Aesthetic and Mathematical Research: A Comparison with two Examples

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Abstract

This paper examines the emerging field of aesthetic research and my experience in two research projects. I would like to outline an argument that suggests that the methods of mathematical and aesthetic research may be more closely related to one another than they are to many other areas of research. In this process I will discuss why it is that the fields are, in my mind, closely related. In particular I would like to underline that research communication is not external to research practice, but an integral part of the research process.

Introduction

Aesthetic research is a field that is gaining awareness over the past few years. Its boundaries remain fluid as it works out its place in the research landscape. I often find myself in a strange position of discussing mathematical research with non-mathematicians and seeing the problems of aesthetic development that are often left by the roadside because of a lack of suitable forums for the communication, discussion and development of aesthetic research. I believe that there are interesting similarities between the areas of mathematical and aesthetic research. I would like to claim that mathematical and aesthetic research have more in common that aesthetic and many other areas of research. This comes, in some sense, from the relative freedom of each area from commitments of realism or applicability. This paper looks at the areas of mathematical and aesthetic research and in particular at the communication of results within these areas. The communication of research is an integral part of these research practices.

The paper is structured as follows. First I outline my understanding of aesthetic research, then introduce two research projects that I have been involved in. Through a description of these research projects we see several similarities and distinctions. In particular I will concentrate upon the role of research communication. In this discussions I will be basing my ideas upon personal experiences, whilst also drawing upon the ideas of Corfield in [8] and the formalisms of aesthetic research within the field of practice as research [1].

Aesthetic Research

Over the past 15 years, I have been involved in many media arts projects which have had a large research component. While it has been accepted that there is some kind of research going on, it has been difficult to work out what sort of research it is. Research paradigms that have been used include engineering research [3], Human Computer Interaction (HCI) research [13], media history [9] or cultural studies [10]. However all of these paradigms miss important parts of what is happening in these creative / artistic projects and more importantly, ignore a large part of what is difficult about the research.

When an artist is working on a project, they spend a lot of time doing "something", then a piece of art emerges. One way of talking about this "something" is to equate it with a research process, where the results leads to products that we see as works of art. A huge amount of work is being done in the studios that is not getting out. This is what we refer to as *aesthetic research*: investigating the aesthetic properties of various systems and materials, processes and tools, which Carter refers to as *Material Thinking*[6].

I claim that mathematical and aesthetic research have a lot in common, perhaps more so that aesthetic research has with many other areas of research. Mathematical research deals with systems of mathematical structures, divorced from any reality in a physical world. Mathematical research defines from the outset its axioms and their structures of implication and causality. Similarly an aesthetic practitioner starts from a given and relatively freely chosen collection of materials, structures, forms and formats within which they choose to operate. An algebraist might choose an axiom system of semigroups and some restrictions such as finiteness or topological properties and investigates the properties of idempotent elements. A choreographer chooses to work on an empty stage using only live percussion and light from video projections of realtime streams from alpine weather cameras, and work with feelings of longing and homeliness with a group of homeless women and a class from the local primary school. Both must respect certain restrictions: there are no nontrivial idempotent anticommutative semigroups of prime order; gravity as well as bedtime for children cannot be ignored. But within these restrictions they are able to follow their own concepts of interest and relevance, unlike chemistry or engineering research where the physical world lays down the laws that are being found, or cultural studies where a given cultural mileu is investigated.

Moreover, mathematical and aesthetic researchers develop new knowledge not by performing experiments or investigating texts or data sets, but rather by creating works, whether musical inventions, specific examples of algebraic structures or a set of acting exercises, before developing larger works. This knowledge acquiring practice is the basis for "Practice as Research" [1] and forms a major paradigm of knowledge acquisition in aesthetic research. The best way to become a better artist is to make more art. Similarly one of the most repeated exhortations heard by young research mathematicians from their supervisors is "Examples!"

Algebraic Research Project

An *automaton* $A = (Q, \Sigma, \phi)$ is a finite set Q of states, a finite set Σ of input symbols and a transition function $\phi : Q \times \Sigma \to Q$. A series of input symbols $s_1, \ldots, s_n$, called a *word*, transfers the automaton in state q_0 through a series of states $q_i = \phi(q_{i-1}, s_i)$ to a final state q_n. A word is called a *reset word* if it transfers all states to a specific state. An example of an automaton with a reset word is C_n for some natural number n. $Q = \mathbb{Z}_n$, $\Sigma = \{a, b\}$ with $\phi(q, a) = q + 1$, $\phi(n - 1, b) = 0$ and $\phi(q, b) = q$ otherwise. The minimal length reset word for this automata is of length $(n - 1)^2$. The reset word is $b(a^{n-1}b)^{n-2}$. Černy conjectured that every automaton with a reset word has a reset word of length at most $(|Q| - 1)^2$[7].

