Mathematical Models for Argentine Tango

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Abstract

In this note we present mathematical models for **Argentine tango**. After giving a simplified description of this dance in Section 1, we introduce **continuous, discrete**, and **group theoretical models** for a couple dancing Argentine tango in Section 2. In Section 3 we compute the **fundamental group class** of a dance**.** We **extend** our models to the case of **several couples**.

1. Argentine Tango. Argentine tango is an exquisite couple dance that is at the same time very sensual and technical. This special form of tango, which originated in Argentina, has rapidly grown in popularity during the last few decades and is, at present, danced all over the world in many different styles and venues. Argentine tango is a complex dance form that incorporates many elements of movement, and highlights the connection with the partner and the music.

Figure 1: *Tango Dancers Darrell Sanchez and Sue Thompson.*

When dancing in a social setting, several couples move simultaneously around the circle of dance generally traveling counterclockwise (but they can also travel clockwise for short lapses). Each couple, usually a man and a woman (but other gender combinations are also possible) keeps their original embrace fixed within dance segments, thus moving as a rigid unit from the waist up. The embrace can be very close or more open depending on the particular tango style practiced. The basic dance steps are modeled on standard walking on music beats, and steps can be taken in any chosen direction, although the couple generally progresses forward (=counterclockwise). The dancers can occasionally pause and stand still, and/or rotate and pivot by a certain angle on the spot. Certain technical movements (such as *volcadas* or *boleos*, see the first and third images of Figure 1 respectively) are sometimes performed during sections of the dance.

The leader (usually the man) leads all the dance movements and the follower (usually the woman) executes her dance movements following the leader's clues, in accord with several technical tango rules and personal style. The leader's inspiration, in addition to the music and technical ability, comes from the couple's connection and the follower's response to the leads.

The most salient feature in the many styles of Argentine tango practiced around the world is perhaps the fact that the distance and the relative position between the leader and the follower's chests remain constant throughout segments of the dance, since the couple keeps their original embrace fixed within dance segments. This implies that we can parametrize the couple's dancing by using the leader's chest; specifically we will use as our parameters the *leader's chest position* in the circle of dance and the *leader's chest horizontal angle of twist* with respect to the circle of dance's tangent line (the positive direction is counterclockwise). Indeed, the leader's steps follow his chest movement (i.e., if his chest moves forward and twists to his left, his free foot advances to his forward left). Moreover, the leader's chest position and twist determine, via technical tango rules, the position of the follower's hips, and what the follower is going to do (next). Hence we can consider the leader's *chest position* and *angle of twist* as the main parameters for Argentine tango. Worth noting is that in a traditional Argentine tango style called *milongero,* the leader's chest twist is close to zero during the whole dance, except for couple's turns. In other styles, such as for instance in *tango nuevo,* leader's chest twists by non-trivial angles are much more common. To fully describe this dance, we also need to take into account the connection with the partner and the music: we will use a color scale for this psychological variable.

2. Continuous and Discrete Models. Since the couple's arms are linked in the tango embrace, their bodies can be modeled in space in a simplified mathematical form by a mathematical solid 2-torus (a mathematical 2-torus is the mathematical abstract version of a doughnut). But, as the dancers also come often into touching contact with their faces, heads, and/or legs, this solid 2-torus during a real dance experience changes often shape to become a 3 (or more in general d)-holed mathematical solid torus. Thus a couple's dancing can be mathematically described as a solid 2-torus flowing in space along a circle, and transforming itself continuously in and out of d-holed solid tori.

