Creating Cartoon Images with Functions: A Pedagogical Project

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Abstract

The aim of this paper is to present how elementary real functions can be used to produce some artistic works and how this can be an efficient method for teaching functions, namely for making students discover and understand, in a recreational way, many facts concerning functions and their graphs.

1. Introduction

In mathematics high school curriculum much time is devoted to the study of functions and their graphs. This study is done either in an excessively informal way, using graphing calculators and intuition to draw conclusions on the basis of "it looks like", or in an overly formal and uninteresting way. Students reach the university showing enormous difficulties in elementary calculus courses, namely in what involves the knowledge of functions, like computing the value of areas, volumes and lengths of lines. As the teaching of mathematics at the university is essentially formal and abstract, poorly prepared students quickly discourage and very often, end up quitting or choosing to study something different that does not involve mathematics.

Lately math programs have gradually come to include a strong emphasis on applications of mathematics and mathematical modelling as a way to motivate students. The problem is that they do not have enough knowledge of other sciences in order to understand the applications or to make any modelling. Thus, these objectives are not fulfilled and the students do not end up handling functions in practice.

The project described in this paper was designed for students finishing high school or starting the university, and its main objective is to interest students in the study of functions in a playful and not conventional way. The idea is to lead students to apply the study of functions to a goal that is achievable: the creation of mathematical models for sketching known objects. For this purpose is usually convenient the use of a dynamic geometry software.

2. Motivation

Everything started in a calculus class, solving the following well known problem (see, e.g., [1]):

Find the total area lying between the curves y=sin(x) and y=cos(x), from x= 3π/4 to x=5π/4.

The first step to solve this kind of problems is to sketch the graphs of the functions, to have an idea of the region whose area we want to find. That was what we did, as it is shown in figure 1.

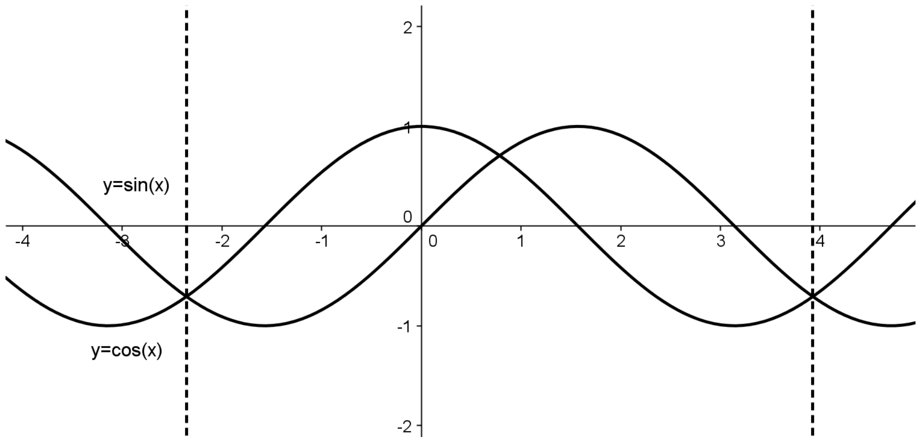


Figure 1: *y=sin(x), y=cos(x), x= 3π/4, x=5π/4.*

Isolating and shading the required region, it is easy to check that, in fact, we are evaluating the area of a wonderful moustache, as shown in figure 2.

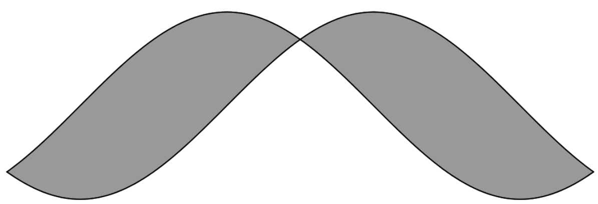


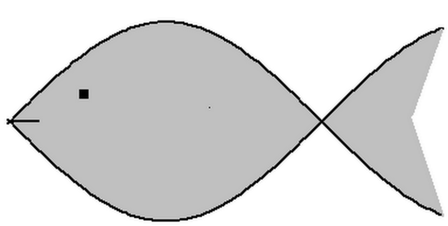
Figure 2: *The Rector's moustache.*

It turns out that this design is an almost perfect mathematical model for the moustache of our University's Rector. For our students this was a pleasant and stimulating discovery, and from that moment on the initial problem simplified to: "*Find the area of the Rector's moustache*". Nowadays, very often students refer to the problems of evaluating areas as *the rector's moustache problems.*

2. Development

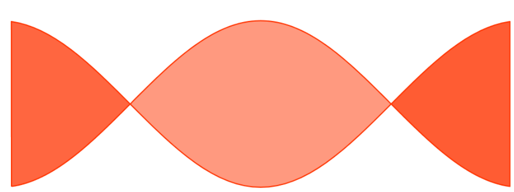
The interest caused by the moustache led us, in a first moment, with the help of students, to use well known elementary functions to model other shapes, as we exemplify in next figures.