The first approach was to look at my own and other examples. Looking at the collection of examples up to order 10 [18] one sees that other than C_n, there are only 8 examples that reach the bound for the length of a reset word. The special properties of these examples have been investigated in detail. A literature review followed, then examples constructed using special techniques. Separable automata, generalising linear automata and related classes were shown to have a tractable reset word length problem.

Next was the application of nearring theory, the nonlinear generalisation of ring theory [15], following up on ring theoretical ideas. Radicals in nearrings and their application to the problem led to a much simpler approach using the ideas of free algebras and their homomorphic images. This was a surprise as one expected the general nonlinearity to lead far away from the area of algebras over fields.

There are a number of levels with this work. There is an ongoing project to apply algebraic structures, notable rings and nearrings, to the field of automata theory [16, 17]. There are conjectures such as that of Černy or the recently solved Road Colouring Problem [19]. Such conjectures capture the imaginations of a large number of people due to their simplicity, elegance, obviousness, clarity and intractability and bring practitioners together to work on and develop new techniques or even whole fields of work. The daily grind at the coal face is more mundane. Working examples, developing, proving or refuting conjectures, identifying relevant allied areas of work; these things are vital work at a direct level.

On the day to day level, communication with colleagues is often along the lines of asking about hazily remembered results or about looking for related structures in similar fields. One of the most vital processes, however, is the regular process of stepping away from the work at hand, summarising it in a written or diagrammatic form and then presenting this in a somewhat formalised form to colleagues. Many of these written and presented summaries are dead-ends, however the process of making these ongoing thought processes available to interested outsiders is vital in the process of clarifying one's own thought processes.

Aesthetic Research Project

The project was not formulated as aesthetic research, however the practices and processes of the project make it clear that is definitely falls within the realm of aesthetic research. It investigates the claims that the concept of space as we know and perceive it is not a given as it is formulated in Newtonian and Einsteinian physics. Rather space is an emergent property of a remarkably simple but tightly interwoven and complex process. This theory is known as Process Physics[5]. At this stage I had been involved in projects that conveyed systems of relations by translating them into a physical space, such as the network audio pieces *Transient Loop*[21]. The basis of the research project was that there should be a direct translation of the process physics ideas into a representation that demonstrated the emergence of three dimensional space from this process, rather than having to resort to a complex mathematical argument [14]. As such the success of the project was not a measurable thing, but rather had to do with the subjective perception of something that "felt like" three dimensional space. The process of investigating the interplay between abstract systems and perceived three dimensionality is most important: it is an aesthetic, subjective criteria.

The research process began with a literature review and discussing the ideas and the way that they were understood by physics practitioners [4]. These ideas are closely allied with aspects of physical intuition tempered by the 20th century cosmology and quantum physics.

I then began to investigate the possibilities for visual and acoustic representations of the processes. The mathematical tools used here are non trivial [11], the representations that I was interested in were screen visualisations where the three dimensionality of the emergent systems should become apparent. After a period I began to doubt the validity of the premise. Not able to understand the complex mathematics used by Nagels, nor able to rely upon some physical intuition, I began to suspect that either there was no interesting way of doing this visualisation or, worse still, that the emerging data sets did not have the right structures for three dimensional display or even that the results did not support the thesis of emergent three dimensional space.

Based upon some discussions with Klinger, I attempted an analytic rather than aesthetic analysis of the representations of the emergent structures, to investigate some statistical properties of the distribution of points. In a structure with a relation of closeness, one can define a relation of distance. One can talk about the distance in that structure; the lines of least distance are called *geodesics*. In a flat plane, these are straight line segments, on the surface of a sphere, these are the great circles. One of the intuitions of conformal topology[12] is that the collection of geodesics of a structure defines the shape of that structure, that two structures with the same collection of geodesics are equivalent. Nagels argues that a random sparse graph has a geodesic distribution the same as the distribution of geodesics of a 3-sphere. Three dimensional space emerges as a relational structure of a random sparse graph, a "bucket of dust."

To investigate this, I took examples of the relational structures produced by the models and embedded them in Euclidean space. The contention was that the emergent space should embed approximately as a three dimensional sphere in four dimensional Euclidean space, which could be observed [2].