Continuous Models. To simplify our description, we will now think of the couple as a point, and we will use as dance parameters the *distance* (along the circle of dance) of the leader's chest from the starting point of the dance, and the leader's chest *horizontal angle of twist* with respect to the line tangent to the circle of the dance at the given point. In our model, the connection with the partner will be quantified by using a color wheel, so that this variable takes values in a color wheel's circle. Thus, the model's *configuration space* (i.e., the domain of its parameters) is a *3-torus* (non-solid!), that is, the *product of three circles*, two of which represent the *two spatial variables* associated to the leader's chest, and the last one the *connection*. The couple's dancing corresponds in our model to a *curve*, parametrized by the time, *on this 3-torus.* If we assume that the dancers return to their original position at the end of the dance, this curve is a *loop*. We will call this the *standard continuous model* for the dancing of a single couple. The *spatial version* of the continuous model is obtained from the standard continuous model by using color to describe the psychological variable. The *spatial configuration space* is the product of the two spatial circles (= a 2-torus), with a color wheel attached to each of its points to give the connection. The dancing couple can then be modeled by a colored curve, parametrized by the time, on the spatial 2-torus. For example, in Figure 2, the vertical green circle corresponds to walking along the line of dance, without any turning. A colored circle along the equator will instead represent a full pivot at the start. Note that in Figure 2 a single color has been chosen for each point.

The continuous models can also be given a *group-theoretical structure*, in parallel with the finite group structure for square dancing outlined in [3]; see also [2]. In fact, we first observe that *the circle* (in where each point is represented say as an angle between 0 and 360 degrees, up to 360) *is a mathematical group* in a standard way. For, the group operation is the sum of angles (defined up to 360 degrees), and the inverse of the point in the circle with angle *t* is the point with angle *(360-t)* up to 360 degrees. The neutral element of the group is the point that corresponds to the angle 0. Since the configuration spaces of the continuous models are products of circles, they can be given a group structure with standard group operations defined componentwise.

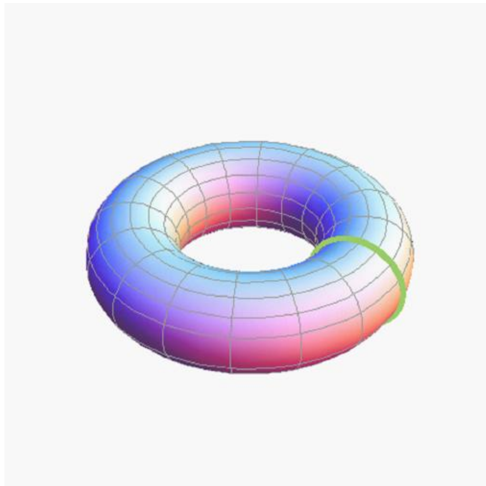

Figure 2: *Cross-Section of the Continuous Mathematical Model. Software: Mathematica.*

Discrete Models. To *discretize* the standard continuous model in a natural way, notice that in most of the dancing, steps are only taken on music beats. We thus associate *to each music beat* the *three values* of the corresponding circular parameters corresponding to it. This amounts to selecting in the standard continuous model only the *points* corresponding to *music beats*. Because of the strict rules governing this dance, the discrete model contains enough information to fully describe the entire dance. To endow the discrete model with a group structure, one would need to introduce some rather technical conditions. Because of this, we are not going to pursue it. Obviously, the spatial version of the continuous model can also be a discretized in a similar way.

3. Fundamental Group Class of a Dance. As we have seen, in the standard continuous model we can mathematically represent the dancing of a couple by a loop on a 3-torus parametrized by the time. We are now going to describe the *fundamental group class* of this loop. Since this entails looking at the loop up to *homotopy*, small variations of parameters are of no importance for our calculations. So, since the connection variable is usually fairly constant throughout the dance, we can ignore it. Hence we only need to consider to the two spatial variables, whose domain is a 2-torus. (This is of course equivalent to considering the spatial version of the loop, while forgetting its color). The fundamental group of the 2-torus is the *group of the integers squared*, and its generators are the vertical circle shown in Figure 2, together with the circle around the equator (both traveled once around) . Hence the *fundamental group class* of a dancing couple is represented by an ordered pair (r,q), where r is the number of times the couple comes back to the original starting point after traveling completely along the circle of dance, and q represents the difference of the number of full counterclockwise turns minus the number of full clockwise turns the couple has made.

