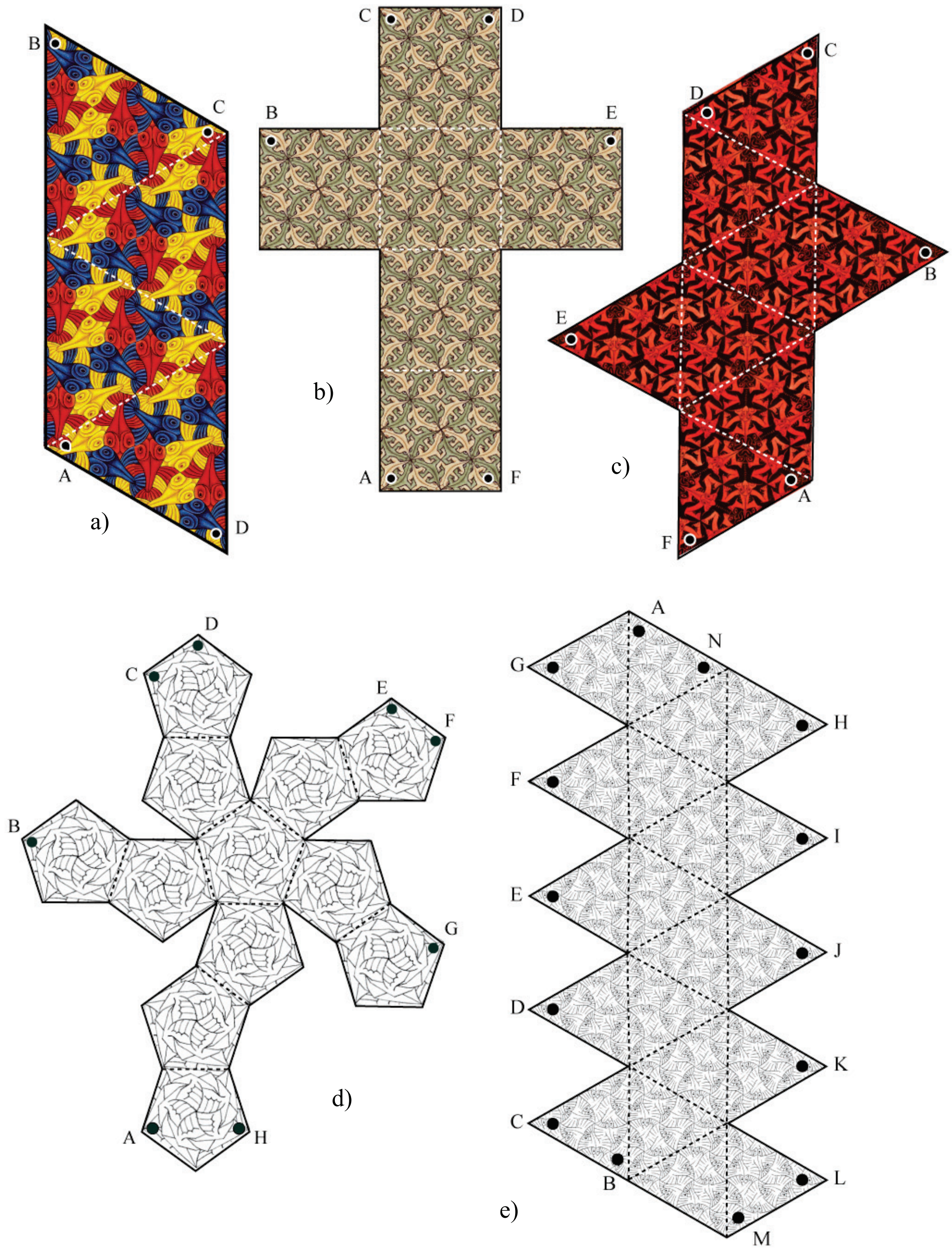


Figure 3: *Pull-up nets for a) the tetrahedron, b) the cube, c) the octahedron, d) the dodecahedron and e) the icosahedron, patterned with manipulated Escher designs*

Summary

These activities presented above are designed to assist in developing pupils' understanding of two- and three-dimensional shapes, symmetry, patterns and tessellations. The teaching of geometry in a visually stimulating way has been shown to engage students from a wide range of abilities [5]. The material presented within this workshop can also be combined in the teaching of a larger cross-curricular project between departments, which could include topics such as symmetry, shape, tessellations and the geometry of Islamic patterns (see [8] for activities linking Islamic tilings patterns and mathematics through paper folding activities).

References

[1] D. Schattschneider. *M,C. Esher: Visions of Symmetry.* London, Thames and Hudson. 2nd ed. 2004.
[2] B.G. Thomas and M.A. Hann. *Patterns in the Plane and Beyond: Symmetry in Two and Three Dimensions*. Monograph no. 37 in the Ars Textrina series, The University of Leeds International Textiles Archive (ULITA). 2007.
[3] D. Schattschneider and W. Walker. *M.C. Escher Kaleidocycles.* Norfolk, Tarquin Publications. 3rd ed. 1982.
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[5] E.B. Meenan and B.G. Thomas. "Pull-up Patterned Polyhedra: Platonic Solids for the Classroom" in R. Sarhangi and C. Séquin (eds.) *Bridges Leeuwarden: Mathematical Connections in Art, Music and Science*, pp.109-116. 2008.
[6] B.G. Thomas and M.A. Hann. "Patterning by Projection: Tiling the Dodecahedron and other Solids" in R. Sarhangi and C. Séquin (eds.) *Bridges Leeuwarden: Mathematical Connections in Art, Music, and Science*, pp.101-108. 2008.
[7] B.G. Thomas. "Counterchange Patterns and Polyhedra" in in C.S. Kaplan and R. Sarhangi (eds.) *Bridges Banff: Mathematics / Music / Art / Architecture / Culture*, pp.177-182. 2009.
[8] E.B. Meenan and B.G. Thomas. "Creating Islamic Patterns from Folded Shapes" in C.S. Kaplan and R. Sarhangi (eds.) *Bridges Banff: Mathematics / Music / Art / Architecture / Culture*, pp.367-374. 2009.

Acknowledgments

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