Alhambra's Nazari Single Tile Patterns Guided Tour

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Abstract

In the School Year 2006-07 our Institution took part in a European Comenius Project called Partners in Patterns. Our work was related to the elaboration of patterns generated by plane movements of simple Escher-style tiles. We took as models the Alhambra's nazari mosaics that were part of our cultural heritage. For that purpose we used Dynamic Geometry Software (DGS) called Geogebra, which was well-accepted by our students. We planned as a first stage to study those tessellations that involve only one tile shape. After that we decided to represent in eight poster scenes the Alhambra's Nazari Single Tile Patterns Guided Tour in order to appreciate in a different way either a visit to the Alhambra's Palaces or the study of Geometry.

Introduction

One of the most current research lines in DM (Didactics of Mathematics) has to do with psychological and educational foundations. Proposals in the eighties by researchers such as Freudenthal (1981) or Nesher and Kilpatrick (1990), were followed and developed by Brousseau and his Theory of Didactic Situations (1990), Godino (1994) and others. Following these, we used art as a didactic tool for teaching Geometry, to prove that artistic contexts can improve the motivation of students toward mathematical tasks. In our case, it certainly did as we have proved (see [7]). In this paper, we would like to show how to look at Art in the context of a mathematical guided tour of the Alhambra's Palaces.

For our project, we used the Dynamic Geometric Software (DGS) Geogebra, a Markus Hohenwarter freeware developed by the University of Salzburg for mathematics teaching. The program can be downloaded at http://www.geogebra.org/cms/.

Secondary Mathematics Curricula and Aims

Our Secondary Mathematics Curricula includes Geometric Transformations, Movements and Isometries in the plane: Translations, Rotations, Reflections, Glide reflections and their compositions. Our aim is to understand the underlying geometry of plane movements as they are regarded in the Mathematics Curricula of Compulsory Education.

Regular Division of the Plane

A tessellation or tiling of the plane is a collection of plane figures that fill the plane with no overlaps and no gaps. There has been an enormous amount of research on tilings worldwide, and they are a source of many teaching materials in mathematics and art. In 1958, the Dutch artist M.C. Escher published a book called *Regular Division of the Plane*, in which he described the systematic buildup of mathematical patterns in his artworks; irregular shapes or combinations of shapes that interlock completely to cover a surface or plane (see [2]). He was greatly surprised by Islamic art after his first journey to the Alhambra in 1922, and further inspired to make his own tessellations in 1936 after another visit to the Alhambra. We now describe briefly some methods called Escher techniques to modify the sides of polygons to create tiles which, by applying geometric movements, will allow us to produce a variety of mosaics.

A) By translations. On a parallelogram or a hexagon, we modify (or cut) a piece of a side and translate the modified or cut pieces to the opposite side.

B) By 180° rotations. Modify (or cut) half of one side of a square, triangle, or hexagon, from one vertex to the midpoint of the side, then rotate this modified piece about the midpoint of the side.

C) By 60 º, 90 º or 120 º rotations. On one side of a polygon, modify (or cut) from one of its vertices to the other, then rotate the modified or cut piece about the first vertex so that it replaces the adjacent side of the polygon.

D) Mixed techniques.

There are other techniques (such as using glide reflections) that are a little more complex.

Art and Mathematics in La Alhambra's Single Tile Tessellations

In Figures 1 – 7 that follow, we illustrate how to obtain nazari tiles from polygons using techniques described in the previous section. Each nazari tile is obtained by cutting out shaded sections from the original polygon and moving them, usually by rotation, to a position outside the polygon, adjoining an edge of the polygon. By repeating the tile, we obtain a mosaic. The movements involved in the construction of a tile affect the placement of the pieces in the mosaic: when rotations are involved in the tile construction, adjacent mosaic pieces will fit through rotations.