4. Lifts and Graphs. It is well known that the *universal cover* of the 3-torus is the Euclidean 3-space, and that the standard covering map from the 3-space to the 3-torus is given in terms of exponential functions. This covering map sends the unit segments on the x-axis, y-axis, and z-axis onto the standard generators of the fundamental group of the 3-torus respectively. To give a more precise model of a dancing couple we will then consider, instead than the loop on the 3-torus of Section 3, its *lift* (via the standard covering map) to the Euclidean 3-space, with starting point at (0, 0, 0). In this way, we can get a more faithful picture of the dance, as the traveled distance and the angle of twist are given *on the nose*, and not up to 360 degrees. If we use the spatial model, thus choosing to use color to represent the connection, we can *lift* our colored dancing couple's curve from the 2-torus to the plane in similar manner. Finally, one can *graph* this lifted colored plane curve in 3-space as a function of the time by introducing a time axis.

5. Several Couples Continuous Model (no Overtakes or Crashes). We are now going to model *several couples dancing together* by coalescing in a suitable manner the *graphs* of the couples' *spatial continuous models*. We further assume here that there are no overtakes or crashes among the couples (we will remove this assumption in the next section). If for example there are a total of n couples, say C1, …. Cn, on the dancing circle, let's assume that the i-th couple Ci starts and ends the dancing at the point in the dancing circle of angle (360) (i-1)/n, and that the leaders' angles of twist are 0 at the beginning and at the end of the dancing. If [0,T] is the time dance interval (with t=0 the beginning of the dance, and t=T the end), and the plane colored curve ci represents the dancing of the couple Ci, we have ci(0)=((360) (i-1)/n, 0), ci(T)=(R+(360) (i-1)/n, Qi), where R and Qi are integer multiples of (360), i=1,…,n. Note that R is assumed to be the same for all couples because there are no overtakes or crashes, and each couple returns to its original starting position at the end of the dance. The n colored curves ci, i=1,…,n. can now be *graphed in 3-space* with the introduction of a *time axis*, as at the end of Section 5. Represented in this latter way, the curves ci do not intersect each other because of the hypotheses we made. (But they could cross.) When projected down on the 2-torus via the exponential covering map, the curve ci becomes a loop ti based at the point (si, 1), where si is the ith n-root of unity (i=1,…,n.). The fundamental group class of ti is an element of the fundamental group of the 2-torus based at (si, 1) that equals (R/(360), qi), with qi= Qi/(360).

6. Several Couples Continuous Model and Braids. We are now going to modify the model detailed in Section 5 by allowing *crashes* and *overtakes*. We will thus assume that two or more couples can occupy the same position in space at the same time (say without the 3-space graphs of their colored curves ci intersecting each other; slightly modify chest angles to achieve this if needed). Moreover, we allow couples to finish in a different place than the one from where they originally started. In particular, we will assume that ci(0)=((360) (i-1)/n, 0), ci(T)=(Ri + (360) m(i)/n, Qi), where Ri and Qi are integer multiples of (360), and m(i) is a permutation of {0,…(n-1)}. The graphs of the colored curves ci, i=1,…,n, when rescaled and considered as a unity in 3-space, form a *braid.* Several interesting mathematical invariants have been associated to braids, see e.g. [1]. For example, one could join in a straightforward manner both ends of each of the colored curves ci (i=1,…,n), to produce an oriented link. Moreover, if we come back to our original idea of each couple being represented as a solid doughnut, see the beginning of Section 2, the dancing can now be mathematically described as n solid 2-tori flowing in space along the braid, while continuously transforming themselves in and out of di-holed solid tori. Of course one could also discretize these models in a straightforward manner by only considering music beats. Moreover, the continuous and spatial discrete models can also be generalized in a straightforward manner to the case of several couples.

References

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