Fish



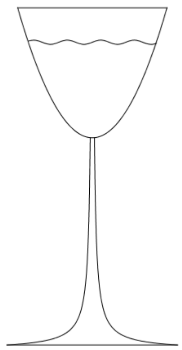
Restriction *of y=cos(x)* and *y=-cos(x) to* [-π/2, π]
Figure 3

Wrapped Candy



Restriction *of y=cos(x)* and *y=-cos(x) to* [-π, π]
Figure 4

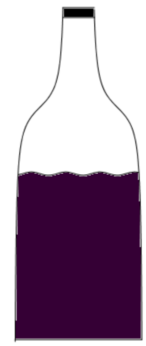
Glass



Restriction of $y=1/(x-12)-14$ to [12.1,16]
Restriction of $y=1/(x-12)-14$ to [8,11.9]
Restriction of $y=-13.75$ to [8.5,16]
Restriction of $y=0.5(x-12)^2-4$ to [8.5,15.5]
Restriction of $y=2.12$ to [8.5,15.5]
Restriction of $y=0.1\sin(4(x-12))+0.5$ to [9,15]

Figure 5

Bottle



Restriction of $y=\tan(x)$ to [-1.4995 , 1.406]
Restriction of $y=1/\tan(x)$ to [3.307,6.212]
Restriction of $y=-14$ to [1.5,6.21]
Restriction *of* $y=6$ to [1.4,3.31]
Restriction of $y=5.4$ to [1.39,3.32]
Restriction of $y=0.1\sin(-3x)-4$ to [-1.3,6]

Figure 6

After these initial drawings, we understood how powerful this approach could be to interest students in the study of functions. We began to combine different functions, defined in appropriate domains, together with other geometric objects like circumferences or ellipses, in order to create more sophisticated drawings. Each drawing follows a "mathematical recipe" and can be exactly reproduced by anyone. In figure 7, we exhibit a cat and the mathematical recipe that generates it.

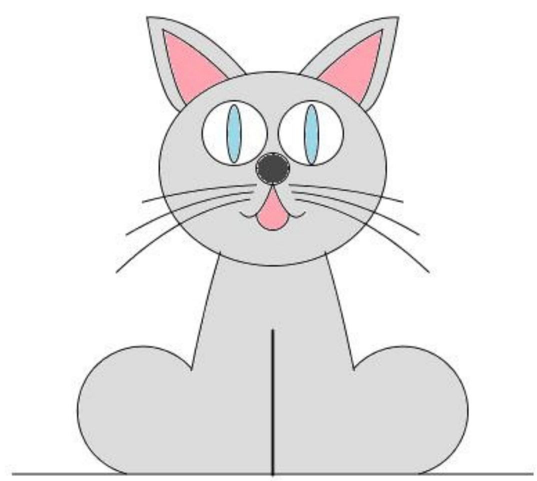
MATHEMATICAL RECIPE	OBJECT
Ellipse $0.08x^2 + 0.11y^2 = 1$ Circumference centred in (-1.2,1.1) and radius1 Circumference centred in (1.2,1.1) and radius1 Ellipse $25x^2 + y^2 + 60x - 2.2y = -36.21$ Ellipse $25x^2 + y^2 - 60x - 2.2y = -36.21$ Circumference centred in (0,0) and radius 0.5 Restriction of $y=2(x - 0.72)^2 - 1.5$ to [0 , 1] Restriction of $y=2(x + 0.72)^2 - 1.5$ to [-1 , 0] Restriction of $y=-(0.25 - x^2)^\wedge(1/2) - 1.4$ to [-0.5 , 0.5] Restriction of $y=-1/30\,x^2 - 0.5$ to [0.5 , 4] Restriction of $y=-1/30\,x^2 - 0.5$ to [-4 , -0.5] Restriction of $y=-1/15\,x^2 - 0.7$ to [0.6 , 4.5] Restriction of $y=-1/15\,x^2 - 0.7$ to [-4.5 , -0.6] Restriction of $y=-1/10\,x^2 - 0.9$ to [0.7 , 4.8] Restriction of $y=-1/10\,x^2 - 0.9$ to [-4.8 , -0.7] Restriction of $y=-(x + 2)^3 + 1.68$ to [-3.38 , -2.65] Restriction of $y=-(x + 2.42)^3 + 1.68$ to [-3.86 , -2.84] Restriction of $y=-1/7\,(x + 5)^2 + 4.68$ to [-3.37 , -1.35] Restriction of $y=-1/5\,(x + 3.78)^2 + 4.68$ to [-3.85 , -0.8] Restriction of $y=(x - 2)^3 + 1.68$ to [2.65 , 3.38] Restriction of $y=-1/7\,(x - 5)^2 + 4.68$ to [1.35 , 3.37] Restriction of $y=-1/5\,(x - 3.78)^2 + 4.68$ to [0.8 , 3.85] Restriction of $y=(x - 2.42)^3 + 1.68$ to [2.84 , 3.86] Restriction of $y=-x^2$ to [-2.5 , -1.6] Restriction of $y=-x^2$ to [1.6 , 2.5] Restriction of $y=(4 - (x - 4)^2)^\wedge(1/2) - 7.5$ to [2.5 , 6] Restriction of $y=-(4 - (x - 4)^2)^\wedge(1/2) - 7.5$ to [4.5 , 6] Restriction of $y=(4 - (x + 4)^2)^\wedge(1/2) - 7.5$ to [-6 , -2.5] Restriction of $y=-(4 - (x + 4)^2)^\wedge(1/2) - 7.5$ to [-6 , -4.5] Restriction of $y=-9.45$ to [-8 , 8] A = (0, -9.45), B = (0, -5) Segment [A,B]	