The core issue that arises here is that the project itself centred around the aesthetic problem of appropriate representations of formal data sets. Outside the foundational works of Tufte [20] I found it difficult to find any place where practitioners spoke of their experiences with such representational problems and techniques they had developed to produce aesthetically relevant representations. The chasm between a formal representation based upon objective criteria and a fanciful 'artists impression' that communicated precisely what the artist intends but not what the data actually says, is one pivotal problem here.

Comparing Research Projects

We see in both these examples the existence of a relatively high level idea, the Černy Conjecture and the emergent space idea, closely coupled with an explorative research process. In both cases the day to day work was the creation of examples, searching for relevant literature, trying the possibilities of the available tools and systems. One aspect that was missing significantly in the Aesthetic Research project was a network of colleagues with whom I could exchange ideas, discuss problems and present work in a way that was relevant to the approach I was taking.

The Communication of Research

It can be claimed that when a practitioner solves a problem, they are undertaking research. However in much the same way that someone who only sings in the shower or sketches in their personal diary, we do not consider their work art until they choose to exhibit it. So as a first restriction it might be useful to call an investigating activity research when it is communicated. If a conjecture is solved but no-one reads the proof, was it proved?

The communication of aesthetic research is an *intersubjective discourse* with an interested audience about these results. It is not objective results about measurable phenomena that are reported, it is a subjective, aesthetic appreciation of the results and the way that the various processes made those results that is communicated to an audience who will also only be able to appreciate them on a subjective level.

The result of such an intersubjective discourse should be a type of "Aha" moment for the recipient where a comprehension of the ideas as a whole emerges. Highly subjective commentary along the lines of "I want to try that too" or "interesting!" are the results of successful intersubjective communication.

A common perception of mathematical research communication is a blackboard filling series of theorems and proofs, interspersed with technical diagrams. This model, a merging of badly written mathematical papers, high school teaching disasters and Hollywood stereotypes, in true only in sad single cases. Such technical communications may occur in discussions with colleagues or some departmental seminars. Larger mathematical forums have other formats.

Visiting the NSAC09 conference, I took an external view of various lectures. Very few speakers presented proofs. What an audience is interested in is not the proof as such. What is important is to understand the ideas and motivations of the speaker, to comprehend what guided the development of the results, how the results exceeded or avoided expectations, some mathematical intuition. The recipient wanted a subjective moment which would motivate them to read the papers.

The process of communicating mathematical results leads to a different view on the results: by anticipating questions, comments and possible extensions, one becomes more aware of the details oneself and often extends the work in the process of (preparing for) communicating it.

Conclusion

In this essay I have attempted to draw out some of the connections between mathematical and what is emerging as aesthetic research. I contend that the two have a lot more in common than practitioners in either camp might be aware of. Aesthetic research is a relatively young research area and is still finding its feet. The area is vitally important and offers a forum for practitioners who wish to move beyond a level of proficiency and productivity into a realm of reflection and communication. This aspect of communication, a intersubjective discourse around an area of common interest, is most vital here.

Most importantly I contend that this is precisely what mathematicians do when communicating. While many mathematicians communicate well within the structures of mathematical correctness and these communications, especially when written, are of importance, the communication around this core of mathematical truth is often of greater mathematical importance. The motivations, dreams, directions and descriptions of beauty, surprise and connectedness make mathematical research communication interesting and vital.

There are two things that might be learnt directly from the process of mathematical research: the two exhortations that are commonly made by the supervisors of young academic mathematicians: 'more examples' and 'write things down.' Contrary to the claims of many aesthetic practitioners, breaking the process of creation is a vital part of successful creativity. The process of creativity is often one that is a form of dialogue, as argued by the constructivist Glanville, within a person or a team, with a constant evolutionary iteration of creation and directing, breeding and culling. The most important part is the duality of being a reader and a writer, a speaker and a listener, even when listening to oneself.

This paper has been inspired by my own experience within these twin realms of mathematical and aesthetic practice. Too often have I seen mathematicians creating work that, once I *get* what they are aiming at, would be of much wider interest if only they were able to communicate their enthusiasm and motivations. Too often I have seen aesthetic practitioners develop bodies of work with ensuing collections of intuitions, technologies and guides for related practitioners that they wish to share but find no place within which this sharing can take place. And too often research is blocked until the attempt is made at communicating the blockage leads to its dissolution. I hope that a field of aesthetics research will continue to emerge and find places that enable and foster these communications. I believe that the field has a lot to learn from mathematics research and communication.

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