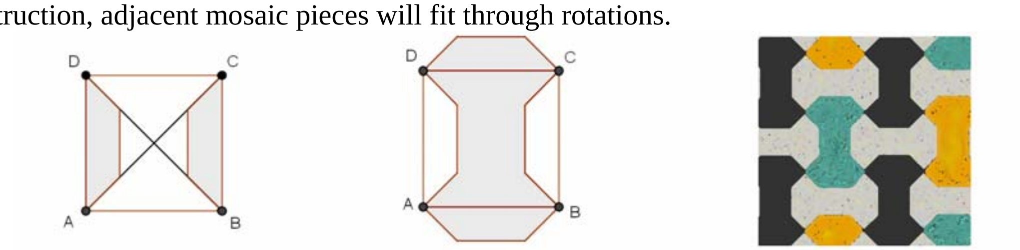

Figure 1: ***Bone tile.*** *90º rotations about B and C.*

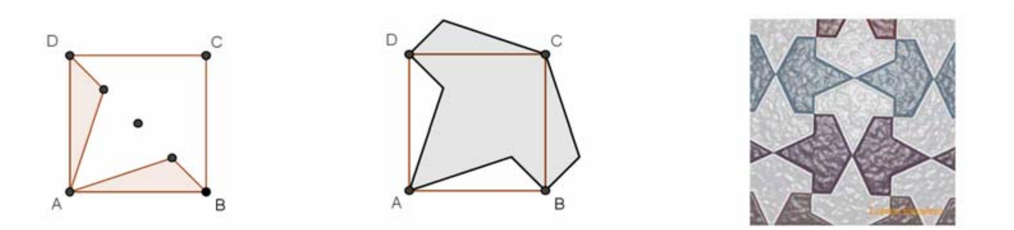

Figure 2: ***Gross Arrow tile***. *90º rotations about D; - 90º rotations about B.*

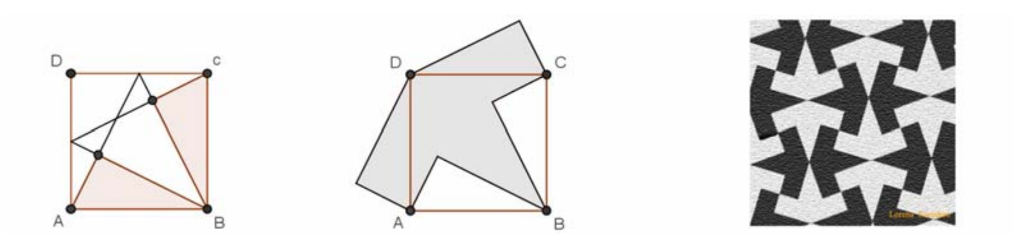

Figure 3: ***Thin Arrow tile***. *90º Rotations about A; - 90º rotations about C.*

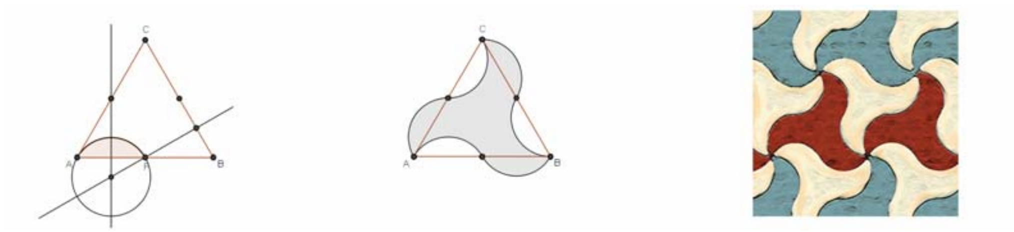

Figure 4: ***Pigeon tile***. *180° Rotation about triangle low side mid point F; repeat construction on side.*