Figure 7: A *recipe* for a pussy-cat

3. Pedagogical Issues

The idea presented can be pedagogically used in many ways. Using functions to model objects is something the students can achieve and can help them to develop several skills. In order to produce recipes as the ones shown above, students must master different concepts related with functions, such as domain, restriction, image, how slight changes in the parameters can affect the function graph and so on. They also need to acquire clear ideas about the behaviour of graphs in certain domains so that they can choose the appropriate function to model some precise shape. Students become aware of the need to know more about functions and realize that the theoretical notions about functions can have an immediate practical application.

From the initial idea, it is possible to propose several activities, according to the students' knowledge of functions, such as:

- Given a mathematical recipe, sketch the corresponding object and freely decorate it.
- Given a figure, create a mathematical model that reproduces it (this was the way the cat in figure 7 was created, reproducing a figure in [2]).
- Given graphs of several functions, use them to identify figures that can be modeled by these functions in appropriate domains. This is the case of the rector's moustache (figure 2) and also of the fish and wrapped candy in figures 3 and 4.
- Given a figure already modeled, try to discover the recipe that originated it. This activity can start by just counting the number of functions involved.
- Create a free sketch and the corresponding recipe.

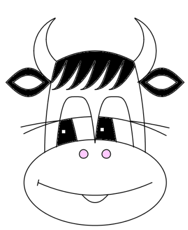
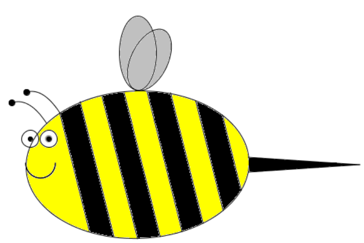
The use of computers to perform such activities is also an opportunity for students to develop computer skills. In fact, after a short while following a mathematical recipe, students that didn't know previously how to work with dynamic geometry software, can easily deal with it without help.

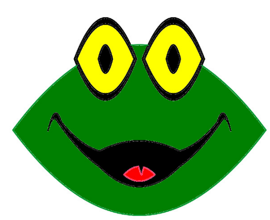
4. Conclusions

This project is in the beginning, but all the experiments either with teachers or with students have proven successful. For instance, during the last University Open Days we suggested to the young visitors activities such as those described above and they were so enthusiastic that they didn't want to leave.

Although any computer software that sketches function graphs can be used, we have used GeoGebra [3], which has proved to be a powerful tool for our purposes. In addition, making mathematical drawings also seems to be a good way to learn to work with this software and some high school teachers have been asking us to teach them to use GeoGebra, but by implementing these designs.

We finish showing some other drawings, without the *recipes*. We challenge the reader to discover them!



References

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[1] Robert A. Adams, *Single –variable Calculus*, Addison Wesley, 1995

[2] http://gatoseoutrostiposdeestofos.blogspot.com/2006/04/13-passos-para-desenhar-um-gato-e-um.html.

[3] www.geogebra.org