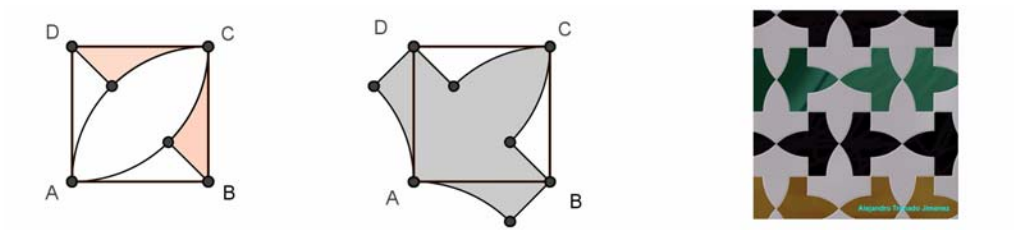

Figure 5: ***Airplane tile***. *90º Rotations about B; - 90º rotations about D.*

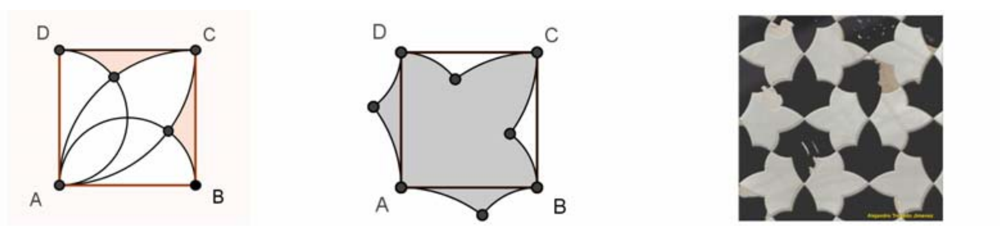

Figure 6: *Leaf tile*. *90º Rotations about B; - 90º rotations about D.*

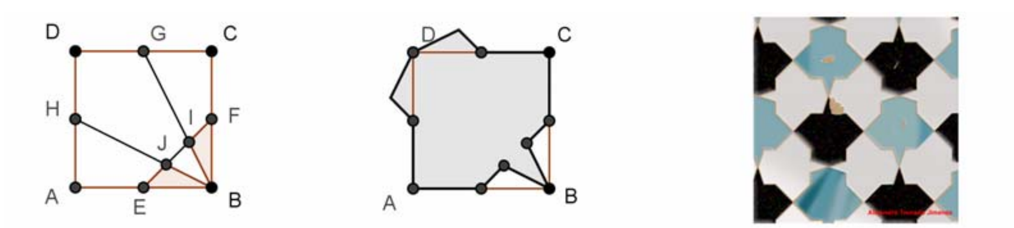

Figure 7: ***Fish tile***. *Glide-reflection of EJB to GD; glide-reflection of BLF to DH.*

The Alhambra's Palaces Guided Tour begins at the entrance in the Mexuar and finishes at the famous Lion's Courtyard Fountain. Each of these places is shown in 8 poster scenes, as we can see, for example, in figure 8. Each scene marks an important place whether there is any of the analyzed mosaics as not. Their dimensions are 50x70 cm and the file size over 100Mb and 300dpi, due to the images high quality. The students made the Geogebra tile models and the single tiles patterns (nazari mosaics reproductions) using Photoshop Then we mixed up all their work, including some original takes, in a Corel Draw file. The posters have been printed and they are hanging on in the School inner walls, acting as a didactical tool for Mathematics, Art and History teachers, available to anyone for educational purposes.

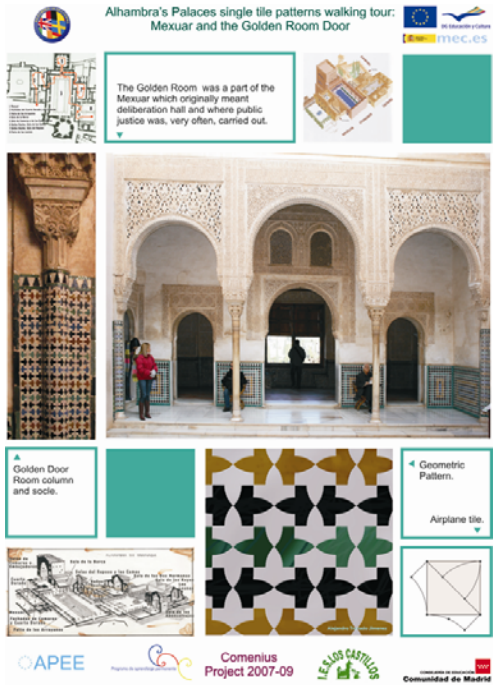

Figure 8: The Golden Room. *It was a part of Mexuar which originally meant deliberation room.*

Conclusion

Artistic contexts have proved to be a powerful tool for the development of mathematical work in Primary and Secondary Education in Spain, and have begun to be used as tools for the teaching of mathematics. Students' perception of artistic situations can motivate their work in mathematics. A dynamic geometry tool such as Geogebra is suitable for modelling and can also be a discovery tool. To use European Historical and Artistical Heritage as a tool for the teaching and learning of Mathematics must be an important goal in the European Dimension of Education perspective.

References

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